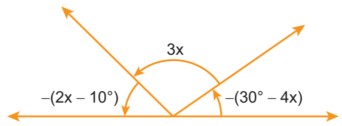
Solucionario Trigonometría 5.°

Unidad 1 SISTEMAS DE MEDICIÓN ANGULAR

APLICAMOS LO APRENDIDO (página 6) Unidad 1

1. Colocamos los ángulos en sentido antihorario, y tenemos:

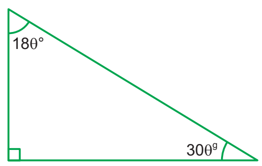


Del gráfico:

$$-(2x - 10°) + (3x) - (30° - 4x) = 180°$$
$$-2x + 10° + 3x - 30° + 4x = 180°$$
$$5x - 20° = 180°$$
$$5x = 200°$$
$$\therefore\ x = 40°$$

Clave D

2.



Del gráfico, se cumple:

$$18\theta° + 30\theta^g = 90°$$
$$18\theta° + 30\theta^g\left(\frac{9°}{10^g}\right) = 90°$$
$$18\theta + 27\theta = 90$$
$$45\theta = 90$$
$$\therefore\ \theta = 2$$

Clave B

3. Del enunciado:

$$30x + \frac{\pi}{2}\text{ rad} = 3\left(90° - \frac{\pi}{6}\text{ rad}\right)$$

Luego:

$$30x = 270° - \frac{3\pi}{6}\text{ rad} - \frac{\pi}{2}\text{ rad}$$
$$30x = 270° - \pi\text{ rad}$$
$$30x = 270° - \pi\text{ rad}\left(\frac{180°}{\pi\text{ rad}}\right)$$
$$30x = 270° - 180°$$
$$30x = 90°$$
$$\therefore\ x = 3°$$

Clave E

4.

$$\frac{810\,000''}{\pi} = \frac{810\,000''}{\pi}\left(\frac{1°}{3600''}\right)$$
$$\frac{810\,000''}{\pi} = \frac{225°}{\pi}\left(\frac{\pi\text{ rad}}{180°}\right) = \frac{5}{4}\text{ rad}$$
$$\therefore\ \frac{810\,000''}{\pi} = \frac{5}{4}\text{ rad}$$

Clave D

5.

$$\frac{R+3}{C+S} = \frac{C+S}{C^2 - S^2}$$
$$\frac{R+3}{C+S} = \frac{C+S}{(C+S)(C-S)}$$
$$\Rightarrow R + 3 = \frac{C+S}{C-S} \qquad \ldots(1)$$

Se sabe: $\frac{S}{180} = \frac{C}{200} = \frac{R}{\pi} = k$

$\Rightarrow S = 180k,\ C = 200k\ \text{ y }\ R = \pi k$

Reemplazando en (1):

$$\pi k + 3 = \frac{200k + 180k}{200k - 180k}$$
$$\pi k + 3 = 19 \Rightarrow \pi k = 16 \Rightarrow R = 16$$

Por lo tanto, el ángulo mide 16 rad.

Clave B

6. Por dato: los ángulos $(3x)°$ y $\left(\frac{20x}{3}\right)^g$ son complementarios.

Entonces: $(3x)° + \left(\frac{20x}{3}\right)^g = 90°$

$$(3x)° + \left(\frac{20x}{3}\right)^g\left(\frac{9°}{10^g}\right) = 90°$$
$$3x + 6x = 90$$
$$9x = 90$$
$$\therefore\ x = 10$$

Clave B

7. Sean los ángulos: α y β $(\alpha > \beta)$

Por dato:

$$\left.\begin{aligned}\alpha + \beta &= \frac{7\pi}{20}\text{ rad}\\ \alpha - \beta &= 30^g\end{aligned}\right\}(+)$$

$$2\alpha = \frac{7\pi}{20}\text{ rad}\left(\frac{200^g}{\pi\text{ rad}}\right) + 30^g$$
$$2\alpha = 70^g + 30^g$$
$$2\alpha = 100^g \Rightarrow \alpha = 50^g$$

Como:

$$\alpha - \beta = 30^g$$
$$(50^g) - \beta = 30^g$$
$$\Rightarrow\ \beta = 20^g$$

Por lo tanto, el menor ángulo mide 20^g.

Clave A

8.

$$50^m = 50^m\left(\frac{1^g}{100^m}\right)$$
$$50^m = 0{,}5^g\left(\frac{9°}{10^g}\right) = 0{,}45°$$
$$50^m = 0{,}45°\left(\frac{3600''}{1°}\right) = 1620''$$
$$\therefore\ 50^m = 1620''$$

Clave D

9.

$$54° = 54°\left(\frac{\pi\text{ rad}}{180°}\right) = \frac{3\pi}{10}\text{ rad}$$
$$54^g = 54^g\left(\frac{\pi\text{ rad}}{200^g}\right) = \frac{27\pi}{100}\text{ rad}$$

El error cometido será:

$$\frac{3\pi}{10}\text{ rad} - \frac{27\pi}{100}\text{ rad} = \frac{3\pi}{100}\text{ rad}$$

Clave A

10. $S = x^2 - 3x - 10 \qquad \ldots(1)$

$C = x^2 - 2x - 4 \qquad \ldots(2)$

Dividiendo (1) y (2):

$$\frac{S}{C} = \frac{x^2 - 3x - 10}{x^2 - 2x - 4}$$
$$\frac{9}{10} = \frac{x^2 - 3x - 10}{x^2 - 2x - 4}$$
$$\Rightarrow 9x^2 - 18x - 36 = 10x^2 - 30x - 100$$
$$x^2 - 12x - 64 = 0$$
$$(x - 16)(x + 4) = 0 \Rightarrow x = 16 \vee x = -4$$
$$\Rightarrow x = 16$$

Reemplazando en (1):

$S = (16)^2 - 3(16) - 10$

$S = 198$

Entonces el ángulo mide 198°

$$\Rightarrow 198°\left(\frac{\pi\text{ rad}}{180°}\right) = \frac{11\pi}{10}\text{ rad}$$

Por lo tanto, el ángulo mide $\frac{11\pi}{10}$ rad.

Clave C

11. Sean S, C, R los números que expresan la medida de un ángulo en los 3 sistemas.

De la fórmula de conversión:

$$\frac{S}{180} = \frac{C}{200} = \frac{R}{\pi} = k$$

Del enunciado:

$SCR = \frac{\pi}{6}$, reemplazando valores

$$(180k)(200k)(k\pi) = \frac{\pi}{6}$$
$$k^3 = \frac{1}{(180)(200)(6)}$$
$$k^3 = \frac{1}{3^3.2^3.10^3}$$
$$k = \frac{1}{3.\,2.\,10}$$

Luego: $S = 180k = 180\left(\frac{1}{60}\right) = 3$

$\therefore$ El ángulo es 3°.

Clave C

12. De la relación:

$$(179x + 185)° = (1 + x)\pi\text{ rad}$$
$$(179x + 185)° = (1 + x)\pi\text{ rad}\underbrace{\left(\frac{180°}{\pi\text{ rad}}\right)}_{\text{factor conversión}}$$
$$(179x + 185)° = (180 + 180x)°$$
$$179x + 185 = 180 + 180x$$
$$x = 5$$

Reemplazando:

$$\alpha = (1 + 5)\pi\text{ rad}$$
$$\alpha = 6\pi\text{ rad}\underbrace{\left(\frac{200^g}{\pi\text{ rad}}\right)}_{\text{factor conversión}}$$
$$\therefore\ \alpha = 1200^g$$

Clave C

13. Sea el error E:

$$E = 315° - 315^g$$
$$E = 315° - 315^g\left(\frac{9°}{10^g}\right)$$
$$E = \frac{315°}{10}$$
$$E = 31{,}5°$$

En el sistema radial

$$E = 31{,}5° = 31{,}5°\left(\frac{\pi\text{rad}}{180°}\right)$$
$$\therefore\ E = \frac{7\pi}{40}\text{ rad}$$

Clave D

14. Se tiene:

$175° = 175°\left(\frac{\pi}{180°}\right)$ rad $= \frac{175\pi}{180}$ rad

$175° = \frac{35\pi}{36}$ rad $\Rightarrow$ n $= \frac{35\pi}{36}$ rad

Reemplazando en M:

$M = \frac{1}{\pi}[36n - 30\pi] = \frac{1}{\pi}\left[36\left(\frac{35\pi}{36}\right) - 30\pi\right]$

$M = \frac{1}{\pi}[35\pi - 30\pi] = \frac{5\pi}{\pi}$

$M = 5$

Luego:

$M° = 5° = 5° \times \frac{\pi}{180°}$ rad $= \frac{\pi}{36}$ rad

$\therefore$ El número de radianes de M° es $\frac{\pi}{36}$.

Clave B

PRACTIQUEMOS
Nivel 1 (página 8) Unidad 1

Comunicación matemática

1. Se sabe:

$9° = 10^g \Rightarrow 1° = 1,\widehat{1}^g > 1^g$
$\Rightarrow 1° > 1^g$ ...(α)

Además:

π rad $= 180° \Rightarrow 1$ rad $= \frac{180°}{\pi}$

$\Rightarrow 1$ rad $\approx 57°19'30'' > 1°$
$\Rightarrow 1$ rad $> 1°$ ...(β)

De (α) y (β):
$\Rightarrow 1$ rad $> 1° > 1^g$
$\Rightarrow 2$ rad $> 2° > 2^g$

Por lo tanto, la relación correcta es la C.

Clave C

2. Del gráfico se observa que:

$y - \angle AOB$; tiene un número entero de vueltas el cual según gráfico es 3, luego:

$y - \angle AOB = 3(m\angle 1$ vuelta$)$
$y = 3(360°) + \angle AOB$ ... (1)

Del gráfico: $\angle BOA = -(-60)° = 60°$
$\angle AOB + \angle BOA = 90°$
$\angle AOB = 90° - \angle BOA$
$\angle AOB = 90° - 60°$
$\angle AOB = 30°$ ... (2)

Reemplazamos (2) en (1):
$y = 1080° + 30°$
$\therefore \; y = 1110°$

Clave B

Razonamiento y demostración

3. Por dato:

$x = \frac{4°}{30} \;\wedge\; y = \frac{2^g}{36}$

Luego:

$\frac{x}{y} = \frac{4°}{2^g}\left(\frac{36}{30}\right)\left(\frac{10^g}{9°}\right) = \frac{8}{3}$

$\Rightarrow x = 8k \;\wedge\; y = 3k$

Piden:

$M = \frac{3x + 4y}{5x - 4y} = \frac{3(8k) + 4(3k)}{5(8x) - 4(3k)} = \frac{9}{7}$

$\therefore \; M = \frac{9}{7}$

Clave E

4. $36° = 36°\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{\pi}{5}$ rad

$36^g = 36^g\left(\frac{\pi \text{ rad}}{200^g}\right) = \frac{9\pi}{50}$ rad

El error cometido será:

$\frac{\pi}{5}$ rad $- \frac{9\pi}{50}$ rad $= \frac{\pi}{50}$ rad

Clave C

5. Sabemos:

$\frac{S}{180} = \frac{C}{200} = \frac{R}{\pi} = k$

$\Rightarrow S = 180k$, $C = 200k$ y $R = \pi k$

Piden:

$\frac{\pi C + \pi S + 20R}{200R} = \frac{\pi(200k) + \pi(180k) + 20(\pi k)}{200(\pi k)}$

$\therefore \; \frac{\pi C + \pi S + 20R}{200R} = \frac{400\pi k}{200\pi k} = 2$

Clave B

6. Por dato: $SCR = \frac{\pi}{162}$

Sabemos: $\frac{S}{180} = \frac{C}{200} = \frac{R}{\pi}$

$\Rightarrow S = \frac{180R}{\pi} \;\wedge\; C = \frac{200R}{\pi}$

Reemplazando en el dato se tiene:

$\left(\frac{180R}{\pi}\right)\left(\frac{200R}{\pi}\right)R = \frac{\pi}{162}$

$R^3 = \frac{\pi^3}{180^3}$

$R = \frac{\pi}{180}$

Por lo tanto, la medida del ángulo es $\frac{\pi}{180}$ rad.

Clave A

7. Por dato: $C^2 - S^2 = 76$

Sabemos: $C = 10k \;\wedge\; S = 9k$

$\Rightarrow (10k)^2 - (9k)^2 = 76$
$19k^2 = 76$
$k^2 = 4$
$\Rightarrow k = 2$

Luego:
$S = 9k = 9(2) = 18 \Rightarrow S = 18$

En radianes:

$18°\left(\frac{\pi}{180°}\right) = \frac{\pi}{10}$ rad

Por lo tanto, la medida del ángulo es $\frac{\pi}{10}$ rad.

Clave B

8. Se sabe: $S = 9k \wedge C = 10k$

Por dato:

$\left(\frac{S}{9} - 1\right)\left(\frac{C}{10} + 1\right) = 15$

$\left(\frac{9k}{9} - 1\right)\left(\frac{10k}{10} + 1\right) = 15$

$(k - 1)(k + 1) = 15$
$k^2 - 1 = 15$
$k^2 = 16 \Rightarrow k = 4$

Luego:
$S = 9k = 9(4) = 36$

Entonces, el ángulo en radianes mide:

$36°\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{\pi}{5}$ rad

Por lo tanto, la medida del ángulo es $\frac{\pi}{5}$ rad.

Clave C

9. $30,5^g = (30,5)^g\left(\frac{9°}{10^g}\right)$

$30,5^g = 27,45°$
$= 27° + 0,45°$
$= 27° + 0,45(60')$
$= 27° + 27'$
$\therefore \; 30,5^g = 27°\,27'$

Clave C

Resolución de problemas

10. Por dato:

60S $\Rightarrow$ n.° de minutos sexagesimales
100C $\Rightarrow$ n.° de minutos centesimales

Entonces:
$60S + 100C = 1540$

$\frac{S}{9} = \frac{C}{10} \Rightarrow S = \frac{9C}{10}$, luego

$60\left(\frac{9C}{10}\right) + 100C = 1540$

$54C + 100C = 1540$
$154C = 1540$
$C = 10$

Entonces:

$\alpha = 10^g = 10^g \times \frac{\pi \text{ rad}}{200^g}$

$\therefore \; \alpha = \frac{\pi}{20}$ rad

Clave C

11. Sea el ángulo: α

Por dato:

$\alpha = 130^g \wedge 180° - \alpha = (8n - 1)°$

$\Rightarrow \alpha = 130^g\left(\frac{9°}{10^g}\right) = 117° \Rightarrow \alpha = 117°$

Luego:
$180° - 117° = (8n - 1)°$
$63° = (8n - 1)°$
$64 = 8n$
$\Rightarrow n = 8$

Piden: n^g en radianes

$$n^g = 8^g\left(\frac{\pi \text{ rad}}{200^g}\right) = \frac{\pi}{25} \text{ rad}$$

$$\therefore\ n^g = 8^g = \frac{\pi}{25} \text{ rad}$$

Clave E

Nivel 2 (página 8) Unidad 1

Comunicación matemática

12. De los datos:

$\alpha = 786{,}75' = 786' + 0{,}75'$

$\alpha = 786' + 0{,}75 \times 60''$

$\alpha = \underbrace{786'} + 45''$

$\alpha = (13 \times 60 + 6)' + 45''$

$\alpha = \underbrace{13 \times 60'} + 6' + 45''$

$\alpha = 13° + 6' + 45''$

$\alpha = 13°\,6'\,45'' = a°\,b'\,c''$

$\therefore\ a = 13;\ b = 6;\ c = 45$

Análogamente

$\beta = 4217{,}09^m$

$\beta = 4217^m + 0{,}09^m$

$\beta = 4217^m + 0{,}09 \times 100^s$

$\beta = \underbrace{4217^m} + 9^s$

$\beta = \underbrace{42 \times 100^m} + 17^m + 9^s$

$\beta = 42^g + 17^m + 9^s$

$\beta = 42^g\,17^m\,9^s = x^g\,y^m\,z^s$

$\therefore\ x = 42;\ y = 17;\ z = 9$

De las expresiones:

I. $a = 13;\ b = 6 \Rightarrow$ a y b no son equivalentes. (F)

II. $\frac{b}{z} = \frac{6}{9} = \frac{2}{3} \Rightarrow$ b y z están en razón de 2 a 3. (V)

III. $c = 45;\ z = 9 \Rightarrow$ z es menor que c. (F)

Clave E

13. Del gráfico se obtiene:

$x° + 90° + \alpha° + (-\beta°) + 90° = 360°$

$x° + \alpha° + 180° - \beta° = 360°$

$x° - \beta° + \alpha° = 180°$

$x = 180° + \beta° - \alpha°$

Luego; el suplemento de x será:

$S(x) = 180° - x$

$S(x) = 180° - (180° + \beta° - \alpha°)$

$\therefore\ S(x) = \alpha° - \beta°$

Clave B

Razonamiento y demostración

14. $40° = \overline{aa}^g\,\overline{aa}^m\,\overline{aa}^s$ ...(1)

$$\Rightarrow 40°\left(\frac{10^g}{9°}\right) = \frac{400^g}{9} = 44^g + \frac{4^g}{9}\left(\frac{100^m}{1^g}\right)$$

$$\Rightarrow 40° = 44^g + 44^m + \underbrace{\frac{4^m}{9}\left(\frac{100^g}{1^m}\right)}_{\approx 44^s}$$

$\Rightarrow 40° = 44^g\,44^m\,44^s$ ...(2)

Comparando (1) y (2):

$\Rightarrow a = 4$

$\therefore\ 2a = 8$

Clave E

15. Del dato:

$$\underbrace{(C+S)+(C+S)+(C+S)+...+(C+S)}_{\text{n términos}} = 3800\frac{R}{\pi}$$

Luego:

$n(C + S) = \frac{3800R}{\pi}$

De la fórmula general:

$\frac{S}{180} = \frac{C}{200} = \frac{R}{\pi}$

$\Rightarrow S = \frac{180R}{\pi}$ y $C = \frac{200R}{\pi}$

Luego:

$n(C + S) = \frac{3800R}{\pi}$

$n\left(\frac{200R}{\pi} + \frac{180R}{\pi}\right) = \frac{3800R}{\pi}$

$n\left(\frac{380R}{\pi}\right) = \frac{3800R}{\pi}$

$n = 10$

Finalmente:

$2n° = 20°\left(\frac{\pi}{180°}\right)$ rad

$\therefore\ 2n° = \frac{\pi}{9}$ rad.

Clave A

16. Por dato:

$160^A = \frac{1}{3}$ (m∠1 vuelta)

$160^A = \frac{1}{3}(360°) = 120°$

$160^A = 120°$

$4^A = 3°$

$\left(\frac{4}{3}\right)^A = 1°$ ... (1)

Además:

$27^B = 90°$

$3^B = 10°$ ... (2)

(1) en (2):

$3^B = 10(1°) = 10\left(\frac{4}{3}\right)^A$

$3^B = \left(\frac{40}{3}\right)^A$

$1^A = \left(\frac{9}{40}\right)^B$ ... (3)

Piden:

$120^A = 120 \times 1^A$

De (3):

$120^A = 120 \times \left(\frac{9}{40}\right)^B$

$\therefore\ 120^A = 27^B$

Clave C

Resolución de problemas

17. Sean: α, β y θ dichos ángulos.

$\alpha + \beta = 20°$ ...(1)

$\beta + \theta = 40^g\left(\frac{9°}{10^g}\right) = 36°$ ...(2)

$\alpha + \theta = \frac{5\pi}{9}\text{rad}\left(\frac{180°}{\pi \text{ rad}}\right) = 100°$ ...(3)

Sumando (1), (2) y (3):

$2(\alpha + \beta + \theta) = 156°$

$\alpha + \beta + \theta = 78°$ ...(4)

De (1) y (4):

$(20°) + \theta = 78° \Rightarrow \theta = 58°$

De (2) y (4):

$\alpha + (36)° = 78° \Rightarrow \alpha = 42°$

De (3) y (4):

$\beta + 100° = 78° \Rightarrow \quad \beta = -22°$

El mayor es: $58°\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{29\pi}{90}$ rad

Clave D

18. Sabemos que S representa el número de grados sexagesimales y C el número se grados centesimales.

Entonces, se cumple:

3600 S: representa el n.° de segundos sexagesimales.

100 C: representa el n.° de minutos centesimales.

Por dato:

$(3600S) - 3(100C) = 29\,400$

$12S - C = 98$

$12\left(\frac{180R}{\pi}\right) - \left(\frac{200R}{\pi}\right) = 98$

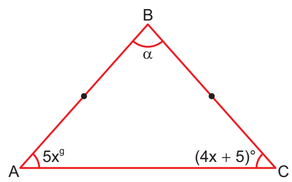
$\frac{1960R}{\pi} = 98$

$\Rightarrow R = \frac{\pi}{20}$

Por lo tanto, el ángulo mide $\frac{\pi}{20}$ rad.

Clave A

19.



Del gráfico: $5x^g = (4x+5)°$

$$5x^g\left(\frac{9°}{10^g}\right) = (4x+5)°$$

$$\frac{9x}{2} = 4x + 5$$

$$\frac{x}{2} = 5$$

$$\Rightarrow x = 10$$

Entonces: $m\angle C = (4 \times 10 + 5)° = 45°$

$$\Rightarrow m\angle A = 45°$$

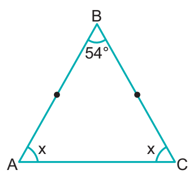
Luego: $45° + \alpha + 45° = 180°$

$$\alpha = 90°\left(\frac{\pi\ \text{rad}}{180°}\right)$$

$$\therefore \alpha = \frac{\pi}{2}\ \text{rad}$$

Clave D

20.



En el triángulo ABC, se cumple:

$x + 54° + x = 180°$

$2x = 126°$

$x = 63°\left(\frac{10^g}{9°}\right) = 70^g$

$\therefore x = 70^g$

Clave C

21. $\frac{3\pi}{13}$ rad $= \overline{4a}°\,\overline{3b}'\,\overline{1c}''$ ...(1)

$$\Rightarrow \frac{3\pi}{13}\ \text{rad}\left(\frac{180°}{\pi\ \text{rad}}\right) = 41° + \frac{7°}{13}\left(\frac{60'}{1°}\right)$$

$$\Rightarrow \frac{3\pi}{13}\ \text{rad} = 41° + 32' + \frac{4'}{13}\left(\frac{60''}{1'}\right)$$

$$\frac{3\pi}{13}\ \text{rad} = 41°32'18'' \quad ...(2)$$

Comparando (1) y (2):

$\Rightarrow a = 1,\ b = 2$ y $c = 8$

Piden:

$J = (a + b)c = (1 + 2)8$

$\therefore J = 24$

Clave D

22. Datos

$x \to$ n.° de minutos centesimales

$y \to$ n.° de minutos sexagesimales

Sean S, C lo convencional para un ángulo α:

$x = 100C$

$y = 60S$

Además:

$x - y = 368 \Rightarrow 100C - 60S = 368$

También:

$$\frac{C}{10} = \frac{S}{9} \Rightarrow S = \frac{9}{10}C$$

Luego:

$$100C - 60S = 100C - 60\left(\frac{9}{10}C\right) = 368$$

$$100C - 54C = 368$$

$$46C = 368$$

$$C = 16$$

Luego, el ángulo será:

$\alpha = 16^g$

$$\alpha = 16^g \times \frac{\pi\ \text{rad}}{200^g}$$

$$\therefore \alpha = \frac{2\pi}{25}\ \text{rad}$$

La medida del ángulo es $\frac{2\pi}{25}$ rad.

Clave B

Nivel 3 (página 9) Unidad 1

Comunicación matemática

23. De la fórmula general de conversión:

$$\frac{S}{180} = \frac{C}{200} = \frac{R}{\pi} \Rightarrow \frac{S}{9} = \frac{C}{10} = \frac{20R}{\pi}$$

Se tiene:

A) $\frac{S}{9} = \frac{C}{10} \Rightarrow \frac{S}{9} - 1 = \frac{C}{10} - 1$

$$\frac{S-9}{9} = \frac{C-10}{10}$$

$\therefore$ A es correcta.

B) Se tiene:

$$\frac{S}{9} = \frac{C}{10} = \frac{20R}{\pi} = k_1$$

$$\frac{S+C}{9+10} = \frac{9(k_1) + 10(k_1)}{9+10} = k_1$$

$\therefore$ B es correcta.

C) $\frac{S}{180} = \frac{C}{200} = \frac{R}{\pi} = k_2$

$$\frac{C-R}{200-\pi} = \frac{200k_2 - \pi k_2}{200-\pi} = k_2 \quad ...(1)$$

$$\frac{C-S}{200-180} = \frac{200k_2 - 180k_2}{200-180} = k_2 \quad ...(2)$$

De (1) y (2):

$$\frac{C-S}{200-\pi} = \frac{C-S}{20}$$

$\therefore$ C es correcta.

D) Sea:

$$\frac{S}{9} = \frac{C}{10} = \frac{20R}{\pi} = k_1$$

$$\left(\frac{S}{9}\right)^2 = k_1^2;\ \frac{C}{10}\left(\frac{20R}{\pi}\right) = k_1^2$$

$$\Rightarrow \frac{S^2}{81} = \frac{2CR}{\pi}$$

$\therefore$ D es correcta.

E) De: $\frac{S}{9} = \frac{C}{10} = \frac{20R}{\pi} = k_1$

$$\frac{S}{9}\left(\frac{C}{10}\right) = k_1^2;\ \left(\frac{20R}{\pi}\right)^2 = k_1^2$$

$$\frac{SC}{40} = \frac{400R^2}{\pi^2}$$

$\therefore$ E es incorrecta.

Clave E

24. Sean V, S, C, R los números que representan las medidas de un ángulo en los sistemas de medición angular:

Del dato:

$$\frac{V}{S} = \frac{7}{6} \Rightarrow \frac{V}{7} = \frac{S}{6}$$

$$\frac{V}{7 \times 30} = \frac{S}{6 \times 30} \Rightarrow \frac{V}{210} = \frac{S}{180} \quad ...(1)$$

En la fórmula general de conversión:

$$\frac{S}{180} = \frac{C}{200} = \frac{R}{\pi} = \frac{V}{210} \quad ...(2)$$

A) Relación entre R y V

De (2)

$$\frac{R}{\pi} = \frac{V}{210} \quad \therefore V = \frac{210R}{\pi}$$

A es correcta.

B) Relación entre V y C

De (2)

$$\frac{C}{200} = \frac{V}{210} \quad \therefore C = \frac{20V}{21}$$

B es correcta.

C) De (1), siendo $S = 360$ para el ángulo de 1 vuelta

$$\frac{V}{7} = \frac{360}{6} \Rightarrow V = 420$$

$\therefore m\angle 1$ vuelta $= 420^V$ ... (3)

C es correcta.

D) De (3)

$m\angle 1$ vuelta $= 420^V = 360°$

$\Rightarrow 420^V = 360°$

$\therefore 42^V = 36°$ ... (4)

D es incorrecta.

E) De (4)

$42^V = 36°$

$7^V = 6°$

$7^V = 6(60')$

$7^V = 360'$

E es correcta.

Clave D

Razonamiento y demostración

25. Reduciendo la expresión tenemos:

$$E = \sqrt{\frac{(\sqrt{C} + \sqrt{S})^2 + (\sqrt{C} - \sqrt{S})^2}{(\sqrt{C})^2 - (\sqrt{S})^2} - 2}$$

$$E = \sqrt{\frac{2(C+S)}{(C-S)} - 2}$$

Sabemos: $\frac{S}{C}=\frac{9}{10}=k$

$\Rightarrow S=9k \wedge C=10k$

Reemplazando en la expresión reducida:

$$E=\sqrt{\frac{2(19k)}{k}-2}=\sqrt{36}$$

$\therefore\ E=6$

Clave C

26. Factorizando tenemos:

$$M=\left[\frac{11^g(1+2+3+...+70)}{2\,rad(1+2+3+...+70)}\right]\frac{400}{\pi}$$

$$M=\frac{2200^g}{\pi\ rad}\left(\frac{\pi\ rad}{200^g}\right)$$

$$M=\frac{2200}{200}=11$$

$\therefore\ M=11$

Clave B

27. Por dato:

$$\left(\frac{a^g\,a^m}{a^m}\right)^g\left(\frac{b^g\,b^m}{b^m}\right)^m=a^g\,b^m;\ (a>b)$$

$$\Rightarrow\left(\frac{(100a^m)+a^m}{a^m}\right)^g\left(\frac{(100b^m)+b^m}{b^m}\right)^m=a^gb^m$$

$$(101)^g(101)^m=a^gb^m$$
$$(102)^g(1)^m=a^gb^m$$

Comparando: $a=102 \wedge b=1$

Piden:

$a+b=102+1=103$

$\therefore\ a+b=103$

Clave A

28. Por dato:

$$1^w=\frac{1}{5}(1°)\ \wedge\ 20^v=10^g$$

$$5^w=1°\qquad\wedge\ 2^v=1^g$$

Se sabe:

$\frac{1°}{1^g}=\frac{10}{9}$; reemplazando:

$$\frac{5^w}{2^v}=\frac{10}{9}\Rightarrow\frac{1^w}{1^v}=\frac{4}{9}$$

$$\therefore\ 1^V=\left(\frac{9}{4}\right)^w\ \vee\ 1^V=2{,}25^w$$

Clave C

29. $$M=\frac{20x°+\left(\frac{3x}{5}\right)\pi\ rad+80x^g}{\frac{2x\pi}{9}\ rad+(50^g)x+15x°}$$

Simplificando, tenemos:

$$M=\frac{20°+\frac{3\pi}{5}\ rad+80^g}{\frac{2\pi}{9}\ rad+50^g+15°}$$

$$M=\frac{20°+\frac{3\pi}{5}\ rad\left(\frac{180°}{\pi\ rad}\right)+80^g\left(\frac{9°}{10^g}\right)}{\frac{2\pi}{9}\ rad\left(\frac{180°}{\pi\ rad}\right)+50^g\left(\frac{9°}{10^g}\right)+15°}$$

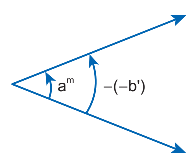
$$M=\frac{20°+108°+72°}{40°+45°+15°}=\frac{200°}{100°}$$

$\therefore\ M=2$

Clave C

Resolución de problemas

30. Colocando los ángulos en sentido antihorario, tenemos:



$\Rightarrow a^m=-(-b')$

$a^m=b'$

$$\frac{a}{b}=\frac{1'}{1^m}\left(\frac{10^g}{9°}\right)\left(\frac{1°}{60'}\right)\left(\frac{100^m}{1^g}\right)$$

$$\Rightarrow\frac{a}{b}=\frac{50}{27}$$

Piden:

$$E=\sqrt{\frac{75a}{2b}}=\sqrt{\frac{75}{2}\left(\frac{a}{b}\right)}=\sqrt{\frac{75}{2}\left(\frac{50}{27}\right)}$$

$$\Rightarrow E=\sqrt{\frac{625}{9}}=\frac{25}{3}$$

$\therefore\ E=\frac{25}{3}$

Clave B

31. Por dato:

$$\sqrt[3]{\frac{180}{S}}+\sqrt[3]{\frac{200}{C}}+\sqrt[3]{\frac{\pi}{R}}=3$$

Sabemos: $S=180k$, $C=200k \wedge R=\pi k$

$$\Rightarrow\sqrt[3]{\frac{180}{180k}}+\sqrt[3]{\frac{200}{200k}}+\sqrt[3]{\frac{\pi}{\pi k}}=3$$

$$\sqrt[3]{\frac{1}{k}}+\sqrt[3]{\frac{1}{k}}+\sqrt[3]{\frac{1}{k}}=3$$

$$3\sqrt[3]{\frac{1}{k}}=3$$

$$\sqrt[3]{\frac{1}{k}}=1$$

$$\Rightarrow k=1$$

Piden:

$$E=\sqrt[3]{6(\sqrt{3}-\sqrt{2})SCR}$$

$$E=\sqrt[3]{6(\sqrt{3}-\sqrt{2})(180k)(200k)(\pi k)}$$

$$E=\sqrt[3]{60^3k^3(\sqrt{3}-\sqrt{2})\pi}=60k\sqrt[3]{(\sqrt{3}-\sqrt{2})\pi}$$

Una aproximación de π es: $\sqrt{3}+\sqrt{2}$

$$\Rightarrow E=60k\sqrt[3]{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}=60k(1)$$

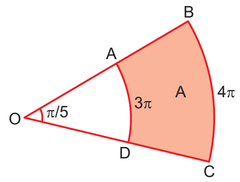
$\therefore\ E=60k=60(1)=60$

Clave E

SECTOR CIRCULAR

APLICAMOS LO APRENDIDO (página 10) Unidad 1

1.



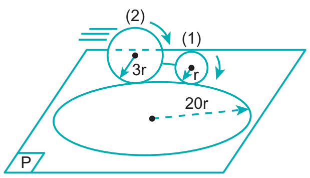
Por propiedad, el área del trapecio circular es:

$$A=\frac{(4\pi)^2-(3\pi)^2}{2\left(\frac{\pi}{5}\right)}=\frac{7\pi^2}{\frac{2\pi}{5}}=\frac{35\pi}{2}$$

$\therefore A=\frac{35}{2}\pi$

Clave A

2.



Por dato: la bicicleta dará 20 vueltas.

Entonces, la longitud recorrida por el centro de la rueda (1) será:

$L_{C(1)}=20(2\pi\,.\,20r)=800\pi\, r$

$\Rightarrow n_{v(1)}=\frac{L_{C(1)}}{2\pi r}=\frac{800\pi r}{2\pi r}=400$

$\Rightarrow n_{v(1)}=400$

Luego, la longitud recorrida por el centro de la rueda (2) será:

$L_{C(2)}=20(2\pi\,.\,20r)=800\pi r$

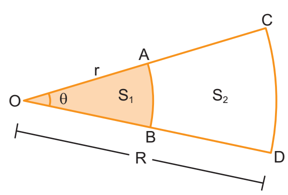
$\Rightarrow n_{v(2)}=\frac{L_{C(2)}}{2\pi(3r)}=\frac{800\pi r}{6\pi r}=\frac{400}{3}$

$\Rightarrow n_{v(2)}=\frac{400}{3}$

Por lo tanto, cada rueda dará $\frac{400}{3}$ y 400 vueltas.

Clave A

3.



Del gráfico:

$S_1=\frac{\theta r^2}{2}$

$S_1+S_2=\frac{\theta R^2}{2}$

$S_2=\frac{\theta R^2}{2}-S_1=\frac{\theta R^2}{2}-\frac{\theta r^2}{2}$

$\Rightarrow S_2=\frac{\theta}{2}(R^2-r^2)$

Por dato: $2S_2=3S_1$

$2\,\frac{\theta}{2}\,(R^2-r^2)=3\,\frac{\theta r^2}{2}$

$2(R^2-r^2)=3r^2$

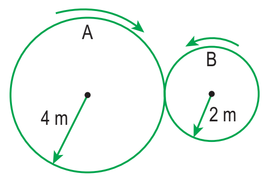
$2R^2=5r^2$

$\Rightarrow \frac{r^2}{R^2}=\frac{2}{5}$

$\therefore \frac{r}{R}=\frac{\sqrt{2}}{\sqrt{5}}$

Clave B

4.



Por dato la rueda mayor gira 18°.

$\Rightarrow \theta_A=18°$

Luego, por estar en contacto las ruedas, se cumple:

$L_1=L_2$

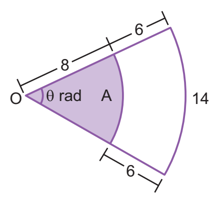
$\Rightarrow \theta_A\,.\,R_A=\theta_B\,.\,R_B$

$(18°)(4)=\theta_B(2)$

$\therefore \theta_B=36°$

Clave D

5. Del gráfico:



Del gráfico:

$\theta(14)=14\Rightarrow\theta=1$

Piden:

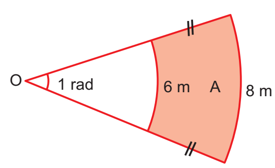
El área del sector sombreado (A):

$A=\frac{\theta R^2}{2}=\frac{(1)(8)^2}{2}=32$

$\therefore A=32$

Clave B

6.



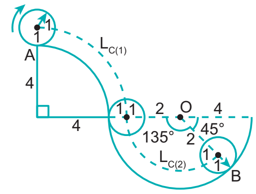
Por propiedad, el área del trapecio circular será:

$A=\frac{(8)^2-(6)^2}{2(1)}=\frac{28}{2}=14$

$\therefore A=14\text{ m}^2$

Clave C

7.



Del gráfico, la longitud que recorre el centro de la rueda al ir de A hasta B será:

$L_{C(AB)}=L_{C(1)}+L_{C(2)}$

$L_{C(AB)}=\left(\frac{\pi}{2}\right).\,(5)+\left(\frac{3\pi}{4}\right).\,(3)$

$\Rightarrow L_{C(AB)}=\frac{19\pi}{4}$

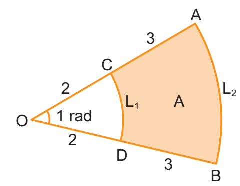
Piden:

$n_{v(AB)}=\frac{L_{C(AB)}}{2\pi R}=\frac{\frac{19\pi}{4}}{2\pi(1)}=\frac{19}{8}$

$\therefore n_{v(AB)}=\frac{19}{8}$

Clave E

8.



Del gráfico:

$L_1=(1)(2)=2$

$L_2=(1)(5)=5$

Piden: el perímetro de la región sombreada ($2p_{somb.}$).

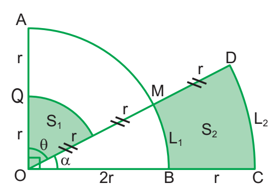
$2p_{somb.}=L_1+3+L_2+3$

$\Rightarrow 2p_{somb.}=2+3+5+3=13$

$\therefore 2p_{somb.}=13$

Clave A

9.



Sea el valor de α expresado en radianes.

Del gráfico:

$\alpha+\theta=\frac{\pi}{2}\text{ rad}\Rightarrow\theta=\frac{\pi}{2}-\alpha\quad ...(1)$

$S_1=\frac{\theta r^2}{2}$

$S_2=\left(\frac{L_1+L_2}{2}\right)r=\left(\frac{\alpha\,.\,2r+\alpha\,.\,3r}{2}\right)r$

$S_2=\left(\frac{5\alpha r}{2}\right)r=\frac{5\alpha r^2}{2}$

Por dato: $\frac{S_1}{S_2}=\frac{1}{2}$

$$\Rightarrow \frac{\frac{\theta r^2}{2}}{\frac{5\alpha r^2}{2}} = \frac{1}{2} \Rightarrow \frac{\theta}{5\alpha} = \frac{1}{2} \Rightarrow \theta = \frac{5\alpha}{2}$$

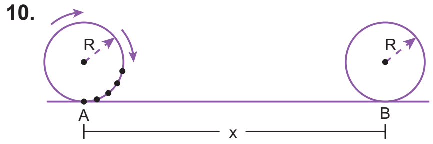
Reemplazando en (1):

$$\left(\frac{5\alpha}{2}\right) = \frac{\pi}{2} - \alpha \Rightarrow \frac{7\alpha}{2} = \frac{\pi}{2}$$

$$\therefore \alpha = \frac{\pi}{7}$$

Clave B

10.



Por dato: la rueda barre un ángulo de $\frac{49\pi}{11}$ rad y $R = 0{,}5$

Se cumple que la longitud que recorre su arco va ser igual a:

$$L_{recorrida} = (\theta_b)R = \left(\frac{49\pi}{11}\right)(0{,}5)$$

$$\Rightarrow L_{recorrida} = \frac{49\pi}{22}$$

A su vez, la longitud que recorre su arco también va ser igual a la distancia entre A y B.

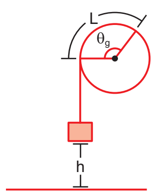
$$\Rightarrow d_{(AB)} = L_{recorrida} = x$$

$$\Rightarrow x = \frac{49\pi}{22} = \frac{49}{22}\left(\frac{22}{7}\right) = 7$$

$$\therefore x = 7$$

Clave A

11. Para la polea:



Cuando la polea gira un ángulo θ_g el bloque se eleva una altura h tal que equivale a la longitud de arco correspondiente al θ_g entonces:

$$\theta_g r = L = h \quad \rightarrow \quad \theta_g = \frac{h}{r} \qquad \dots (1)$$

Además:

$$n_v = \frac{\theta_g}{2\pi} \quad \rightarrow \quad n_v = \frac{h}{2\pi r}$$

Reemplazando datos:

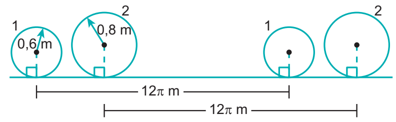
$$n_v = \frac{\sqrt{75} + \sqrt{50}}{2\pi(1)} = \frac{5\cancel{(\sqrt{3}+\sqrt{2})}}{2\cancel{(\sqrt{3}+\sqrt{2})}}$$

$$n_v = 2{,}5$$

$\therefore$ El número de vueltas que da la polea es 2,5

Clave A

12. Del enunciado, la bicicleta recorre 12π m; por lo tanto, cada rueda de la bicicleta también recorre 12π m entonces:



Luego:

$$n_1 = \frac{\ell_1}{2\pi r_1}\ ; \ n_1\text{: n.° de vueltas de radio 1}$$

$$n_2 = \frac{\ell_2}{2\pi r_2}\ ; \ n_2\text{: n.° de vueltas del radio 2}$$

Nos piden:

$$n_1 + n_2 = \frac{\ell_1}{2\pi r_1} + \frac{\ell_2}{2\pi r_2}$$

Reemplazando:

$$n_1 + n_2 = \frac{12\cancel{\pi}}{2\cancel{\pi}(0{,}6)} + \frac{12\cancel{\pi}}{2\cancel{\pi}(0{,}8)}$$

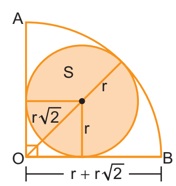
$$n_1 + n_2 = 6\left(\frac{10}{6} + \frac{10}{8}\right)$$

$$n_1 + n_2 = 17{,}5$$

$\therefore$ La suma del número de vueltas de las ruedas es igual a 17,5.

Clave C

13. Del gráfico:



Por dato:

$$S = \cancel{\pi} r^2 = \cancel{\pi}(3 - 2\sqrt{2})$$

$$r^2 = (\sqrt{2})^2 - 2\sqrt{2} + 1$$

$$r^2 = (\sqrt{2} - 1)^2$$

$$r = (\sqrt{2} - 1)\,\text{m}$$

Luego:

$$AO = OB = r + r\sqrt{2} = r(1 + \sqrt{2})$$
$$= (\sqrt{2} - 1)(\sqrt{2} + 1)$$

$$AO = 1\text{ m}$$

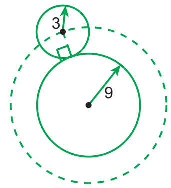
Nos piden

$$AO + OB + L_{\overset{\frown}{AB}} = 1 + 1 + \left(\frac{\pi}{2}\right)(1)$$

$$\therefore AO + OB + L_{\overset{\frown}{AB}} = \frac{4 + \pi}{2}\,\text{m}$$

Clave E

14. Del grafico:



Sabemos:

$$n_v = \frac{\theta(R + r)}{2\pi r}$$

Para el problema

$$\theta = 2\pi\ ;\ R = 9u\ ;\ r = 3u$$

$$n_v = \frac{\cancel{2\pi}(9 + 3)}{\cancel{2\pi}(3)}$$

$$\therefore n_v = 4$$

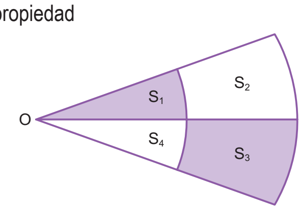
Clave B

PRACTIQUEMOS

Nivel 1 (página 12) Unidad 1

Comunicación matemática

1. Por propiedad



Se cumple:

$$S_1S_3 = S_2S_4 \qquad \dots (1)$$

I. Si : $S_3 = S_4$:

De (1):

$$S_3S_1 = S_2S_4$$
$$S_4S_1 = S_2S_4$$
$$S_1 = S_2$$

$\therefore$ Si $S_3 = S_4 \Rightarrow S_1 = S_2$

(verdadero)

II. Se tiene del gráfico:

$$S_1 = \frac{1}{2}\alpha r^2;\ S_2 = \frac{1}{2}\alpha R^2 - S_1$$

$$S_2 = \frac{1}{2}\alpha R^2 - \frac{1}{2}\alpha r^2$$

$$S_2 = \frac{1}{2}\alpha(R^2 - r^2)$$

Si : $S_1 = S_2$:

$$S_1 = S_2$$

$$\cancel{\tfrac{1}{2}}\cancel{\alpha}\, r^2 = \cancel{\tfrac{1}{2}}\cancel{\alpha}(R^2 - r^2)$$

$$r^2 = R^2 - r^2$$

$$2r^2 = R^2$$

$$r\sqrt{2} = R$$

$\therefore$ Si : $S_1 = S_2 \Rightarrow R = r\sqrt{2}$

(falso)

III. Si : $S_3 = 4S_4$

De (1):

$S_1S_3 = S_2S_4$

$S_1 4\cancel{S_4} = S_2\cancel{S_4}$

$4S_1 = S_2$

$\therefore$ Si: $S_3 = 4S_4 \Rightarrow 4S_1 = S_2$

(verdadero)

Clave C

2. De las propiedades de engranajes y ejes:

Sean: n_A, n_B, n_C, n_D, n_E, n_F; los números de vueltas:

I. Del gráfico:

D y C mismo eje:

$n_D = n_C$ ... (1)

D y E engranaje:

$n_D \cdot \cancel{r} = n_E 4\cancel{r}$

$n_D = 4n_E$ ... (2)

(1) en (2):

$n_C = 4n_E$

$\therefore$ El número de vueltas de C es igual a 4 veces el número de vueltas de E

(falsa)

II. A y B unidas por un mismo eje:

$n_A = n_B$

$\therefore$ A y B dan un mismo número de vueltas

(verdadera)

III. Del gráfico:

D y C mismo eje:

$n_D = n_C$ ... (1)

B y C unido por una banda:

$n_B \cdot r = n_C \cdot 2r$

$n_B = 2n_C$ ... (2)

(1) en (2):

$n_B = 2n_D$

Si: n_B es igual a 2

$2 = 2n_D$

$n_D = 1$

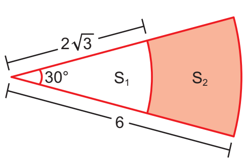
$\therefore$ Si B da 2 vueltas ($n_B = 2$), D da 1 vuelta ($n_D = 1$)

(verdadera)

Clave E

Razonamiento y demostración

3.



30° a radianes:

$$\frac{S}{9} = \frac{20R}{\pi}$$

$$\frac{30}{9} = \frac{20R}{\pi}$$

$$R = \frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{6} \text{ rad}$$

$$S_1 = \frac{\pi}{6}\frac{(2\sqrt{3})^2}{2} = \frac{\pi}{6}\cdot\frac{12}{2} = \pi$$

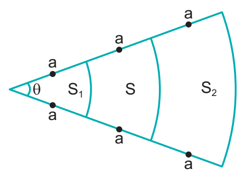
$$S_1 + S_2 = \frac{\pi}{6}\frac{(6)^2}{2} = \frac{36\pi}{12} = 3\pi$$

$$\Rightarrow \pi + S_2 = 3\pi$$

$$\therefore S_2 = 2\pi$$

Clave B

4.



$$S_1 = \frac{\theta a^2}{2} \quad ...(1)$$

$$S_1 + S + S_2 = \frac{\theta(3a)^2}{2}$$

$$\frac{\theta(2a)^2}{2} + S_2 = \frac{9\theta a^2}{2}$$

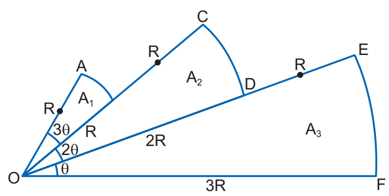
$$\frac{4\theta a^2}{2} + S_2 = \frac{9\theta a^2}{2}$$

$$\Rightarrow S_2 = \frac{5\theta a^2}{2} \quad ...(2)$$

$$\therefore \frac{S_1}{S_2} = \frac{\frac{\theta a^2}{2}}{\frac{5\theta a^2}{2}} = \frac{1}{5}$$

Clave D

5.



Del gráfico:

$$A_1 = \frac{(3\theta)R^2}{2} = \frac{3\theta R^2}{2}$$

$$A_2 = \frac{(2\theta)(2R)^2}{2} = 4\theta R^2$$

$$A_3 = \frac{\theta(3R)^2}{2} = \frac{9\theta R^2}{2}$$

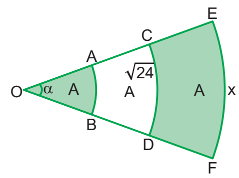
Piden:

$$J = \frac{A_2 - A_1}{A_3 - A_2} = \frac{4\theta R^2 - \frac{3\theta R^2}{2}}{\frac{9\theta R^2}{2} - 4\theta R^2} = \frac{\frac{5\theta R^2}{2}}{\frac{\theta R^2}{2}} = 5$$

$$\therefore J = 5$$

Clave E

6.



Del gráfico:

$$S_{\triangleleft COD} = 2A = \frac{L_1^2}{2\theta} = \frac{(\sqrt{24})^2}{2\theta} = \frac{24}{2\theta}$$

Entonces:

$$\cancel{2}A = \frac{\cancel{24}}{\cancel{2}\theta} \Rightarrow A = \frac{6}{\theta} \quad ... (1)$$

Luego:

$$S_{\triangleleft EOF} = 3A = \frac{L_2^2}{2\theta} = \frac{x^2}{2\theta}$$

$$\Rightarrow 3A = \frac{x^2}{2\theta}$$

De (1):

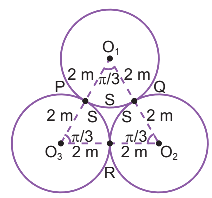
$$3\left(\frac{6}{\theta}\right) = \frac{x^2}{2\theta}$$

$$x^2 = 36$$

$$\therefore x = 6$$

Clave C

7. Del gráfico:



- El triángulo $O_1O_2O_3$ es equilátero.
- Las regiones (sectores circulares) PO_3R; RO_2Q; QO_1P son congruentes (áreas iguales).

Sea el área sombreada S_x.

Se tiene que:

$A_{\triangle ABC} = S_x + 3S$ ... (1)

$\triangle ABC$: equilátero

$$A_{\triangle ABC} = \frac{\ell^2\sqrt{3}}{4};\ S = \frac{1}{2}\left(\frac{\pi}{3}\right)(2)^2$$

$$A_{\triangle ABC} = \frac{(4)^2\sqrt{3}}{4};\ S = \frac{2\pi}{3}m^2$$

$$A_{\triangle ABC} = 4\sqrt{3}\,m^2$$

En (1):

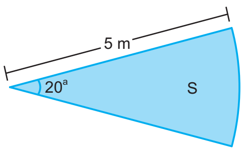
$$4\sqrt{3} = S_x + \cancel{3}\left(\frac{2\pi}{\cancel{3}}\right)$$

$$\therefore S_x = (4\sqrt{3} - 2\pi)m$$

Clave E

Resolución de problemas

8. Del enunciado:



Por dato:

$1^a = 3^g$

$1^a = 3^{\cancel{g}} \dfrac{\pi \text{rad}}{200^{\cancel{g}}}$

$1^a = \dfrac{3\pi}{10}\text{rad}$

$\Rightarrow 20^a = \dfrac{3\pi}{200}\text{rad}$

El área del sector será:

$S = \dfrac{1}{2}\theta R^2 = \dfrac{1}{2}\left(\dfrac{3\pi}{10}\right)(5)^2 = \dfrac{15\pi}{4}$

$\therefore S = \dfrac{15\pi}{4}\ \text{m}^2$

Clave B

9. Sabemos:

$n_V = \dfrac{\ell}{2\pi r}$

Para la rueda C:

$n_{VC} = \dfrac{\ell_c}{2\pi(5)} \rightarrow \ell_c = 10\pi n_{vc}$ ... (1)

Por dato:

$n_{VC} = n_{VA} + n_{VB} = \dfrac{\ell_1}{2\pi(3)} + \dfrac{\ell_2}{2\pi(4)}$

Pero: $\ell_1 = \ell_2 = 24$ m, entonces:

$n_{VC} = \dfrac{24}{6\pi} + \dfrac{24}{8\pi} = \dfrac{4}{\pi} + \dfrac{3}{\pi}$

$n_{VC} = \dfrac{7}{\pi}$ ... (2)

(2) en (1):

$\ell_C = 10\cancel{\pi}\left(\dfrac{7}{\cancel{\pi}}\right)$

$\therefore \ell_C = 70$ m

Clave B

10. De la figura se cumple:

$n_A r_A = n_B r_B$

n_A, n_B : n.° de vueltas

$r_A; r_B$: radios

Por dato:

$(n - 4)6 = n(3)$

$3n = 24$

$n = 8$

$\therefore n + 3 = 11$

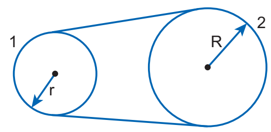
Clave D

Nivel 2 (página 12) Unidad 1

Comunicación matemática

11.

I. Para dos ruedas unidas por una banda:



Se cumple:

$rn_1 = Rn_2$

$\dfrac{n_1}{n_2} = \dfrac{R}{r} = k$

$\therefore$ La razón de los radios (k) es igual a la inversa de la razón $\left(\dfrac{1}{k}\right)$ entre su número de vueltas.

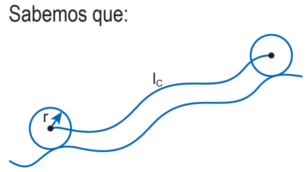
I es falsa.

II. Del enunciado:

$\ell = \theta r$ cm

ℓ: longitud que la rueda recorre.

Sabemos que:



$\ell_C = \theta r$

θ: ángulo que gira la rueda

r: radio de la rueda

ℓ_C: longitud que recorre el centro de la rueda

$\therefore$ θ . r cm es igual a la longitud que recorre el centro de la rueda.

II es falsa.

III. Sean 2 poleas unidas por un eje:

Sabemos que:

$\theta_1 = \theta_2$

θ_i: ángulo que gira la polea

El número de vueltas de una polea (n_i) está definido:

$n_i = \dfrac{\theta_i}{2\pi}$

Luego:

$\dfrac{\theta_1}{2\pi} = \dfrac{\theta_2}{2\pi} \Rightarrow n_1 = n_2$

$\dfrac{n_1}{n_2} = 1$

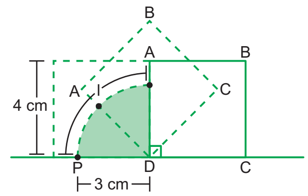
$\therefore$ La razón de su número de vueltas es igual a 1

III es verdadera.

Clave C

12.

I. De la condición:



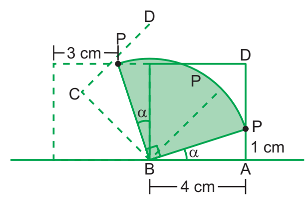
La región sombreada corresponde a un sector circular cuya longitud de arco es la longitud que recorre P cuando el cuadrado gira desde el instante dado hasta que C toca el piso por primera vez.

Luego, del gráfico:

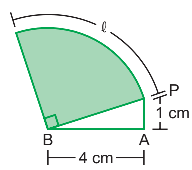
$\ell_P = 3.\dfrac{\pi}{2}$

$\therefore \ell_P = \dfrac{3\pi}{2}$ cm

II. De la condición:



Trayectoria de P cuando el cuadrado gira según las condiciones luego, el área sombreada, sector circular y ℓ longitud de arco donde:



$BP^2 = 1^2 + 4^2$

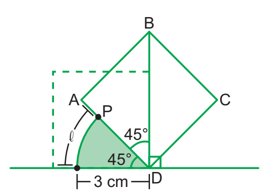
$BP = \sqrt{17}$ cm

Entonces:

$\ell = \dfrac{\pi}{2}\sqrt{17}$

$\therefore \ell = \dfrac{\sqrt{17}\,\pi}{2}$

III. De la condición:



ℓ, longitud de arco del sector circular sombreado de ángulo central 45° y radio 3 cm, entonces:

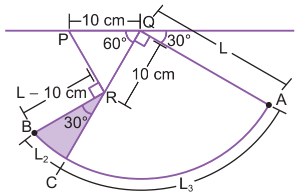
$\ell = \dfrac{\pi}{4}.3;\ 45° = \dfrac{\pi}{4}\text{rad}$

$\therefore \ell = \dfrac{3\pi}{4}$ cm

Clave E

Razonamiento y demostración

13. De la figura, sea L la longitud del péndulo:



QAC, RCB sectores circulares donde:

L_2, L_3: longitudes de arco

Por dato:

$L_2 + L_3 = 13\pi$ cm ... (1)

Del gráfico:

$L_3 = \frac{\pi}{2}L$; $L_2 = (L - 10)\theta$; $\theta = 30°$

$L_3 = \frac{L\pi}{2}$ $\theta = 30°\frac{\pi}{180}$

$\theta = \frac{\pi}{6}$

$L_2 = (L - 10)\frac{\pi}{6}$

En (1):

$L_2 + L_3 = \frac{(L-10)\pi}{6} + \frac{L\pi}{2} = 13\pi$

$\frac{(L-10)\pi + 3\pi L}{6} = 13\pi$

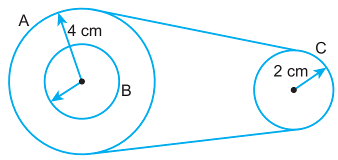
$4\pi L - 10\pi = 78\pi$

$\cancel{4}\cancel{\pi}L = 88\cancel{\pi}$

$\therefore L = 22$ cm

Clave C

14. Del grafico:



Datos:

$n_B + n_C = 18$; n_B, n_C: número de vueltas

Del gráfico:

A y B unidos por el mismo eje:

$n_A = n_B$

A y C unidos por una banda

$n_A r_A = n_C r_C \Rightarrow n_C = \frac{n_A r_A}{r_C}$

Luego:

$n_B + n_C = n_A + \frac{n_A r_A}{r_C} = \frac{n_A r_C + n_A r_A}{r_C}$

$\frac{n_A r_C + n_A r_A}{r_C} = 18$

$\frac{n_A.2 + n_A.4}{2} = 18$

$6n_A = 36$

$n_A = 6$

Finalmente:

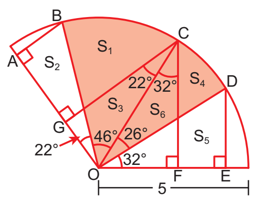
$n_A = \frac{\theta_A}{2\pi}$; θ_A: ángulos que gira la rueda A

$\theta_A = 2\pi\, n_A$

$\therefore \theta_A = 12\pi$

Clave B

15. Del gráfico:



$\triangle OAB \cong \triangle CGO$; $\triangle CFO \cong \triangle DEO$

Entonces:

$S_2 = S_3$; $S_5 = S_6$

Nos piden:

$A_{\triangleleft BOD} = \frac{\theta R^2}{2}$ ... (1)

Por dato:

θ rad $= 72° = 72°\frac{\pi \text{rad}}{180°}$

θ rad $= \frac{2\pi}{5}$rad

$\theta = \frac{2\pi}{5}$

R = 5 u

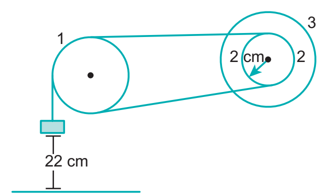
En (1):

$A_{\triangleleft BOD} = \frac{\cancel{2}\pi}{5}.\frac{1}{\cancel{2}}.(5)^2$

$\therefore A_{\triangleleft BOD} = 5\pi\ u^2$

Clave E

16.



Para la polea (1)
La longitud que gire la polea 1 será igual a la longitud que recorre el bloque al descender.

Entonces:
$L_1 = 22$ cm

La polea (1) y (2) unidas por una banda:
$L_1 = L_2 = 22$ cm

Polea (2) y (3) unidas por el eje de giro:
$\theta_2 = \theta_3$ ... (1)

Se sabe:
$L_2 = \theta_2 r_2$

De (1)

$L_2 = \theta_3 r_2$

De: $n_v = \frac{\theta}{2\pi} \Rightarrow \theta = 2\pi\, n_v$

$L_2 = n_3(2\pi)r_2$; $r_2 = 2$ cm

$22 = \cancel{2}\pi\, n_3(2)$

$n_3 = \frac{11}{2\pi} = \frac{\cancel{11}}{\cancel{2}.\frac{\cancel{22}}{7}}$

$\therefore n_3 = 1{,}75$

Clave C

Resolución de problemas

17. Para los engranajes se cumple que:

$n_1 r_1 = n_2 r_2$ ... (1)

Por dato:

$r_1 = 5$ u ; $n_2 = 1{,}25$

$r_2 = 1$ u

En (1)

$n_1(5) = 1{,}25\ (1)$

$n_1 = 0{,}25$

$n_1 = \frac{1}{4}$

Luego:

Para el punto A en el engranaje (1)

$n_1 = \frac{\theta_A}{2\pi} \Rightarrow \theta_A = \cancel{2}\pi\left(\frac{1}{\cancel{4}}\right)$

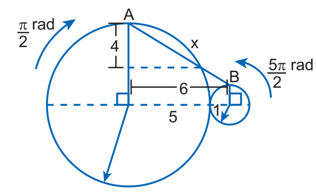
$\theta_A = \frac{\pi}{2}$

Para el punto B en el engranaje (2)

$n_2 = \frac{\theta_B}{2\pi} \Rightarrow \theta_B = 2\pi(1{,}25)$

$\theta_B = \frac{5\pi}{2}$

Finalmente:



$x^2 = 4^2 + 6^2$

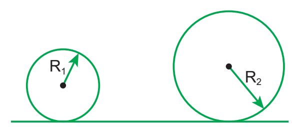
$x^2 = 52$

$x = 2\sqrt{13}$

$\therefore$ La distancia será igual a $2\sqrt{13}$ u.

Clave A

18. Del enunciado:



Dato:

$\frac{R_1}{R_2} = \frac{8}{15}$; $R_1 = 8k$

$R_2 = 15k$

En la bicicleta se cumple:

$\ell_1 = \ell_2 \Rightarrow n_1R_1 = n_2R_2$

Luego:

$n_1(8k) = n_2(15k)$; $n_1 = \frac{3}{8}$

$\frac{\cancel{3}}{\cancel{8}}(\cancel{8}\cancel{k}) = n_2(\cancel{15}\cancel{k})$

$n_2 = \frac{1}{5}$

Además:

$\theta_g = 2\pi\, n_2$; θ_g: ángulo que gira la rueda

$\theta_g = 2\pi\left(\frac{1}{5}\right)$

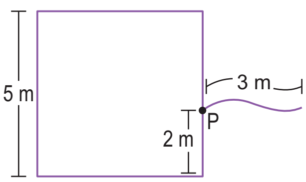
$\theta_g = \frac{2\pi}{5}\text{ rad} = \frac{2\cancel{\pi}}{5}\cancel{\text{rad}}\,\frac{180°}{\cancel{\pi\text{ rad}}}$

$\theta_g = 72°$

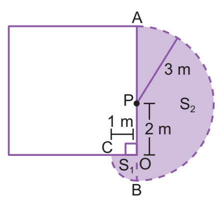
$\therefore$ Cualquier punto sobre la superficie de la rueda gira un ángulo de 72°.

Clave D

19. Del enunciado



La cabra podrá pastar hasta los puntos donde la cuerda esté totalmente estirada luego:



La zona sombreada representa la región en la que la cabra puede pastar donde:
APB y BOC son sectores circulares.

Del gráfico:

$S_1 + S_2 = \left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right)(1)^2 + \frac{1}{2}\,.\,\pi\,.\,3^2$

$= \frac{\pi}{4} + \frac{9\pi}{2}$

$S_1 + S_2 = \frac{19\pi}{4}\text{ m}^2$

$\therefore$ La cabra puede pastar en un área de $\frac{19\pi}{4}\text{ m}^2$.

Clave A

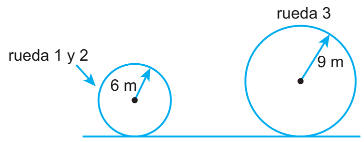
20. Para el triciclo, si se traslada una distancia d; cada rueda recorrerá la misma distancia d.

De la expresión:

$n_v = \frac{\ell}{2\pi r}$

Para 2 ruedas de radios iguales, si recorren la misma distancia. Entonces el número de vueltas que dan las ruedas son iguales.

Luego:



Se cumple:

$n_1r_1 = n_3r_3$;

$n_1(6) = n_3(9)$

$2n_1 = 3n_3$ ... (1)

Por dato:

$(n_1 + n_2) - n_3 = 8$;

Pero:

$n_1 = n_2$; radios iguales

$2n_1 - n_3 = 8$ ... (2)

De (1) y (2):

$3n_3 - n_3 = 8$

$2n_3 = 8$

$n_3 = 4$

Finalmente, de:

$n_v = \frac{\ell}{2\pi r}$

Para la rueda 3

$n_3 = \frac{d}{2\pi r_3}$

$4 = \frac{d}{2\pi(9)}$

$\therefore d = 72\pi$ m

Clave A

Nivel 3 (página 13) Unidad 1

Comunicación matemática

21. Tenemos 2 expresiones para el cálculo del número de vueltas de una rueda.

$n_V = \frac{\theta_g}{2\pi}$; $n_V = \frac{\ell_c}{2\pi r} = \frac{\theta(R + r)}{2\pi r}$

Para la primera expresión, solo es necesario conocer el ángulo que la rueda gira para calcular el número de vueltas.

De la segunda expresión, es necesario conocer θ, R y r; o conocer ℓ_c y r para el cálculo del número de vueltas.

Clave B

22. Del dato I:

$S_1 + S_2 = \frac{1}{2}\alpha R^2 + \frac{1}{2}\theta R^2$

$S_1 + S_2 = \frac{1}{2}(\alpha + \theta)R^2$

$3\pi = \frac{1}{2}(\alpha + \theta)R^2$ ... (1)

Del dato II:

$L_1 - L_2 = \alpha R - \theta R$

$L_1 - L_2 = (\alpha - \theta)R$

$\frac{2\pi}{5} = (\alpha - \theta)R$ ... (2)

Del dato III:

$m\angle AOB = \pi - (\alpha + \theta)$

$\frac{\pi}{6} = \pi - (\alpha + \theta)$

$(\alpha + \theta) = \pi - \frac{\pi}{6}$

$\alpha + \theta = \frac{5\pi}{6}$ ... (3)

De (1) (2) y (3) se observa que solo (1) y (3) son necesarios, tal que presentan 2 ecuaciones y 2 incógnitas:

$3\pi = \frac{1}{2}(\alpha + \theta)R^2$ ... (1)

$\alpha + \theta = \frac{5\pi}{6}\text{rad}$ ... (3)

$3\cancel{\pi} = \frac{1}{2}\left(\frac{5\cancel{\pi}}{6}\right)R^2$

$\therefore R = \frac{6\sqrt{5}}{5}$

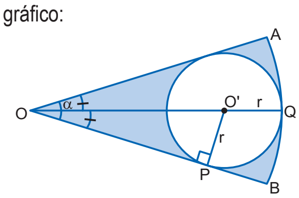
Clave D

Razonamiento y demostración

23. Sea S, área de la circunferencia de radio r

$S = \pi r^2$ ... (1)

En el gráfico:



$\alpha = \frac{200^g}{3} = \frac{20\cancel{0}^g}{3}\cdot\frac{9°}{1\cancel{0}^g}$

$\alpha = 60°$

$\triangle O'PO$ 60° y 30°

$OO' = 2r$

Entonces:

$OQ = OO' + O'Q$

$OQ = 2r + r$

$OQ = 3r$

El área sombreada (S_1) será igual:

$S_1 = S_{\triangleleft AOB} - S$

$S_1 = \theta(3r)^2 - \pi r^2$... (1)

Pero:

$\theta\text{ rad} = 60° = \cancel{60°}.\frac{\pi}{\cancel{180°}}\text{rad} = \frac{\pi}{3}\text{rad}$

$\theta = \frac{\pi}{3}$

En (1):

$S_1 = \frac{1}{2}\left(\frac{\pi}{\cancel{3}}\right)\cancel{9}r^2 - \pi r^2$

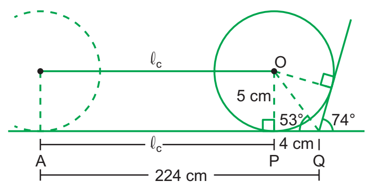
$S_1 = \frac{3\pi}{2}r^2 - \pi r^2$

$S_1 = \frac{\pi}{2}r^2 = \frac{\pi}{\cancel{2}}\left(\frac{\cancel{8}}{\pi}\right)$

$\therefore\ S_1 = 4\ cm^2$

Clave C

24. En el instante que choca con la superficie inclinada.



Del gráfico:

$\ell_c + 4 = 224$

$\ell_c = 220$ cm

Luego:

$n_v = \frac{\ell_c}{2\pi r}$

$n_v = \frac{\cancel{220}}{\cancel{2}\pi(\cancel{5})}$

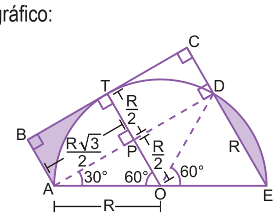
$n_v = \frac{22}{\pi} = \frac{\cancel{22}}{\frac{\cancel{22}}{7}}$

$n_v = 7$

$\therefore$ La rueda da 7 vueltas desde A hasta chocar con la superficie.

Clave E

25. En el gráfico:



Sea:

$2P_1$: perímetro de la región ABT

$2P_1 = AB + BT + L_{\widehat{AT}}$

Del gráfico:

$2P_1 = \frac{R}{2} + \frac{R\sqrt{3}}{2} + \theta R$

Donde:

$\theta\ rad = 60° = 60°\ .\ \frac{\pi}{180°}rad$

$\theta\ rad = \frac{\pi}{3}rad$

$\theta = \frac{\pi}{3}$

$2P_1 = \frac{R}{2} + \frac{R\sqrt{3}}{2} + \frac{\pi}{3}R \quad ...\ (1)$

$2P_2$: Perímetro de la región ED

$2P_2 = ED + L_{\widehat{ED}}$

Del gráfico:

$2P_2 = R + \alpha R;\ \alpha\ rad = 60° = \frac{\pi}{3}rad$

$2P_2 = R + \frac{\pi}{3}R \quad ...\ (2)$

Nos piden:

$2P_1 + 2P_2$

De (1) y (2):

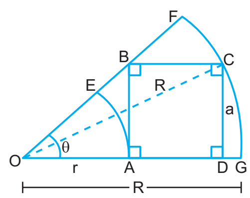
$2P_1 + 2P_2 = \frac{R}{2} + \frac{R\sqrt{3}}{2} + \frac{\pi}{3}R + R + \frac{\pi}{3}R$

$\therefore\ 2P_1 + 2P_2 = \frac{(4\pi + 9 + 3\sqrt{3})}{6}R$

Clave C

Resolución de problemas

26.



Por dato: $L_{\widehat{GF}} = \sqrt{5}\,L_{\widehat{AE}}$

$\Rightarrow \theta R = \sqrt{5}\,(\theta r) \Rightarrow R = \sqrt{5}\,r$

En el ⊿ODC por el teorema de Pitágoras:

$R^2 = a^2 + (r + a)^2$

$(\sqrt{5}\,r)^2 = a^2 + r^2 + 2ra + a^2$

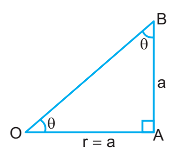
$\Rightarrow a^2 + ar - 2r^2 = 0$

$(a - r)(a + 2r) = 0$

$\Rightarrow a = r\ \vee\ a = -2r$

Como: $a > 0\ \wedge\ r > 0 \Rightarrow a = r$

Entonces el ⊿OAB resulta isósceles.



$\Rightarrow \theta + \theta = 90°$

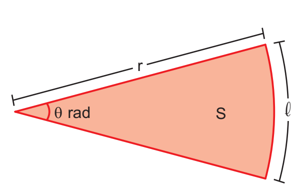
$2\theta = 90°$

$\theta = 45°$

$\therefore \theta = \frac{\pi}{4}$ rad

Clave B

27.



De:

$S = \frac{1}{2}\theta r^2;\ \ell = \theta r$

En la expresión:

$\frac{5(\theta r)^2}{\pi} + 11\left(\frac{1}{2}\theta \cancel{r^2}\right) = 3\pi \cancel{r^2}$

$\frac{5\theta^2}{\pi} + \frac{11\theta}{2} = 3\pi$

$10\theta^2 + 11\theta\pi = 6\pi^2$

$10\theta^2 + 11\theta\pi - 6\pi^2 = 0$

$2\theta \quad\quad +3\pi$

$5\theta \quad\quad -2\pi$

$(2\theta + 3\pi)(5\theta - 2\pi) = 0$

$\theta = -\frac{3\pi}{2} \quad ;\ \theta = \frac{2\pi}{5}$

Entonces:

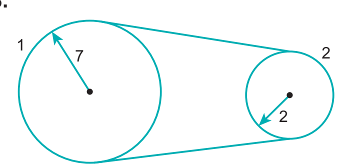
$\theta\ rad = \frac{2\pi}{5}rad = \frac{2\cancel{\pi}}{5}.\frac{180°}{\cancel{\pi}}$

$\theta\ rad = 72°$

$\therefore$ El ángulo del sector es igual a 72°.

Clave D

28.



Del enunciado:

$\theta_1 + \theta_2 = 486°$

θ_i: ángulo que gira la rueda i

Sabemos:

$\theta = n_v 2\pi$

θ: ángulo que gira la rueda en radianes

Para las ruedas 1 y 2:

$\theta_1 + \theta_2 = 486° = 486°\frac{\pi rad}{180°}$

$\theta_1 + \theta_2 = \frac{486\pi}{180}rad$

$n_1\,2\cancel{\pi} + n_2\,2\cancel{\pi} = \frac{\cancel{486}\cancel{\pi}}{\cancel{180}}$

$n_1 + n_2 = \frac{27}{20} \quad ...\ (1)$

Las ruedas están unidas por una faja, se cumple:

$n_1(7) = n_2(2)$

$n_2 = \frac{7n_1}{2} \quad ...\ (2)$

(2) en (1):

$n_1 + \frac{7}{2}n_1 = \frac{27}{20}$

$\frac{\cancel{9}n_1}{\cancel{2}} = \frac{\cancel{27}}{\cancel{20}}$

$n_1 = \frac{3}{10}$

En (2):

$n_2 = \frac{7}{2}.\frac{3}{10}$

$n_2 = \frac{21}{20}$

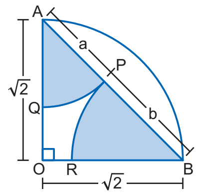
Nos piden:

$n_2 - n_1 = \frac{21}{20} - \frac{3}{10}$

$\therefore\ n_2 - n_1 = \frac{3}{4}$

Clave E

29.

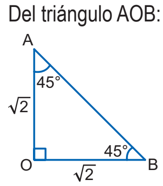


Sea:

$S = A_{\triangleleft PAQ} + A_{\triangleleft PBR}$

$S = \frac{1}{2}\theta_1 a^2 + \frac{1}{2}\theta_2 b^2 \qquad \ldots (1)$

Del triángulo AOB:



$m\angle BAO = m\angle ABO = 45°$

Además

$45° = 45° \frac{\pi}{180°}$ rad

$45° = \frac{\pi}{4}$ rad

En (1):

$S = \frac{1}{2}\frac{\pi}{4}a^2 + \frac{1}{2}\frac{\pi}{4}b^2$

$S = \frac{\pi}{8}(a^2 + b^2) \quad \ldots (2)$

Por desigualdades:

Si a y b $\in \mathbb{R}$, se cumple:

$a^2 + b^2 \geq 2ab$

Entonces:

Si S es mínimo:

$a^2 + b^2 = 2ab$

$a^2 + b^2 - 2ab = 0$

$(a - b)^2 = 0$

$a - b = 0$

$a = b$

Además del ⊿ AOB (⊿ 45°)

$a + b = (\sqrt{2})(\sqrt{2})$

$a + b = 2$

$\cancel{2}a = \cancel{2}$

$\therefore\ a = b = 1$

En (2):

$S = \frac{\pi}{8}(1^2 + 1^2)$

$S = \frac{\pi}{4}\ u^2$

Clave B

30. Del enunciado:

Donde:

θ_g: ángulo que gira la rueda en radianes.

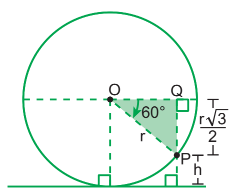
Sabemos que:

$\theta_g = n_v 2\pi$

Por dato:

$n_v = \frac{2}{3} \Rightarrow \theta_g = \frac{4\pi}{3}\text{rad} = \frac{4\cancel{\pi \text{rad}}}{3}\,\frac{180°}{\cancel{\pi \text{rad}}}$

$\theta_g = 240°$



Del ⊿OQP(⊿30° y 60°)

$QP = \frac{r\sqrt{3}}{2}$

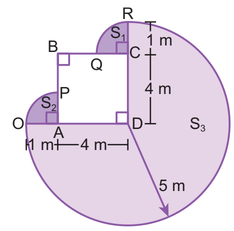
Además, nos piden h, donde:

$h + QP = r$

$\therefore\ h = r - \frac{r\sqrt{3}}{2}$

Clave A

31. Sea la cerca cuadrada ABCD, con la cabra atada al punto D:



La cabra podrá pastar hasta los puntos en que la cuerda está totalmente estirada.

Por lo tanto, todos los puntos en que la cuerda se estira totalmente forman arcos de circunferencia por lo que encierren sectores circulares (S_1, S_2, S_3).

Luego, del gráfico:

$S_1 = \frac{1}{2}\left(\frac{\pi}{2}\right)(1)^2$

$S_1 = \frac{\pi}{4}\ m^2$

$S_2 = \frac{1}{2}\left(\frac{\pi}{2}\right)(1)^2$

$S_2 = \frac{\pi}{4}\ m^2$

$S_3 = \frac{1}{2}\left(\frac{3\pi}{2}\right)(5)^2$

$S_3 = \frac{75\pi}{4}\ m^2$

Nos piden:

$S_1 + S_2 + S_3 = \frac{\pi}{4} + \frac{\pi}{4} + \frac{75\pi}{4}$

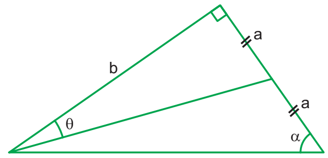
$S_1 + S_2 + S_3 = \frac{77\pi}{4}\ m^2$

$\therefore$ La cabra puede pastar en un área de $\frac{77\pi}{4}\ m^2$.

Clave E

APLICAMOS LO APRENDIDO (página 15) Unidad 1

1. Por dato: $\tan\alpha = 6$



Entonces: $\dfrac{b}{2a} = 6 \Rightarrow \dfrac{a}{b} = \dfrac{1}{12}$

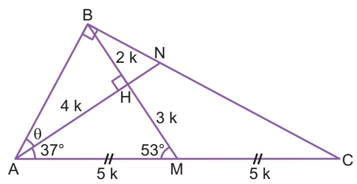
Piden:

$\tan\theta = \dfrac{a}{b} = \dfrac{1}{12}$

$\therefore \tan\theta = \dfrac{1}{12}$

Clave B

2.



Del gráfico:

El ⊿AHM es notable de 37° y 53°

$\Rightarrow AM = 5k,\ HM = 3k$ y $AH = 4k$

$\overline{BM}$: mediana relativa a la hipotenusa.

Entonces por propiedad: $BM = AM = MC$

$\Rightarrow BM = 5k \Rightarrow BH = 2k$

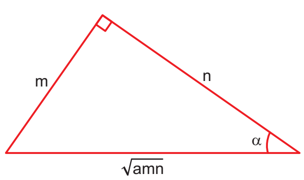
Piden:

$\cot\theta = \dfrac{AH}{BH} = \dfrac{4k}{2k} = 2$

$\therefore \cot\theta = 2$

Clave E

3.



Piden:

$L = \tan\alpha + \cot\alpha$

$\Rightarrow L = \dfrac{m}{n} + \dfrac{n}{m} = \dfrac{m^2+n^2}{mn}$ ...(1)

Por el teorema de Pitágoras:

$m^2 + n^2 = (\sqrt{amn})^2$

$m^2 + n^2 = amn$ ...(2)

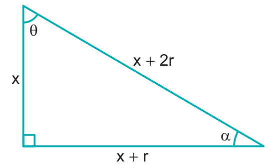
Reemplazando (2) en (1):

$L = \dfrac{m^2+n^2}{mn} = \dfrac{amn}{mn} = a$

$\therefore L = a$

Clave D

4.



Por el teorema de Pitágoras:

$x^2 + (x+r)^2 = (x+2r)^2$

$x^2 - 2xr - 3r^2 = 0$

x ⟍⟋ $-3r$

x ⟋⟍ r

$(x-3r)(x+r) = 0$

$\Rightarrow x = 3r \ \vee\ x = -r$

Del gráfico: $x > 0 \Rightarrow x = 3r$

Además: $\theta > \alpha$

Piden: la cosecante del menor ángulo agudo.

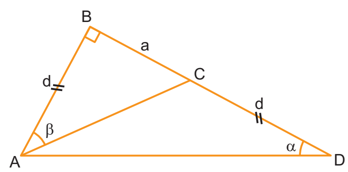
$\csc\alpha = \dfrac{x+2r}{x} = \dfrac{(3r)+2r}{(3r)}$

$\Rightarrow \csc\alpha = \dfrac{5r}{3r} = \dfrac{5}{3}$

$\therefore \csc\alpha = \dfrac{5}{3}$

Clave C

5.



$\cot\alpha = \dfrac{a+d}{d}$

$\tan\beta = \dfrac{a}{d}$

Piden:

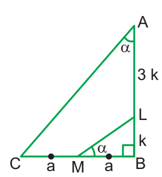
$\cot\alpha - \tan\beta = \dfrac{a+d}{d} - \dfrac{a}{d}$

$\Rightarrow \cot\alpha - \tan\beta = \dfrac{a+d-a}{d} = \dfrac{d}{d} = 1$

$\therefore \cot\alpha - \tan\beta = 1$

Clave A

6.



Por dato: $AL = 3LB$

Si: $LB = k \Rightarrow AL = 3k$

En el ⊿MBL: $\tan\alpha = \dfrac{k}{a}$ ...(1)

En el ⊿ABC: $\tan\alpha = \dfrac{2a}{4k} = \dfrac{a}{2k}$ ...(2)

De (1) y (2):

$\dfrac{k}{a} = \dfrac{a}{2k} \Rightarrow a^2 = 2k^2 \Rightarrow a = \sqrt{2}\,k$

En el ⊿MBL, por el teorema de Pitágoras:

$(ML)^2 = a^2 + k^2 = 2k^2 + k^2 = 3k^2$

$\Rightarrow ML = \sqrt{3}\,k$

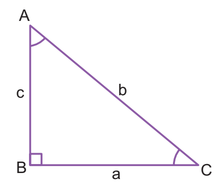
Piden:

$\cos\alpha = \dfrac{MB}{ML} = \dfrac{a}{\sqrt{3}\,k} = \dfrac{\sqrt{2}\,k}{\sqrt{3}\,k}$

$\therefore \cos\alpha = \sqrt{\dfrac{2}{3}}$

Clave A

7.



Piden:

$K = \dfrac{\operatorname{sen}A}{\sec C} + \dfrac{\operatorname{sen}C}{\sec A}$

$K = \dfrac{\left(\frac{a}{b}\right)}{\left(\frac{b}{a}\right)} + \dfrac{\left(\frac{c}{b}\right)}{\left(\frac{b}{c}\right)}$

$\Rightarrow K = \dfrac{a^2}{b^2} + \dfrac{c^2}{b^2} = \dfrac{a^2+c^2}{b^2}$

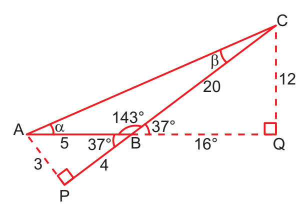
Por el teorema de Pitágoras: $a^2 + c^2 = b^2$

$\Rightarrow K = \dfrac{b^2}{b^2} = 1$

$\therefore K = 1$

Clave A

8.



Trazamos $\overline{AP} \perp \overline{CB} \ \wedge\ \overline{CQ} \perp \overline{AB}$

$m\angle ABP = m\angle CBQ = 37°$

⊿APB notable de 37° y 53°:

$AP = 3 \ \wedge\ PB = 4$

⊿CQB notable de 37° y 53°:

$CQ = 12 \ \wedge\ BQ = 16$

Luego:

$\cot\alpha = \dfrac{AQ}{QC} = \dfrac{21}{12} = \dfrac{7}{4}$

$\cot\beta = \dfrac{PC}{AP} = \dfrac{24}{3} = 8$

$\therefore \cot\alpha\cot\beta = \dfrac{7}{4} \cdot 8 = 14$

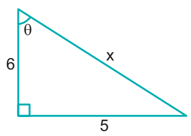
Clave D

9. $\cot\theta = \cos16°\sec37°$

$\cot\theta = \frac{24}{25}\,\frac{5}{4}$

$\cot\theta = \frac{6}{5}$

Luego:



Por T. de Pitágoras

$x^2 = 6^2 + 5^2$

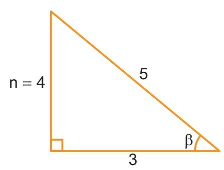
$x^2 = 61$

$x = \sqrt{61}$

$\therefore\ \sec\theta = \frac{\sqrt{61}}{6}$

Clave D

10.



Por dato: $\cos\beta = 0{,}6 = \frac{3}{5}$

Por el teorema de Pitágoras: $n = 4$

Piden:

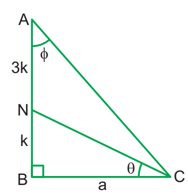
$K = \csc\beta + \cot\beta$

$\Rightarrow K = \frac{5}{4} + \frac{3}{4} = \frac{8}{4}$

$\therefore\ K = 2$

Clave B

11.



Piden:

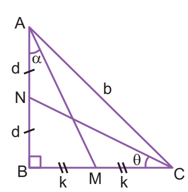
$P = \cot\theta\cot\phi$

$P = \left(\frac{a}{k}\right)\cdot\left(\frac{4k}{a}\right) = 4$

$\therefore\ P = 4$

Clave C

12.



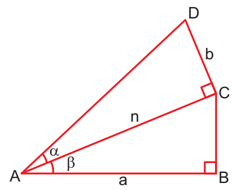
Piden:

$\tan\theta\tan\alpha = \left(\frac{d}{2k}\right)\cdot\left(\frac{k}{2d}\right)$

$\therefore\ \tan\theta\tan\alpha = \frac{1}{4}$

Clave D

13.



Piden:

$P = \cos\beta\cot\alpha + \tan\alpha\sec\beta$

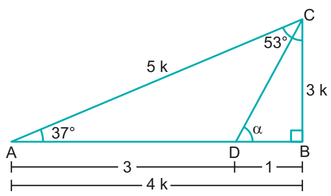
$\Rightarrow P = \left(\frac{a}{n}\right)\cdot\left(\frac{n}{b}\right) + \left(\frac{b}{n}\right)\cdot\left(\frac{n}{a}\right)$

$\Rightarrow P = \frac{a}{b} + \frac{b}{a} = \frac{a^2+b^2}{ab}$

$\therefore\ P = \frac{a^2+b^2}{ab}$

Clave B

14.



El ◺ABC es notable de 37° y 53°.

Del gráfico: $4k = 4 \Rightarrow k = 1$

Piden:

$\tan\alpha = \frac{3k}{1} = 3(1)$

$\therefore\ \tan\alpha = 3$

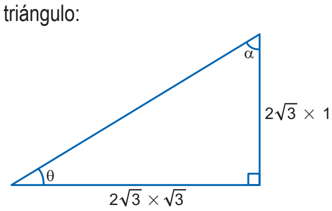
Clave C

PRACTIQUEMOS

Nivel 1 (página 17) Unidad 1

Comunicación matemática

1. Del triángulo:



◺notable de 30° y 60°

I. $\theta = 30° \wedge \alpha = 60°$

θ es la mitad de α

... (Verdadera)

II. $\text{sen}\theta = \text{sen}30° = \frac{1}{2}$

... (Falsa)

III. $\alpha = 60° = \frac{\pi\,\text{rad}}{180°}$

$\Rightarrow\ \alpha = \frac{\pi}{3}\text{rad}$

... (Falsa)

Clave C

2. De la expresión:

$\sec\alpha = \frac{1}{\text{sen}\theta}$

$\text{sen}\theta\ .\ \sec\alpha = 1$

θ y α complementarios:

$\cos\alpha\ .\ \sec\alpha = 1$

Razones recíprocas:

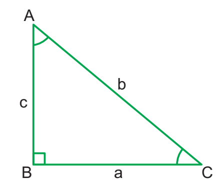
$\therefore\ \sec\alpha = \frac{1}{\text{sen}\theta}$

... (Correcto)

Clave B

Razonamiento y demostración

3.



Piden:

$K = \frac{\text{sen}A}{\sec C} + \frac{\text{sen}C}{\sec A}$

$K = \frac{\left(\frac{a}{b}\right)}{\left(\frac{b}{a}\right)} + \frac{\left(\frac{c}{b}\right)}{\left(\frac{b}{c}\right)}$

$\Rightarrow K = \frac{a^2}{b^2} + \frac{c^2}{b^2} = \frac{a^2+c^2}{b^2}$

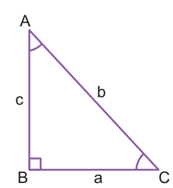
Por el teorema de Pitágoras: $a^2 + c^2 = b^2$

$\Rightarrow K = \frac{b^2}{b^2} = 1$

$\therefore\ K = 1$

Clave A

4.



Piden:

$J = (\sec^2C - \cot^2A)(\text{sen}^2C + \text{sen}^2A)$

$J = \left(\left(\frac{b}{a}\right)^2 - \left(\frac{c}{a}\right)^2\right)\left(\left(\frac{c}{b}\right)^2 + \left(\frac{a}{b}\right)^2\right)$

$\Rightarrow J = \left(\frac{b^2-c^2}{a^2}\right)\left(\frac{c^2+a^2}{b^2}\right)$

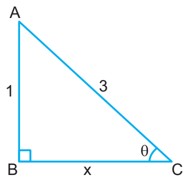
Por el teorema de Pitágoras: $b^2 = a^2 + c^2$

$\Rightarrow J = \left(\frac{a^2}{a^2}\right)\cdot\left(\frac{b^2}{b^2}\right)$

$\therefore\ J = 1$

Clave A

5.



Por el teorema de Pitágoras:

$3^2 = 1^2 + x^2$

$9 = 1 + x^2$

$\sqrt{8} = x$

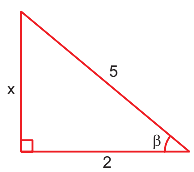
Piden:

$\cot\theta = \frac{x}{1} = \sqrt{8} = 2\sqrt{2}$

$\therefore \cot\theta = 2\sqrt{2}$

Clave B

6.



Por el teorema de Pitágoras:

$5^2 = x^2 + 2^2$

$25 = x^2 + 4$

$\sqrt{21} = x$

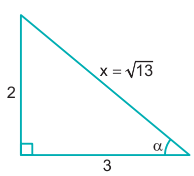
Piden:

$\text{sen}\beta = \frac{x}{5} = \frac{\sqrt{21}}{5}$

$\therefore \text{sen}\beta = \frac{\sqrt{21}}{5}$

Clave A

7.



Por el teorema de Pitágoras:

$x^2 = 2^2 + 3^2$

$x^2 = 13$

$x = \sqrt{13}$

Piden:

$J = \sec\alpha\csc\alpha$

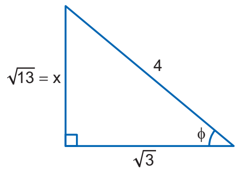
$J = \frac{\sqrt{13}}{3} \cdot \frac{\sqrt{13}}{2} = \frac{13}{6}$

$\therefore J = \frac{13}{6}$

Clave C

Resolución de problemas

8.



Por el teorema de Pitágoras:

$4^2 = x^2 + (\sqrt{3})^2$

$16 = x^2 + 3$

$\sqrt{13} = x$

Piden:

$J = 13\csc^2\phi + 3\tan^2\phi$

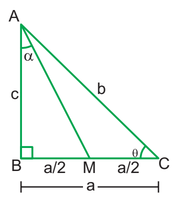
$J = 13\left(\frac{4}{\sqrt{13}}\right)^2 + 3\left(\frac{\sqrt{13}}{\sqrt{3}}\right)^2$

$J = 16 + 13 = 29$

$\therefore J = 29$

Clave D

9.



Piden:

$Q = \tan\alpha \,.\, \tan\theta$

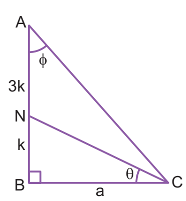
$Q = \frac{\frac{a}{2}}{c} \cdot \frac{c}{a}$

$\Rightarrow Q = \frac{a}{2} \cdot \frac{1}{a} = \frac{1}{2}$

$\therefore Q = \frac{1}{2}$

Clave E

10.



Piden:

$P = \cot\theta \,.\, \cot\phi$

$P = \left(\frac{a}{k}\right) \cdot \left(\frac{4k}{a}\right) = 4$

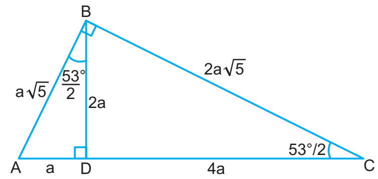
$\therefore P = 4$

Clave C

Nivel 2 (página 17) Unidad 1

Comunicación matemática

11. Del triángulo, sea BD = 2a



BDC notable de $\frac{53°}{2}$ y $\frac{127°}{2}$:

$DC = 4a \wedge BC = 2a\sqrt{5}$

◺BDA notable de $\frac{53°}{2}$ y $\frac{127°}{2}$:

$AD = a \wedge AB = a\sqrt{5}$

I. $\frac{AB}{DC} = \frac{a\sqrt{5}}{4a} = \frac{\sqrt{5}}{4}$ ... (Falso)

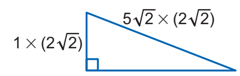
II. $\frac{DC}{AD} = \frac{4a}{a} = 4$ ... (Falso)

III. $\frac{BD}{AC} = \frac{2a}{5a} = \frac{2}{5}$ ... (Verdadero)

Clave A

12.

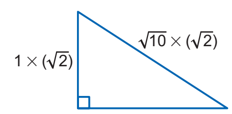
I.



Notable de 8° y 82°

...(a)

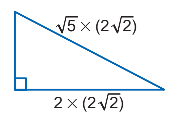
II.



Notable de $\frac{37°}{2}$ y $\frac{143°}{2}$

...(c)

III.



Notable de $\frac{53°}{2}$ y $\frac{127°}{2}$

...(b)

Clave E

Razonamiento y demostración

13. $\tan(a + b + y)\tan(2y - a - b) = 1$

$\tan(a + b + y) = \cot(2y - a - b)$

tan y cot, co–razones:

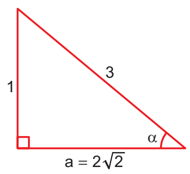
$a + b + y + 2y - a - b = 90°$

$3y = 90°$

$\therefore y = 30°$

Clave B

14.



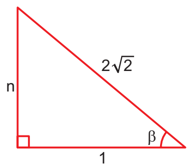
Por dato: $\text{sen}\alpha = \dfrac{1}{3}$

Por el teorema de Pitágoras: $a = 2\sqrt{2}$

Además: $\cos\beta = \tan\alpha$

$\Rightarrow \cos\beta = \dfrac{1}{a} = \dfrac{1}{2\sqrt{2}}$

Luego:



Por el teorema de Pitágoras: $n = \sqrt{7}$

$\Rightarrow \tan\beta = \dfrac{n}{1} = \dfrac{\sqrt{7}}{1}$

$\Rightarrow \tan\beta = \sqrt{7}$

Piden:

$Q = \sqrt{2}\cot\alpha + \sqrt{7}\tan\beta$

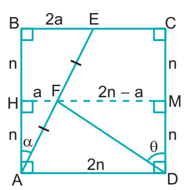
$\Rightarrow Q = \sqrt{2}\left(\dfrac{2\sqrt{2}}{1}\right) + \sqrt{7}(\sqrt{7})$

$\Rightarrow Q = 4 + 7 = 11$

$\therefore Q = 11$

Clave E

15.



Piden:

$Q = \tan\alpha + \tan\theta$

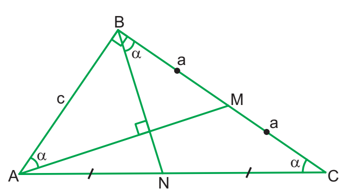
$\Rightarrow Q = \left(\dfrac{a}{n}\right) + \left(\dfrac{2n-a}{n}\right)$

$\Rightarrow Q = \dfrac{2n+a-a}{n} = \dfrac{2n}{n} = 2$

$\therefore\ Q = 2$

Clave B

16.



Por propiedad: $BN = AN = NC$

$\Rightarrow m\angle\ MCN = m\angle\ MBN = \alpha$

Del gráfico:

En el ⊿ABC: $\tan\alpha = \dfrac{c}{2a}$ ...(1)

En el ⊿ABM: $\tan\alpha = \dfrac{a}{c}$ ...(2)

Multiplicando (1) y (2):

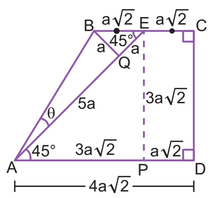
$\tan^2\alpha = \left(\dfrac{c}{2a}\right)\cdot\left(\dfrac{a}{c}\right) = \dfrac{1}{2}$

$\Rightarrow \tan\alpha = \dfrac{1}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\therefore\ \tan\alpha = \dfrac{\sqrt{2}}{2}$

Clave D

17.



Trazamos $\overline{EP} \perp \overline{AD}$.

Sea: $BE = EC = a\sqrt{2}$

$AD = 2BC$

$AD = 4a\sqrt{2}\ \wedge\ PD = a\sqrt{2}$

$\Rightarrow AP = 3a\sqrt{2}$

⊿ APE notable de 45°:

$AE = 6a$

Trazamos $\overline{BQ} \perp \overline{AE}$.

⊿ BQE notable de 45°:

$BQ = QE = a$

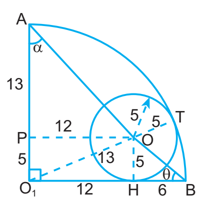
En ⊿ AQB:

$AQ = 5a \wedge BQ = a$

$\therefore\ \tan\theta = \dfrac{a}{5a} = \dfrac{1}{5}$

Clave D

18.



Del gráfico: $AO_1 = O_1B = 18$

Luego se deduce: $O_1O = 13$

En el ⊿O_1HO por el teorema de Pitágoras: $O_1H = 12$

Piden:

$\tan\theta + \cot\alpha = \dfrac{OH}{HB} + \dfrac{AP}{OP}$

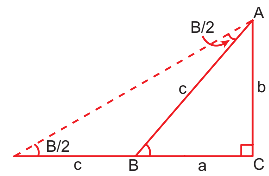
$\Rightarrow \tan\theta + \cot\alpha = \dfrac{5}{6} + \dfrac{13}{12}$

$\therefore \tan\theta + \cot\alpha = \dfrac{23}{12}$

Clave B

Resolución de problemas

19.



Prolongamos $\overline{CB}$ una distancia igual a AB.

$\Rightarrow \tan\dfrac{B}{2} = \dfrac{b}{c+a}$

Por dato:

$3 + 4\tan\dfrac{B}{2} = 3\csc A$

$3 + 4\left(\dfrac{b}{c+a}\right) = 3\left(\dfrac{c}{a}\right)$

$\dfrac{4b}{c+a} = 3\left(\dfrac{c}{a} - 1\right)$

$4b = \dfrac{3(c-a)(c+a)}{a}$

$\Rightarrow 4ba = 3(c^2 - a^2)$

En el ⊿ACB por el teorema de Pitágoras:

$a^2 + b^2 = c^2 \Rightarrow c^2 - a^2 = b^2$

Entonces:

$4ba = 3(b^2)$

$4a = 3b$

$\Rightarrow \dfrac{b}{a} = \dfrac{4}{3}$

Piden:

$M = \text{sen}B\sec A\cos B\csc A\tan B$

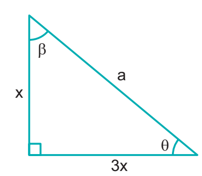
$M = \left(\dfrac{b}{c}\right)\cdot\left(\dfrac{c}{b}\right)\cdot\left(\dfrac{a}{c}\right)\cdot\left(\dfrac{c}{a}\right)\cdot\left(\dfrac{b}{a}\right)$

$\Rightarrow M = \dfrac{b}{a} = \dfrac{4}{3}$

$\therefore M = \dfrac{4}{3}$

Clave A

20.



Del gráfico: $\beta > \theta$

Por el teorema de Pitágoras:

$a^2 = x^2 + (3x)^2$

$a^2 = 10x^2$

$\Rightarrow a = x\sqrt{10}$

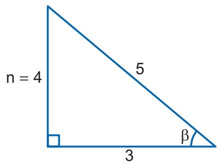
Piden:

$\csc\beta = \dfrac{a}{3x} = \dfrac{x\sqrt{10}}{3x} = \dfrac{\sqrt{10}}{3}$

$\therefore \csc\beta = \dfrac{\sqrt{10}}{3}$

Clave B

21.



Por dato: $\cos\beta = 0{,}6 = \dfrac{3}{5}$

Por el teorema de Pitágoras: $n = 4$

Piden:

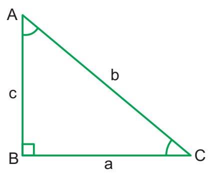
$K = \csc\beta + \cot\beta$

$\Rightarrow K = \dfrac{5}{4} + \dfrac{3}{4} = \dfrac{8}{4}$

$\therefore K = 2$

Clave B

22.



Por dato: $\tan A = 4\tan C$

$\Rightarrow \dfrac{a}{c} = 4\left(\dfrac{c}{a}\right)$

$a^2 = 4c^2 \Rightarrow a = 2c$

Por el teorema de Pitágoras:

$b = \sqrt{5}\,c$

Piden:

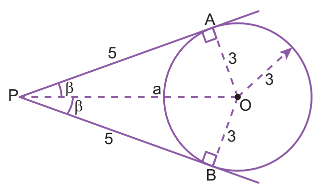
$\operatorname{sen}A\operatorname{sen}C = \left(\dfrac{a}{b}\right)\cdot\left(\dfrac{c}{b}\right)$

$\Rightarrow \operatorname{sen}A\operatorname{sen}C = \left(\dfrac{2c}{\sqrt{5}\,c}\right)\cdot\left(\dfrac{c}{\sqrt{5}\,c}\right) = \dfrac{2}{5}$

$\therefore \operatorname{sen}A\operatorname{sen}C = \dfrac{2}{5} = 0{,}4$

Clave D

23.



En el $\triangle$ PAO por el teorema de Pitágoras:

$a^2 = 5^2 + 3^2$

$a^2 = 34 \Rightarrow a = \sqrt{34}$

Piden:

$\operatorname{sen}\beta\cos\beta = \left(\dfrac{3}{a}\right)\cdot\left(\dfrac{5}{a}\right)$

$\Rightarrow \operatorname{sen}\beta\cos\beta = \dfrac{15}{a^2} = \dfrac{15}{34}$

$\therefore \operatorname{sen}\beta\cos\beta = \dfrac{15}{34}$

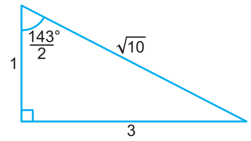
Clave B

Nivel 3 (página 18) Unidad 1

Comunicación matemática

24. En un $\triangle$ notable de $\dfrac{143^\circ}{2}$ y $\dfrac{37^\circ}{2}$:

$\operatorname{sen}\dfrac{143^\circ}{2} = \dfrac{3}{\sqrt{10}} = \dfrac{3\sqrt{10}}{10}$



$\therefore$ C es la correcta.

Clave C

25. De la expresión:

$2\cos^2\alpha + 2\tan^2\theta = 2\cos\alpha + 2\tan\theta - 1$

$\times 2$ $\quad 2\cos^2\alpha - 2\cos\alpha + 2\tan^2\theta - 2\tan\theta + 1 = 0$

$4\cos\alpha^2 - 4\cos\alpha + 1 + 4\tan^2\theta - 4\tan\theta + 1 = 0$

$(2\cos\alpha - 1)^2 + (2\tan\theta - 1)^2 = 0$

$\Rightarrow 2\cos\alpha - 1 = 0 \ \wedge\ 2\tan\theta - 1 = 0$

$\cos\alpha = \dfrac{1}{2} \ \wedge\ \tan\theta = \dfrac{1}{2}$

$\Rightarrow \alpha = 60^\circ \ \wedge\ \theta = \dfrac{53^\circ}{2}$

I. α es igual a 45°.

... (Incorrecta)

II. $\cot\theta$ es igual a 1.

... (Incorrecta)

III. El complemento de $\alpha = 60^\circ$ es igual a 30°, luego:

$r = \dfrac{60}{30} = 2$

r: razón de α y su complemento

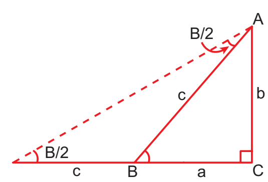
... (Incorrecta)

Clave E

Razonamiento y demostración

26. Primero:

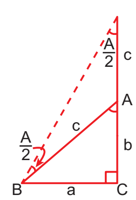
Prolongamos $\overline{CB}$ una distancia igual a AB.



$\Rightarrow \cot\dfrac{B}{2} = \dfrac{c+a}{b}$

Luego:

Prolongamos $\overline{CA}$ una distancia igual a AB.



$\Rightarrow \cot\dfrac{A}{2} = \dfrac{c+b}{a}$

Piden:

$K = \left(\dfrac{\cot\frac{B}{2} + 1}{\cot\frac{A}{2} + 1}\right)\tan B$

$K = \left(\dfrac{\left(\frac{c+a}{b}\right) + 1}{\left(\frac{c+b}{a}\right) + 1}\right)\left(\dfrac{b}{a}\right)$

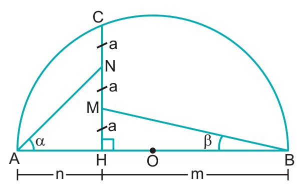
$K = \dfrac{\frac{c+a+b}{b}}{\frac{c+b+a}{a}}\cdot\dfrac{b}{a}$

$\Rightarrow K = \dfrac{a(a+b+c)}{b(a+b+c)}\cdot\dfrac{b}{a} = 1$

$\therefore K = 1$

Clave C

27.



Por propiedad: $CH^2 = AH\,.\,HB$

$\Rightarrow (3a)^2 = (n)\,.\,(m) \Rightarrow 9a^2 = nm$

Piden:

$\tan\alpha\tan\beta = \left(\dfrac{2a}{n}\right)\cdot\left(\dfrac{a}{m}\right)$

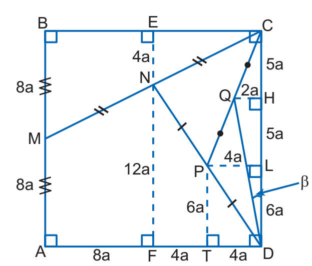
$\Rightarrow \tan\alpha\tan\beta = \dfrac{2a^2}{nm} = \dfrac{2a^2}{(9a^2)}$

$\therefore \tan\alpha\tan\beta = \dfrac{2}{9}$

Clave D

28. Sea el lado del cuadrado ABCD: 16a

Empleando el teorema de los puntos medios y la base media se obtiene la proporción de los segmentos.



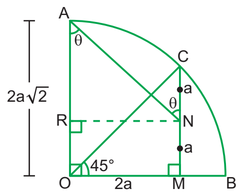
Piden: $\tan\beta$

En el $\triangle$DHQ: $\tan\beta = \dfrac{2a}{11a}$

$\therefore \tan\beta = \dfrac{2}{11}$

Clave B

29.



$m\overset{\frown}{AC} = m\overset{\frown}{CB} \Rightarrow m\angle COH = 45°$

⊿Trazamos $\overline{NR} \perp \overline{AO}$, si RO = a:

CM = 2a

OMC notable de 45°:

$AO = OC = 2a\sqrt{2}$

En ⊿ ARN; $\overline{AR} \,//\, \overline{NC}$; $m\angle NAR = \theta$

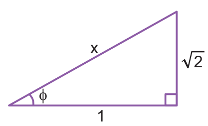
$AR = 2a\sqrt{2} - a \quad \wedge \quad RN = 2a$

$\therefore \cot\theta = \dfrac{2a\sqrt{2} - a}{2a} = \dfrac{2\sqrt{2} - 1}{2}$

Clave C

30. Por dato:

$\tan\phi = \sqrt{2}$



Por T. de Pitágoras.

$x^2 = (\sqrt{2})^2 + 1$

$x^2 = 3$

$x = \sqrt{3}$

En M:

$$M = \frac{\cos\phi\cot 60° + \csc^2\phi \,\mathrm{sen}^2 45°}{\cot\phi \sec 45° + \sec\phi \sec 30°} \cdot \frac{\csc^2\phi}{\tan 30°}$$

$$M = \frac{\frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} + \left(\frac{\sqrt{3}}{\sqrt{2}}\right)^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2}{\frac{1}{\sqrt{2}} \cdot \sqrt{2} + \sqrt{3} \cdot \frac{2}{\sqrt{3}}} \cdot \frac{\left(\frac{\sqrt{3}}{\sqrt{2}}\right)^2}{\frac{1}{\sqrt{3}}}$$

$$M = \frac{\frac{1}{3} + \frac{3}{4}}{1 + 2} \cdot \frac{3\sqrt{3}}{2}$$

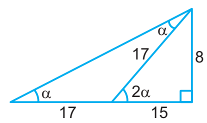
$$M = \frac{\frac{13}{12}}{3} \cdot \frac{3\sqrt{3}}{2}$$

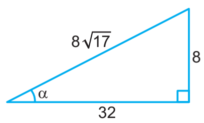
$$\therefore M = \frac{13\sqrt{3}}{24}$$

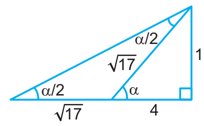
Clave C

31. $0 < \alpha < 45°$

$\cot 2\alpha = \dfrac{15}{8} = \dfrac{\text{c.a.}}{\text{c.o.}} \quad \Rightarrow \quad h = 17$







$E = (\sqrt{17} - 4)\cot\dfrac{\alpha}{2}$

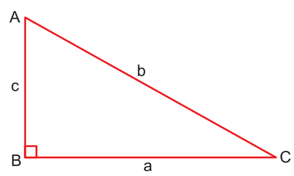
$E = (\sqrt{17} - 4)(\sqrt{17} + 4) = (\sqrt{17})^2 - 4^2$

$\therefore E = 1$

Clave A

Resolución de problemas

32. Del enunciado; sea ⊿ ABC:



donde $a > c$;

Datos:

$$\begin{array}{rcl} a + b &=& 27 \\ a - c &=& 3 \\ \hline b + c &=& 24 \\ b &=& 24 - c \end{array} \quad \ominus \qquad ... (1)$$

También:

$a - c = 3$

$a = c + 3 \qquad ... (2)$

Por teorema de Pitágoras:

$a^2 + c^2 = b^2$

De (1) y (2)

$(c + 3)^2 + c^2 = (24 - c)^2$

$c^2 = (24 - c)^2 - (c + 3)^2$

$c^2 = (24 - c + c + 3)(24 - c - c - 3)$

$c^2 = 27(21 - 2c)$

$c^2 = 27 \,.\, 21 - 27 \,.\, 2c$

$c^2 + 2 \,.\, 27c = 27 \,.\, 21$

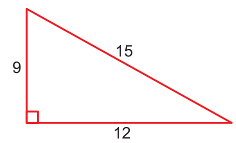
$c^2 + 2 \,.\, 27c + 27^2 = 27 \,.\, 21 + 27^2$

$(c + 27)^2 = 27(48)$

$c + 27 = 36$

$\Rightarrow c = 9$

En ⊿ ABC:

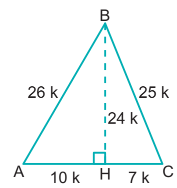


⊿ Notable de 37° y 53°

$\therefore \tan 37° = \dfrac{3}{4}$

Clave A

33. Del enunciado:



Trazamos $\overline{BH} \perp \overline{AC}$

$\tan A = 2{,}4 = \dfrac{12}{5} \wedge \cos C = 0{,}28 = \dfrac{7}{25}$

Luego:

⊿BHC notable de 16° y 74°; para BC = 25k:

$HC = 7k \wedge BH = 24k$

En el ⊿ AHB:

$\overline{BH} = 24k \wedge AH = 10k \wedge AB = 26k$

Sea:

2p: perímetro de ABC

$2p = 26k + 25k + 10k + 7k$

$204 = 68k$

$k = 3$

Nos piden:

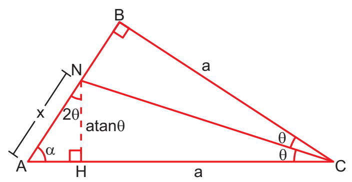
$AB = 26k$

$\therefore AB = 26 \,.\, 3 = 78$ cm

Clave B

APLICAMOS LO APRENDIDO (página 20) Unidad 1

1.



Por el teorema de la bisectriz: BC = HC = a

En el ◺ ABC: $2\theta + \alpha = 90°$

$\Rightarrow m\angle\ ANH = 2\theta$

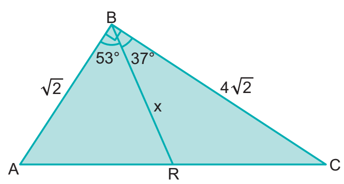
En el ◺ AHN: $\sec 2\theta = \dfrac{x}{a\tan\theta}$

$\Rightarrow a\tan\theta\sec 2\theta = x$

$\therefore\ x = a\tan\theta\sec 2\theta$

Clave B

2.



Del gráfico:

$A_{\triangle ABR} + A_{\triangle RBC} = A_{\triangle ABC}$

$$\frac{(\sqrt{2})(x)}{2}.\text{sen}53° + \frac{(x)(4\sqrt{2})}{2}.\ \text{sen}37° = \frac{(\sqrt{2})(4\sqrt{2})}{2}$$

$$\frac{\sqrt{2}x}{2}.\frac{4}{5} + 2\sqrt{2}x.\frac{3}{5} = 4$$

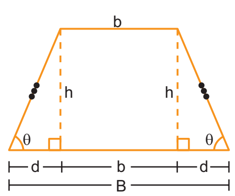
$$\frac{2\sqrt{2}x}{5} + \frac{6\sqrt{2}x}{5} = 4$$

$$\frac{8\sqrt{2}x}{5} = 4$$

$$\therefore x = \frac{5\sqrt{2}}{4}$$

Clave C

3.



Piden el área del trapecio isósceles (A).

$A = \left(\dfrac{B+b}{2}\right)h$ ...(1)

Del gráfico: 2d + b = B

$\Rightarrow 2d = B - b \Rightarrow d = \dfrac{B-b}{2}$

Luego: $h = d\tan\theta$

$\Rightarrow h = \left(\dfrac{B-b}{2}\right)\tan\theta$ ...(2)

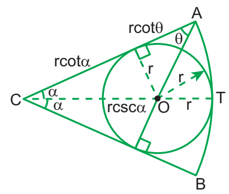
Reemplazando (2) en (1):

$$A = \left(\frac{B+b}{2}\right)\left(\frac{B-b}{2}\right)\tan\theta$$

$$\therefore\ A = \left(\frac{B^2-b^2}{4}\right)\tan\theta$$

Clave C

4.



Sea r: el radio de la circunferencia.

Del gráfico, se cumple: CA = CT

$\Rightarrow r\cot\alpha + r\cot\theta = r\csc\alpha + r$

$\cot\alpha + \cot\theta = \csc\alpha + 1$

$$\frac{\cot\alpha + \cot\theta}{\csc\alpha + 1} = 1$$

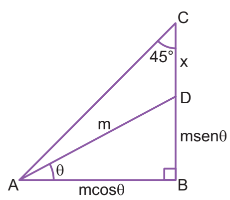
Piden:

$$K = \frac{\cot\alpha + \cot\theta}{1 + \csc\alpha} = 1$$

$\therefore\ K = 1$

Clave A

5.



El ◺ABC es isósceles, entonces: AB = BC

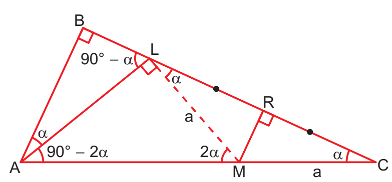
$\Rightarrow m\cos\theta = m\,\text{sen}\theta + x$

$x = m\cos\theta - m\,\text{sen}\theta$

$\therefore\ x = m(\cos\theta - \text{sen}\theta)$

Clave D

6.



$\overline{MR}$: mediatriz de $\overline{LC}$.

$\Rightarrow LM = MC = a$

Además, se deduce: $m\angle ALM = 90°$

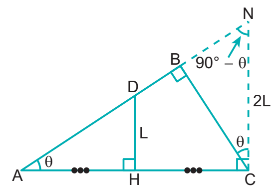
En el ◺MLA: $\sec 2\alpha = \dfrac{AM}{LM}$

$\Rightarrow \sec 2\alpha = \dfrac{AM}{a}$

$\therefore\ AM = a\sec 2\alpha$

Clave A

7.



Por dato: $\overline{HD}$ es mediatriz de $\overline{AC}$.

$\Rightarrow AH = HC$

Por C trazamos una paralela a $\overline{HD}$.

Entonces; por el teorema de la base media:

$DH = \dfrac{NC}{2} \Rightarrow L = \dfrac{NC}{2} \Rightarrow NC = 2L$

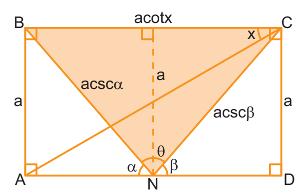
En el ◺CBN: $\cos\theta = \dfrac{BC}{2L}$

$\Rightarrow BC = \cos\theta(2L)$

$\therefore\ BC = 2L\cos\theta$

Clave A

8.



Del gráfico:

$$A_{\triangle BNC} = \frac{(\text{base})(\text{altura})}{2} = \frac{(a\cot x)(a)}{2}$$

$$\Rightarrow A_{\triangle BNC} = \frac{a^2}{2}\cot x \qquad ...(1)$$

$$A_{\triangle BNC} = \frac{(BN)(NC)}{2}.\ \text{sen}\theta = \frac{(a\csc\alpha)(a\csc\beta)}{2}.\ \text{sen}\theta$$

$$\Rightarrow A_{\triangle BNC} = \frac{a^2}{2}(\csc\alpha\ .\ \csc\beta\ .\ \text{sen}\theta) \qquad ...(2)$$

De (1) y (2):

$$\frac{a^2}{2}\cot x = \frac{a^2}{2}(\csc\alpha\ .\ \csc\beta\ .\ \text{sen}\theta)$$

$$\Rightarrow \cot x = \csc\alpha\ .\ \csc\beta.\ \text{sen}\theta$$

$$\left(\frac{\cos x}{\text{sen}x}\right) = \csc\alpha\ .\ \csc\beta\ .\ \left(\frac{1}{\csc\theta}\right)$$

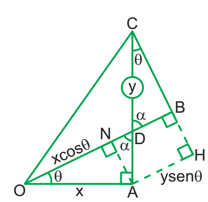
$$\left(\frac{1}{\text{sen}x}\right).\ \csc\theta = \csc\alpha\ .\ \csc\beta\ .\ \left(\frac{1}{\cos x}\right)$$

$$\csc x\ .\ \csc\theta = \underbrace{\csc\alpha\ .\ \csc\beta\ .\ \sec x}_{P}$$

$\therefore\ P = \csc x\csc\theta$

Clave E

9.

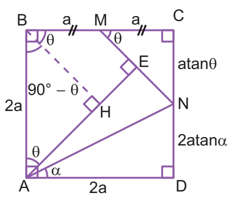


En el ⊿OAD: $\theta + \alpha = 90°$
Además: $NB = AH = y\,\text{sen}\theta$

Piden:
$OB = ON + NB$
$\Rightarrow OB = ON + AH = x\cos\theta + y\,\text{sen}\theta$
$\therefore\ OB = x\cos\theta + y\,\text{sen}\theta$

Clave D

10.

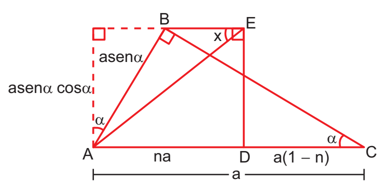


Se traza:
$\overline{BH} \perp \overline{AE} \Rightarrow \overline{BH} \,//\, \overline{MN}$
Entonces: $m\angle HBM = m\angle EMC = \theta$
Luego: $CD = CN + ND$
$\Rightarrow 2a = a\tan\theta + 2a\tan\alpha$
$2 = \tan\theta + 2\tan\alpha$
$\Rightarrow \tan\theta = 2(1 - \tan\alpha)$

Piden:
$$\frac{\tan\theta}{1-\tan\alpha} = \frac{2(1-\tan\alpha)}{1-\tan\alpha} = 2$$
$$\therefore \frac{\tan\theta}{1-\tan\alpha} = 2$$

Clave C

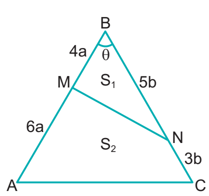
11.



$$n\tan x = \cancel{n}\left(\frac{\cancel{a}\text{sen}\alpha\cos\alpha}{\cancel{n}\cancel{a}}\right) = \text{sen}\alpha\cos\alpha$$

Clave B

12.



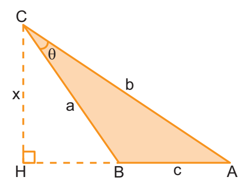
Del gráfico:
$$S_1 = \frac{(4a)(5b)}{2}\,\text{sen}\theta \Rightarrow S_1 = 10ab\,\text{sen}\theta$$
$$S_1 + S_2 = \frac{(10a)\cdot(8b)}{2}\,\text{sen}\theta$$
$$S_1 + S_2 = 40ab\,\text{sen}\theta$$

Entonces:
$(10ab\,\text{sen}\theta) + S_2 = 40ab\,\text{sen}\theta$
$\Rightarrow S_2 = 30ab\,\text{sen}\theta$

Piden:
$$\frac{S_2}{S_1} = \frac{30ab\,\text{sen}\theta}{10ab\,\text{sen}\theta} = \frac{3}{1}$$
$$\therefore \frac{S_2}{S_1} = 3$$

Clave C

13.



Empleando áreas:
- $A_{\Delta ABC} = \dfrac{(AB)\cdot(CH)}{2} = \dfrac{(c)\cdot(x)}{2}$

$\Rightarrow A_{\Delta ABC} = \dfrac{cx}{2}$ ...(I)

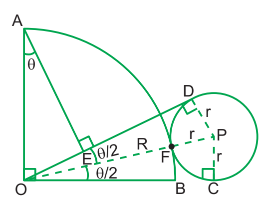
- $A_{\Delta ABC} = \dfrac{(CB)\cdot(CA)}{2}\text{sen}\theta$

$A_{\Delta ABC} = \dfrac{(a)\cdot(b)}{2}\text{sen}\theta$ ...(II)

Igualando (I) y (II):
$$\frac{cx}{2} = \frac{ab}{2}\text{sen}\theta$$
$$\therefore x = \frac{ab}{c}\text{sen}\theta$$

Clave C

14.



⊿ODP: $OP = DP\csc\dfrac{\theta}{2}$
$OF + FP = r\csc\dfrac{\theta}{2}$
$R + r = r\csc\dfrac{\theta}{2}$
$R = r\left(\csc\dfrac{\theta}{2} - 1\right)$

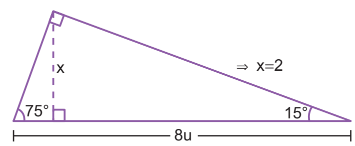
⊿AEO: $AO = r$
$AO = r\left(\csc\dfrac{\theta}{2} - 1\right)$
$AE = AO\cos\theta$
$AE = r\left(\csc\dfrac{\theta}{2} - 1\right)\cos\theta$

Clave E

PRACTIQUEMOS
Nivel 1 (página 22) Unidad 1

Comunicación matemática

1.
I.

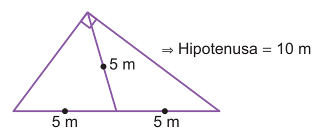


(Falsa)

II. La medida de la hipotenusa siempre es mayor que la medida de los catetos.
(Falsa)

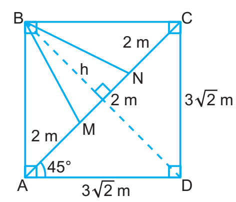
III. Existen más de 367 formas para demostrar el teorema de Pitágoras.
(Falsa)

IV.



(Verdadera)

2.

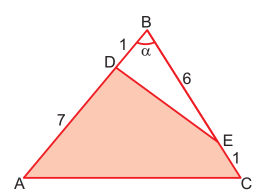


- $AC = 3\sqrt{2}\sec 45°$
  $AC = 6$ m
- $MN = \dfrac{AC}{3} = 2$ m
  $h = 3$ m

$\Rightarrow S = \dfrac{2.3}{2}$
$S = 3\ m^2$ $\quad\therefore S_{\Delta MBN} = 3\ m^2$

Razonamiento y demostración

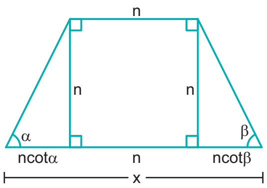
3.



Del gráfico:
$A_{somb.} = A_{\Delta ABC} - A_{\Delta DBE}$
$A_{somb.} = \dfrac{8.7}{2}\,.\,\text{sen}\alpha - \dfrac{1.6}{2}\,.\,\text{sen}\alpha$
$A_{somb.} = 28\text{sen}\alpha - 3\text{sen}\alpha$
$\therefore A_{somb.} = 25\text{sen}\alpha$

Clave C

4.



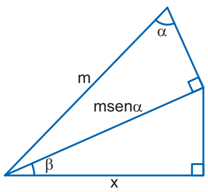
Del gráfico:

$x = n\cot\alpha + n + n\cot\beta$

$\therefore x = n(\cot\alpha + \cot\beta + 1)$

Clave B

5.



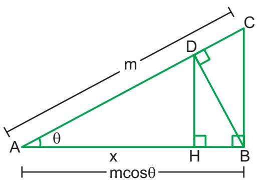
Del gráfico:

$x = (m\,\text{sen}\alpha)\cos\beta$

$\therefore x = m\,\text{sen}\alpha\cos\beta$

Clave B

6.



En el ⊿ADB:

$AD = (m\cos\theta)\cos\theta \Rightarrow AD = m\cos^2\theta$

En el ⊿AHD:

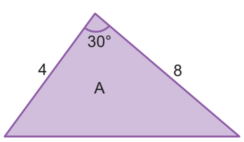
$x = AD\cos\theta$

$\Rightarrow x = (m\cos^2\theta)\cos\theta$

$\therefore x = m\cos^3\theta$

Clave D

7.



Piden:

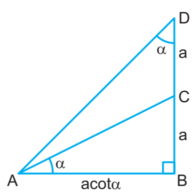
$A = \dfrac{4 . 8}{2} . \text{sen}30°$

$A = 16 . \left(\dfrac{1}{2}\right) = 8$

$\therefore A = 8$

Clave C

8.



Del gráfico:

$BD = AB\cot\alpha$

$\Rightarrow 2a = (a\cot\alpha)\cot\alpha$

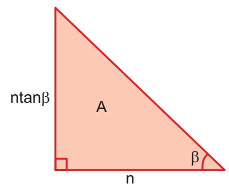
$2 = \cot^2\alpha$

$\therefore \cot\alpha = \sqrt{2}$

Clave C

Resolución de problemas

9.



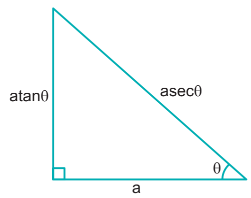
Piden:

$A = \dfrac{(n\tan\beta) . (n)}{2} = \dfrac{n^2}{2}\tan\beta$

$\therefore A = \dfrac{n^2}{2}\tan\beta$

Clave A

10.



Piden: el perímetro del triángulo (2p).

$2p = a + a\tan\theta + a\sec\theta$

$\therefore 2p = a(\tan\theta + \sec\theta + 1)$

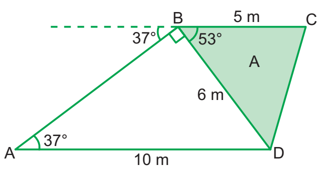
Clave B

Nivel 2 (página 23) Unidad 1

Comunicación matemática

11.

12.



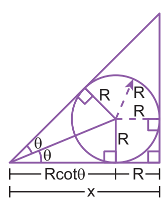
$\Rightarrow A = \dfrac{5.6}{2}\ \text{sen}53°$

$A = 15 . \dfrac{4}{5}$

$\therefore A = 12\ m^2$

Razonamiento y demostración

13.



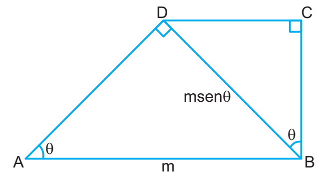
Del gráfico:

$x = R\cot\theta + R$

$\therefore x = R(\cot\theta + 1)$

Clave C

14.



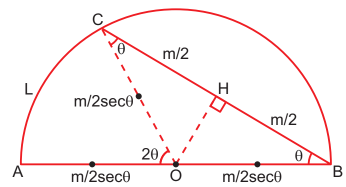
Del gráfico:

$CD = BD\text{sen}\theta = (m\text{sen}\theta)\text{sen}\theta$

$\therefore CD = m\text{sen}^2\theta$

Clave D

15.



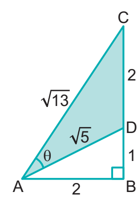
Piden:

$L = (2\theta) . \left(\dfrac{m}{2}\sec\theta\right)$

$\therefore L = \theta m\sec\theta$

Clave C

16.



Por el teorema de Pitágoras:

$AD = \sqrt{5} \ \wedge\ AC = \sqrt{13}$

Luego:

$A_{\Delta ADC} = \dfrac{(\text{base})(\text{altura})}{2} = \dfrac{(2)(2)}{2}$

$\Rightarrow A_{\Delta ADC} = 2 \qquad ...(1)$

$A_{\Delta ADC} = \dfrac{(AD)(AC)}{2} . \text{sen}\theta = \dfrac{(\sqrt{5})(\sqrt{13})}{2} . \text{sen}\theta$

$\Rightarrow A_{\Delta ADC} = \dfrac{\sqrt{65}}{2}\text{sen}\theta \qquad ...(2)$

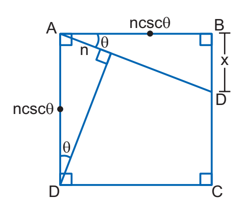
De (1) y (2):

$2 = \dfrac{\sqrt{65}}{2}\text{sen}\theta$

$\therefore \text{sen}\theta = \dfrac{4}{\sqrt{65}}$

Clave D

17.



En el ◺ABD:

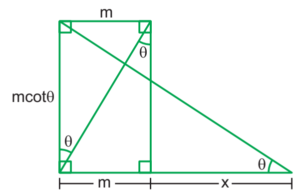
$BD = AB\tan\theta$

$\Rightarrow x = (n\csc\theta)\tan\theta$

$\therefore x = n\tan\theta\csc\theta$

Clave C

18.



Del gráfico:

$\cot\theta = \dfrac{m + x}{m\cot\theta}$

$\Rightarrow m + x = m\cot^2\theta$

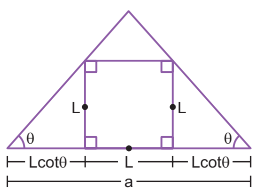
$x = m\cot^2\theta - m$

$\therefore x = m(\cot^2\theta - 1)$

Clave D

Resolución de problemas

19.



Del gráfico:

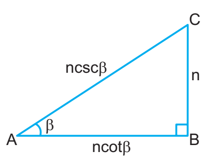
$L\cot\theta + L + L\cot\theta = a$

$L(2\cot\theta + 1) = a$

$\therefore L = \dfrac{a}{2\cot\theta + 1}$

Clave B

20.



Piden: el perímetro (2p) del triángulo.

$2p = CB + AB + AC$

$\Rightarrow 2p = n + n\cot\beta + n\csc\beta$

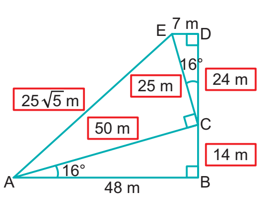
$\therefore 2p = n(1 + \cot\beta + \csc\beta)$

Clave D

Nivel 3 (página 24) Unidad 1

Comunicación matemática

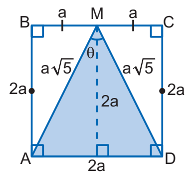
21.



22. V; V; V; F

Razonamiento y demostración

23.



Por el teorema de Pitágoras:

$AM = MD = a\sqrt{5}$

Luego:

$A_{\Delta AMD} = \dfrac{(\text{base})(\text{altura})}{2} = \dfrac{(2a)(2a)}{2}$

$\Rightarrow A_{\Delta AMD} = 2a^2$ ...(1)

$A_{\Delta AMD} = \dfrac{(AM)(MD)}{2} \cdot \operatorname{sen}\theta = \dfrac{(a\sqrt{5})(a\sqrt{5})}{2} \cdot \operatorname{sen}\theta$

$\Rightarrow A_{\Delta AMD} = \dfrac{5a^2}{2}\operatorname{sen}\theta$ ...(2)

De (1) y (2):

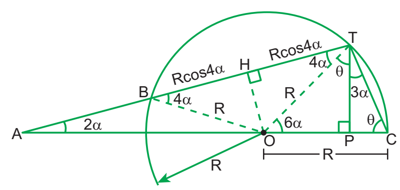
$2a^2 = \dfrac{5a^2}{2}\operatorname{sen}\theta$

$\Rightarrow 4 = 5\operatorname{sen}\theta$

$\therefore \operatorname{sen}\theta = \dfrac{4}{5}$

Clave C

24.



En el ◺TPC: $3\alpha + \theta = 90°$

En el ΔTOC: $2\theta + m\angle TOC = 180°$

$\Rightarrow 2(90° - 3\alpha) + m\angle TOC = 180°$

$\Rightarrow m\angle TOC = 6\alpha$

En el ΔATO: $2\alpha + m\angle ATO = 6\alpha$

$\Rightarrow m\angle ATO = 4\alpha$

Piden:

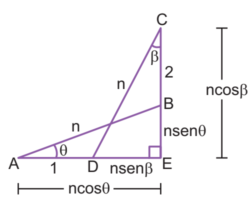
$BT = BH + HT$

$\Rightarrow BT = R\cos 4\alpha + R\cos 4\alpha$

$\therefore BT = 2R\cos 4\alpha$

Clave A

25.



Por dato: $AB = CD = n$

Del gráfico:

$n\cos\theta = 1 + n\operatorname{sen}\beta$

$\Rightarrow n\cos\theta - n\operatorname{sen}\beta = 1$ ...(I)

$n\cos\beta = 2 + n\operatorname{sen}\theta$

$\Rightarrow n\cos\beta - n\operatorname{sen}\theta = 2$ ...(II)

Dividiendo (I) y (II):

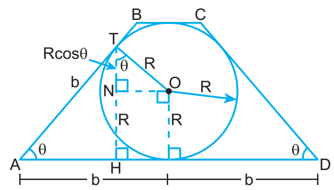
$\dfrac{n(\cos\theta - \operatorname{sen}\beta)}{n(\cos\beta - \operatorname{sen}\theta)} = \dfrac{1}{2}$

$\Rightarrow \dfrac{\cos\theta - \operatorname{sen}\beta}{\cos\beta - \operatorname{sen}\theta} = \dfrac{1}{2}$

$\therefore E = \dfrac{1}{2}$

Clave C

26.



En el ◺AHT:

$\operatorname{sen}\theta = \dfrac{TH}{AT} \Rightarrow TH = AT\operatorname{sen}\theta$

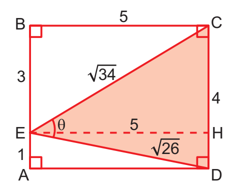
$\Rightarrow R\cos\theta + R = (b)\operatorname{sen}\theta$

$R(\cos\theta + 1) = b\operatorname{sen}\theta$

$\therefore R = \dfrac{b\operatorname{sen}\theta}{1 + \cos\theta}$

Clave D

27.



Por el teorema de Pitágoras:

$EC = \sqrt{34} \wedge ED = \sqrt{26}$

Luego:

$A_{\Delta CED} = \dfrac{(\text{base})(\text{altura})}{2} = \dfrac{(4)\cdot(5)}{2}$

$\Rightarrow A_{\Delta CED} = 10$ ...(1)

$A_{\Delta CED} = \dfrac{(EC)(ED)}{2} \cdot \operatorname{sen}\theta = \dfrac{(\sqrt{34})(\sqrt{26})}{2} \cdot \left(\dfrac{1}{\csc\theta}\right)$

$\Rightarrow A_{\Delta CED} = \dfrac{\sqrt{221}}{\csc\theta}$ ...(2)

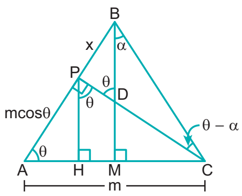
De (1) y (2):

$10 = \dfrac{\sqrt{221}}{\csc\theta} \Rightarrow \csc\theta = \dfrac{\sqrt{221}}{10} = \sqrt{2{,}21}$

$\therefore \csc\theta = \sqrt{2{,}21}$

Clave E

28.



Del gráfico:

$\overline{PH} \;//\; \overline{BM} \Rightarrow m\angle HPD = m\angle PDB = \theta$

Luego:

$PC = m\,\text{sen}\theta$

En el $\triangle BCD$: $\alpha + m\angle BCD = \theta$

$\Rightarrow m\angle BCD = \theta - \alpha$

En el $\triangle BPC$:

$PB = PC\tan(\theta - \alpha)$

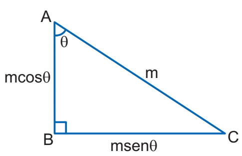
$\Rightarrow x = (m\,\text{sen}\theta)\tan(\theta - \alpha)$

$\therefore x = m\,\text{sen}\theta\tan(\theta - \alpha)$

Clave B

Resolución de problemas

29.



Piden: el perímetro del triángulo (2p).

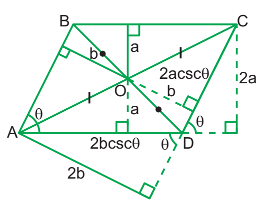
$2p = AC + BC + AB$

$\Rightarrow 2p = m + m\,\text{sen}\theta + m\cos\theta$

$\therefore 2p = m(1 + \text{sen}\theta + \cos\theta)$

Clave B

30.



Por dato: ABCD es un paralelogramo.

$\Rightarrow BC = AD = 2b\csc\theta$

$\Rightarrow AB = CD = 2a\csc\theta$

Piden: el perímetro del paralelogramo (2p).

$2p = AB + BC + CD + AD$

$2p = AB + BC + AB + BC = 2(AB + BC)$

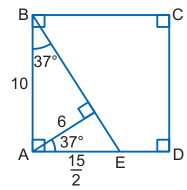
$\Rightarrow 2p = 2(2a\csc\theta + 2b\csc\theta)$

$\therefore 2p = 4(a + b)\csc\theta$

Clave A

MARATÓN MATEMÁTICA (página 25) Unidad 1

1.



$ED = 10 - \dfrac{15}{2}$

$\therefore\; ED = \dfrac{5}{2}$

Clave E

2. De la condición:

$n + m = 2p \;\Rightarrow\; n = 2p - m$

Sabemos:

$n^2 = m^2 + p^2 \;\Rightarrow\; (2p - m)^2 = m^2 + p^2$

$4p^2 - 4pm + m^2 = m^2 + p^2$

$3p^2 = 4pm$

$\dfrac{3}{4}p = m$

Luego:

$n = 2p - m = 2p - \dfrac{3}{4}p$

$n = \dfrac{5}{4}p \Rightarrow \dfrac{p}{n} = \cos M = \dfrac{4}{5}$

Clave A

3. Sabemos:

$\dfrac{S}{C} = \dfrac{9}{10} \Rightarrow \dfrac{mx - n}{mx + n} = \dfrac{9}{10}$

$10mx - 10n = 9mx + 9n$

$mx = 19n$

$\Rightarrow\; S = 18n$

Luego:

$18n\left(\dfrac{\pi}{180}\text{ rad}\right) = \dfrac{n\pi}{10}\text{ rad}$

El suplemento:

$\pi\text{ rad} - n\dfrac{\pi}{10}\text{ rad} = \left(\dfrac{10 - n}{10}\right)\pi\text{ rad}$

Clave B

4. Sean x e y los ángulos.

$\Rightarrow\; y + 60x = 1845$ ... (1)

$\dfrac{60}{\pi}.\, x\left(\dfrac{\pi}{180}\right) - \dfrac{y}{9} = 5 \Rightarrow \dfrac{x}{3} - \dfrac{y}{9} = 5$

$\Rightarrow\; 3x - y = 45$ ... (2)

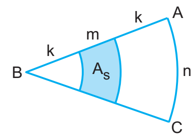
(1) + (2):

$63x = 1890 \quad\Rightarrow\; x = 30°$

$y = 45° \qquad \therefore\; x = 30°$

Clave B

5.



$(2k + m)\theta = n$

$A_S = \dfrac{1}{2}(k + m)^2\theta - \dfrac{1}{2}\theta k^2$

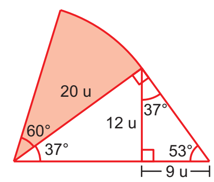
$A_S = \dfrac{1}{2}\theta((k + m)^2 - k)$

$A_S = \dfrac{1}{2}\theta(2k + m)m$

$A_S = \dfrac{1}{2}m\,.\,n$

Clave A

6.



$A_S = \dfrac{1}{2} \times \dfrac{\pi}{3} \times (20)^2 = 200\dfrac{\pi}{3}\text{ u}^2$

Clave D

7.

$\dfrac{S}{C} = \dfrac{9}{10} = k \Rightarrow S = 9k$

$C = 10k$

Reemplazamos:

$2(9k) - 18 = 10k + 30$

$18k = 10k + 48$

$8k = 48 \;\Rightarrow\; k = 6$

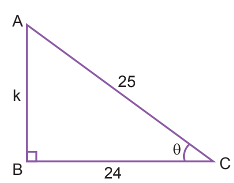
$S = 54°$

Luego:

$R = 54°\left(\dfrac{\pi}{180°}\right)\text{ rad} \;\Rightarrow\; R = \dfrac{3}{10}\pi\text{ rad}$

Clave C

8.



(θ: menor ángulo)

$25^2 = 24^2 + k^2 \Rightarrow k = 7$

$x = \sec\theta + \tan\theta$

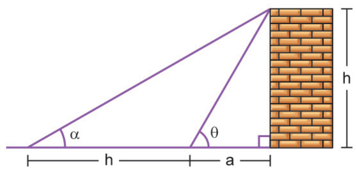
$x = \dfrac{25}{24} + \dfrac{7}{24} = \dfrac{32}{24} = \dfrac{4}{3}$

$\therefore\; x = \dfrac{4}{3}$

Clave C

APLICAMOS LO APRENDIDO (página 27) Unidad 2

1.



Por dato: $\tan\theta = 2$

$\Rightarrow \dfrac{h}{a} = 2 \Rightarrow h = 2a$

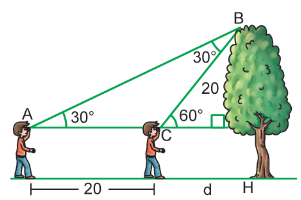
Piden:

$\cot\alpha = \dfrac{h+a}{h} = \dfrac{(2a)+a}{(2a)} = \dfrac{3a}{2a}$

$\therefore \cot\alpha = \dfrac{3}{2}$

Clave B

2.



Del gráfico:

El $\triangle ACB$ es isósceles: $AC = CB = 20$ m

En el ⊿CHB notable de 30° y 60°:

$CH = \dfrac{CB}{2} \Rightarrow d = \dfrac{20}{2} = 10$ m

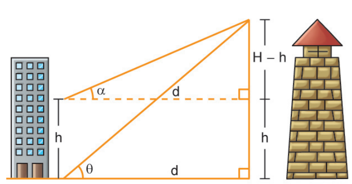
Piden:

$AH = 20 + d = 20 + 10$

$\therefore AH = 30$ m

Clave E

3. Sea la altura de la torre: H



Del gráfico:

$d = (H - h)\cot\alpha$ ...(1)

$d = H\cot\theta$ ...(2)

Igualando (1) y (2):

$(H - h)\cot\alpha = H\cot\theta$

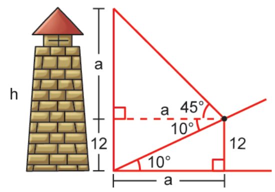
$H\cot\alpha - h\cot\alpha = H\cot\theta$

$H(\cot\alpha - \cot\theta) = h\cot\alpha$

$\therefore H = \dfrac{h\cot\alpha}{\cot\alpha - \cot\theta}$

Clave B

4. Sea la altura de la torre: h



Del gráfico:

$a = 12\cot 10°$

$a = 12(5{,}67) = 68{,}04$

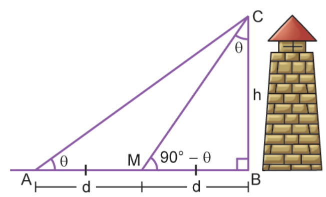
Además:

$h = a + 12$

$h = 68{,}04 + 12 \Rightarrow \therefore h = 80{,}04$ m

Clave C

5. Sea la altura de la torre: h



Del gráfico:

En el ⊿ABC: $\tan\theta = \dfrac{h}{2d}$ ...(I)

En el ⊿MBC: $\tan\theta = \dfrac{d}{h}$ ...(II)

Igualando (I) y (II):

$\dfrac{h}{2d} = \dfrac{d}{h}$

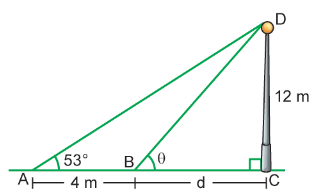
$h^2 = 2d^2 \Rightarrow h = \sqrt{2}\,d$

Reemplazando en (I):

$\Rightarrow \tan\theta = \dfrac{h}{2d} = \dfrac{\sqrt{2}\,d}{2d} = \dfrac{\sqrt{2}}{2} \Rightarrow \therefore \tan\theta = \dfrac{\sqrt{2}}{2}$

Clave B

6.



Del gráfico:

$d + 4 = 12 \,.\, \cot 53°$

$d + 4 = 12\left(\dfrac{3}{4}\right) = 9 \Rightarrow d = 5$

Luego en el ⊿BCD:

Por el teorema de Pitágoras: $BD = 13$

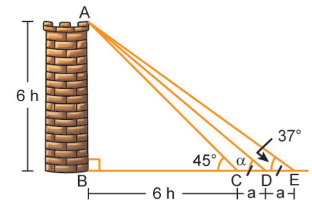
Piden: $\sec\theta$

$\sec\theta = \dfrac{BD}{d} = \dfrac{13}{5} = 2{,}6$

$\therefore \sec\theta = 2{,}6$

Clave C

7.



Sea la altura de la torre: 6h

Del ⊿ABC, notable 45°:

$AB = BC = 6h$

Del ⊿ABE, notable 37° y 53°:

$BE = 8h \Rightarrow a = h$

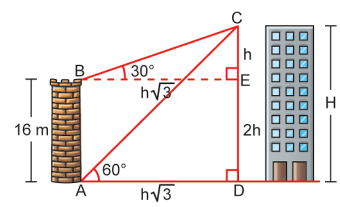
Piden: $\tan\alpha$

$\tan\alpha = \dfrac{AB}{BD} = \dfrac{6h}{6h + a} = \dfrac{6h}{6h + (h)} = \dfrac{6h}{7h}$

$\therefore \tan\alpha = \dfrac{6}{7}$

Clave E

8.



Sea la altura del edificio: H

Del ⊿BEC, notable 30° y 60°:

Si: $CE = h \Rightarrow BE = h\sqrt{3}$

Del ⊿ADC, notable 30° y 60°:

$AD = h\sqrt{3} \Rightarrow CD = 3h$

Del gráfico: $2h = 16$

$h = 8$

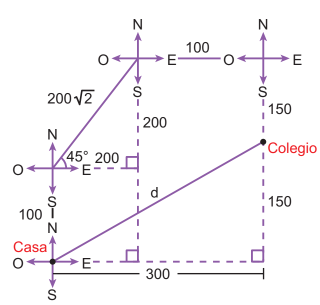
Piden:

$H = h + 2h = 8 + 2(8) = 24$

$\therefore H = 24$ m

Clave E

9.



Sea la distancia entre la casa y el colegio: d

Por el teorema de Pitágoras:

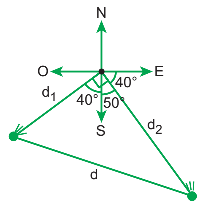
$d^2 = (150)^2 + (300)^2$

$d^2 = 112\,500 = 150^2 \,.\, 5$

$\therefore d = 150\sqrt{5}$ m

Clave C

10.



Por dato:

$d_1 = \left(15\dfrac{km}{h}\right)(4h) = 60$ km

$d_2 = \left(20\dfrac{km}{h}\right)(4h) = 80$ km

Sea d: la distancia de separación después de 4 horas.

Por el teorema de Pitágoras:

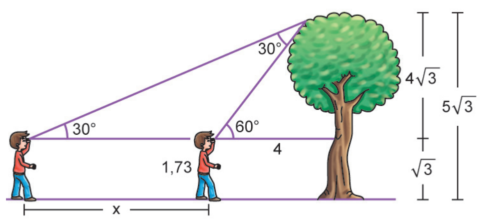
$d^2 = d_1^2 + d_2^2$

$d^2 = (60)^2 + (80)^2 = 10\ 000$

$\therefore d = 100$ km

Clave D

11.



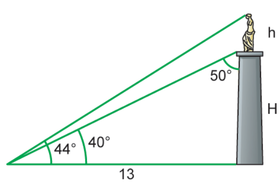
Del gráfico: $\tan 30° = \dfrac{4\sqrt{3}}{x+4} = \dfrac{\sqrt{3}}{3}$

$12 = x + 4$

$x = 8$ m

Clave B

12.



Del gráfico:

$H = 13\cot 50° = 13(0,84) = 10,92$ m

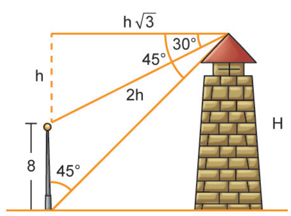
$h + H = 13\tan 44°$

$h = 13(0,97) - 10,92$

$h = 1,69$ m

Clave D

13.



- Del gráfico $\quad h + 8 = h\sqrt{3}$

$$h = \frac{8}{\sqrt{3}-1}\,\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)} = 4(\sqrt{3}+1)$$

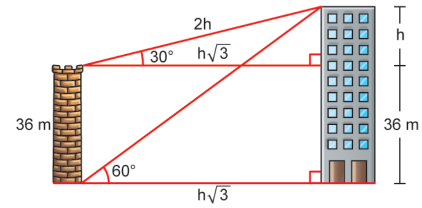
- Luego $\quad H = h + 8$

$H = 4(\sqrt{3}+1) + 8$

$H = 4(3+\sqrt{3})$ m

Clave A

14. Altura del edificio: H



- Del gráfico: $\quad h\sqrt{3} = (h+36)\cot 60°$

$h\sqrt{3} = \dfrac{\sqrt{3}}{3}(h+36)$

$3h = h + 36$

$3h = 36 \Rightarrow h = 18$

- Luego: $\quad H = 18 + 36 = 54$ m

Clave C

PRACTIQUEMOS

Nivel 1 (página 29) Unidad 2

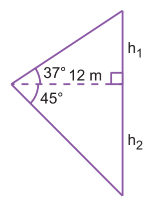
Comunicación matemática

1.

2.

Razonamiento y demostración

3.



- $h_1 = 12 \cdot \tan 37°$

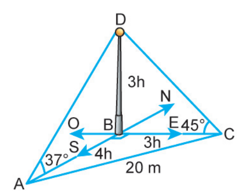
  $h_1 = 9$ m

- $h_2 = 12 \cdot \tan 45°$

  $h_2 = 12$ m

$\therefore\ H = h_1 + h_2 = 21$ m

4.



Sea la altura del poste: 3h

En el ⊿ABC, por el teorema de Pitágoras:

$(4h)^2 + (3h)^2 = 20^2$

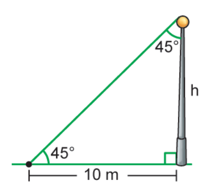
$h^2 = 16$

$h = 4$ m

$\therefore\ 3h = 12$ m

Resolución de problemas

5.



Sea h: la altura del poste

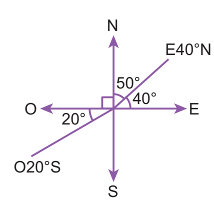
Del gráfico: $h = 10\tan 45°$

$\Rightarrow h = 10(1) = 10$

$\therefore\ h = 10$ m

Clave A

6.

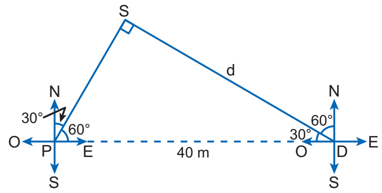


El menor ángulo formado por las direcciones será:

$20° + 90° + 50° = 160°$

Clave E

7.



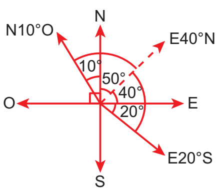
Del gráfico:
El ⊿PSD resulta ser notable de 30° y 60°.

$\Rightarrow d = 40\cos30° = 40\left(\dfrac{\sqrt{3}}{2}\right)$

$\therefore d = 20\sqrt{3} \approx 34{,}6$ m

Clave C

8.

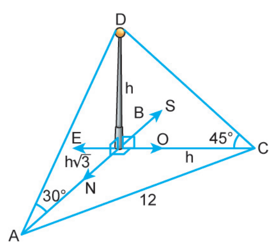


El menor ángulo formado por las direcciones N10°O y E20°S es 120°.

Luego la bisectriz de dicho ángulo tiene la dirección E40°N.

Clave D

9. Altura del poste: h



En el ⊿ABC por el teorema de Pitágoras:

$(h.\sqrt{3})^2 + (h)^2 = 144$

$3h^2 + h^2 = 144$

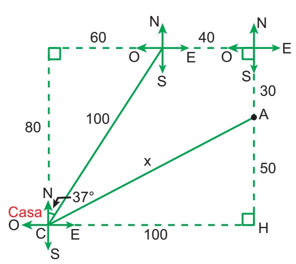
$4h^2 = 144$

$h^2 = 36$

$\therefore h = 6$ m

Clave C

10.



En el ⊿CHA, por el teorema de Pitágoras:

$x^2 = 50^2 + 100^2$

$\Rightarrow x^2 = 12\,500$

$\therefore x = 50\sqrt{5}$ m

Clave C

Nivel 2 (página 29) Unidad 2

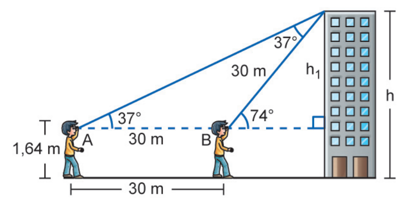
Comunicación matemática

11.

12.

Razonamiento y demostración

13. Del gráfico:

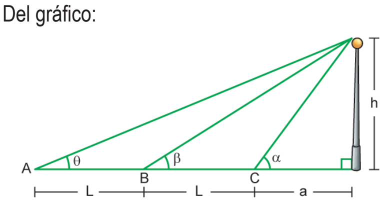


$h_1 = 30\text{sen}74° = 28{,}8$ m

$\therefore\ h = h_1 + 1{,}64 = 30{,}44$ m

Clave A

14. Del gráfico:



$C = (\cot\alpha + \cot\theta)\ .\ \tan\beta$

$C = \left(\dfrac{a}{h} + \dfrac{2L + a}{h}\right) . \dfrac{h}{L + a}$

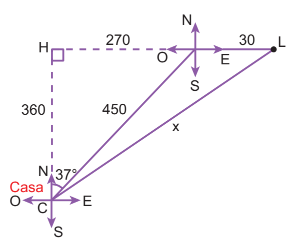
$C = \dfrac{2(L + a)}{h} \cdot \dfrac{h}{(L + a)}$

$\therefore\ C = 2$

Clave B

Resolución de problemas

15.



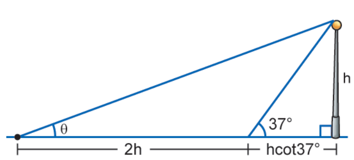
En el ⊿CHL, por el teorema de Pitágoras:

$x^2 = 360^2 + 300^2 \Rightarrow x^2 = 219\,600$

$\therefore x = 60\sqrt{61}$ m

Clave B

16.



Del gráfico.

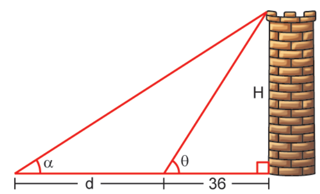
$\tan\theta = \dfrac{h}{2h + h\cot37°} = \dfrac{h}{h(2 + \cot37°)}$

$\Rightarrow \tan\theta = \dfrac{1}{2 + \left(\dfrac{4}{3}\right)} = \dfrac{1}{\dfrac{10}{3}} = \dfrac{3}{10}$

$\therefore \tan\theta = \dfrac{3}{10} = 0{,}3$

Clave E

17.



Por dato:

$\tan\theta = \dfrac{7}{12}\ \wedge\ \tan\alpha = \dfrac{1}{4}$

Del gráfico:

$H = 36\tan\theta = (d + 36)\tan\alpha$

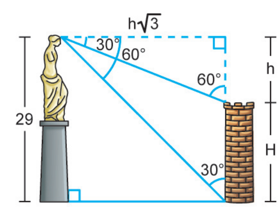
$\Rightarrow 36\left(\dfrac{7}{12}\right) = (d + 36)\left(\dfrac{1}{4}\right)$

$84 = d + 36$

$\therefore d = 48$ m

Clave E

18.



Del gráfico:

$H + h = h\sqrt{3}\cot30°$

$H + h = h\sqrt{3}(\sqrt{3}) = 3h$

$\Rightarrow H = 2h \Rightarrow h = \dfrac{H}{2}$

Luego:

$H + h = 29$

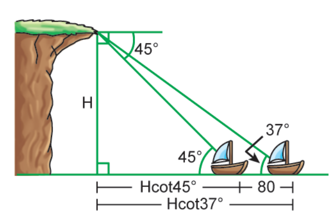
$\Rightarrow H + \dfrac{H}{2} = 29$

$\dfrac{3H}{2} = 29$

$\therefore H = \dfrac{58}{3}$ m

Clave B

19.



Del gráfico:

$H\cot 45° + 80 = H\cot 37°$

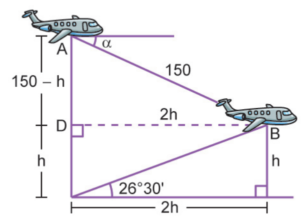
$H(1) + 80 = H\left(\frac{4}{3}\right)$

$80 = \frac{4H}{3} - H \Rightarrow 80 = \frac{H}{3}$

$\therefore H = 240$ m

Clave D

20.



Sabemos: $26°30' = \frac{53°}{2}$

Del gráfico:

En el ⊿ADB por el teorema de Pitágoras:

$(150 - h)^2 + (2h)^2 = 150^2$

$150^2 - 300h + 5h^2 = 150^2 \quad \Rightarrow h^2 = 60\,h$

$\therefore h = 60$ m

Clave B

Nivel 3 (página 30) Unidad 2

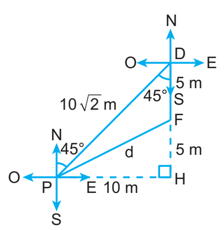
Comunicación matemática

21.

22.

Razonamiento y demostración

23.

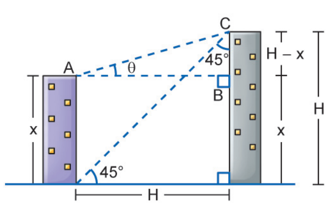


Aplicando T. de Pitágoras en el ⊿ PHF:

$d^2 = (10\text{ m})^2 + (5\text{ m})^2 \quad \therefore\ d = 5\sqrt{5}$ m

Clave E

24.



En el ⊿ABC: CB = (AB)tanθ

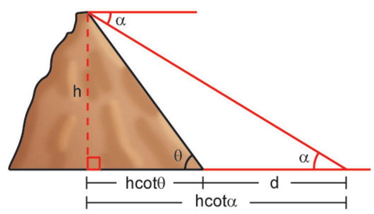
$\Rightarrow H - x = H\tan\theta$

$x = H - H\tan\theta \quad \Rightarrow \therefore\ x = H(1 - \tan\theta)$

Clave D

Resolución de problemas

25.



Del gráfico:

$h\cot\alpha = h\cot\theta + d$

$\Rightarrow h\cot\alpha - h\cot\theta = d$

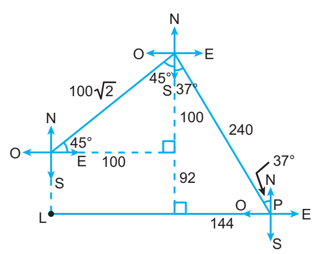
$h(\cot\alpha - \cot\theta) = d$

$h = \dfrac{d}{\cot\alpha - \cot\theta}$

$\therefore\ h = d(\cot\alpha - \cot\theta)^{-1}$

Clave A

26.



Piden la distancia del punto de partida al punto de llegada (PL).

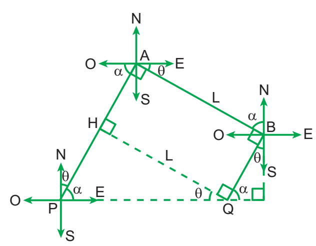
Del gráfico:

PL = 144 + 100 = 244

$\therefore$ PL = 244 km

Clave B

27.



Del gráfico: $\alpha + \theta = 90°$

Luego se deduce que:

$m\angle PAB = m\angle ABQ = 90°$

Trazamos: $\overline{QH} \perp \overline{PA}$

Entonces se forma el rectángulo ABQH.

En el ⊿PHQ: PQ = (HQ)secθ

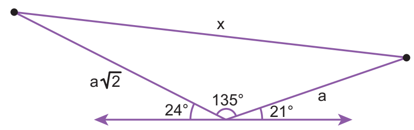
$\Rightarrow$ PQ = (L)secθ

$\therefore$ PQ = Lsecθ

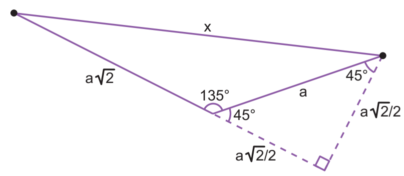
Clave D

28. Debemos considerar cuerdas rígidas sin deformación.

Del siguiente gráfico:



Entonces:



Por el teorema de Pitágoras:

$x^2 = \left(\frac{a\sqrt{2}}{2}\right)^2 + \left(\frac{3a\sqrt{2}}{2}\right)^2$

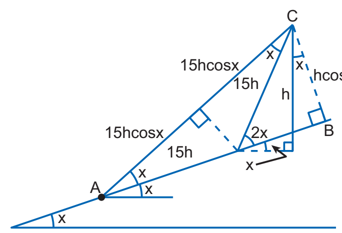
$\Rightarrow x^2 = \frac{2a^2}{4} + \frac{18a^2}{4} = \frac{20a^2}{4}$

$x^2 = 5a^2$

$\therefore\ x = a\sqrt{5}$ m

Clave D

29.



En el ⊿ABC: $\csc x = \dfrac{AC}{CB}$

$\Rightarrow \csc x = \dfrac{30h\cos x}{h\cos x} = 30 \Rightarrow \csc x = 30$

Piden:

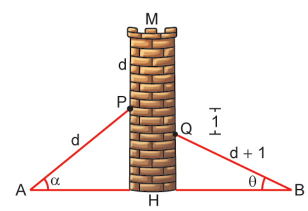
E = cscx – 15

$\Rightarrow$ E = (30) – 15

$\therefore$ E = 15

Clave D

30.



Del gráfico: la altura de la torre es MH.

$\Rightarrow$ MH = MP + PH

MH = d + dsenα = d(1 + senα) ...(1)

En el ⊿QHB: QH = (d + 1)senθ

Luego:

$PH = PQ + QH$

$d\text{sen}\alpha = 1 + (d+1)\text{sen}\theta$

$d\text{sen}\alpha = 1 + d\text{sen}\theta + \text{sen}\theta$

$\Rightarrow d\text{sen}\alpha - d\text{sen}\theta = \text{sen}\theta + 1$

$d(\text{sen}\alpha - \text{sen}\theta) = \text{sen}\theta + 1$

$$\Rightarrow d = \frac{\text{sen}\theta + 1}{\text{sen}\alpha - \text{sen}\theta} \qquad ...(2)$$

Reemplazando (2) en (1):

$$\Rightarrow MH = \left(\frac{\text{sen}\theta + 1}{\text{sen}\alpha - \text{sen}\theta}\right)(1 + \text{sen}\alpha)$$

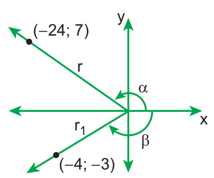
$$\therefore MH = \frac{(\text{sen}\theta + 1)(\text{sen}\alpha + 1)}{\text{sen}\alpha - \text{sen}\theta}$$

Clave B

RAZONES TRIGONOMÉTRICAS DE ÁNGULOS DE CUALQUIER MAGNITUD

APLICAMOS LO APRENDIDO (página 32) Unidad 2

1.



Del gráfico:

$r^2 = 7^2 + (-24)^2 \Rightarrow r = 25$

${r_1}^2 = (-3)^2 + (-4)^2 \Rightarrow r_1 = 5$

Piden: $P = \cos\beta - 5\cos\alpha$

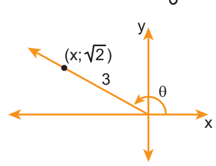
$$P = \left(\frac{x_1}{r_1}\right) - 5\left(\frac{x}{r}\right) = \left(\frac{-4}{5}\right) - 5\left(-\frac{24}{25}\right)$$

$$P = -\frac{4}{5} + \frac{24}{5} = \frac{20}{5}$$

$\therefore P = 4$

Clave C

2. Por dato: $\text{sen}\theta = \dfrac{\sqrt{2}}{3} \wedge \theta \in IIC$



Por radio vector:

$x^2 + (\sqrt{2})^2 = (3)^2$

$x^2 = 7 \Rightarrow x = \sqrt{7} \;\vee\; x = -\sqrt{7}$

Como: $x < 0 \Rightarrow x = -\sqrt{7}$

Piden:

$M = 2\cot^2\theta - \sqrt{7}\sec\theta$

$$M = 2\left(\frac{x}{y}\right)^2 - \sqrt{7}\left(\frac{r}{x}\right)$$

$$M = 2\left(\frac{-\sqrt{7}}{\sqrt{2}}\right)^2 - \sqrt{7}\left(\frac{3}{-\sqrt{7}}\right) = 7 + 3$$

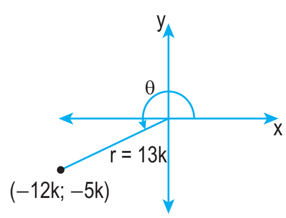
$\therefore M = 10$

Clave B

3. $\tan\theta = \dfrac{5}{12} > 0$

$\text{sen}\theta < 0$

Entonces: $\theta \in IIIC$



Piden:

$$R = 13\text{sen}\theta + 5\cot\theta = 13\left(\frac{y}{r}\right) + 5\left(\frac{x}{y}\right)$$

$$R = 13\left(\frac{-5k}{13k}\right) + 5\left(\frac{-12k}{-5k}\right) = -5 + 12 = 7$$

$\therefore R = 7$

Clave C

4. El radio vector en P será:

$r = \sqrt{(-3)^2 + 5^2} = \sqrt{34}$

$\Rightarrow x = -3;\ y = 5;\ r = \sqrt{34}$

Entonces:

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{34}}{-3} = -\frac{\sqrt{34}}{3}$$

$$\tan\theta = \frac{y}{x} = \frac{5}{-3} = -\frac{5}{3}$$

Reemplazando:

$$A = (\sqrt{34} - 5)\left[\frac{-\sqrt{34}}{3} + \left(\frac{-5}{3}\right)\right]$$

$$A = -(\sqrt{34} - 5)\left(\frac{\sqrt{34} + 5}{3}\right) = -\frac{1}{3}\left[(\sqrt{34})^2 - 5^2\right]$$

$$A = -\frac{9}{3} = -3$$

$\therefore A = -3$

Clave A

5. $$R = \frac{\tan180° + \cos360° + \text{sen}360°}{\text{sen}90° + \cos270°}$$

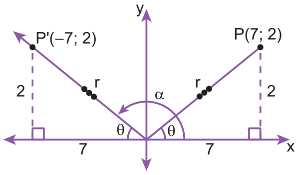
Reemplazando los valores correspondientes:

$$R = \frac{(0) + (1) + (0)}{(1) + (0)} = \frac{1}{1}$$

$\therefore R = 1$

Clave B

6.



Por simetría con respecto al eje y:
P' es (−7; 2)

Luego, por radio vector:

$r^2 = (-7)^2 + 2^2 = 49 + 4$

$r^2 = 53 \Rightarrow r = \sqrt{53}$

Piden:

$\cos\alpha = \frac{x}{r} = -\frac{7}{\sqrt{53}} \cdot \frac{\sqrt{53}}{\sqrt{53}}$

$\therefore \cos\alpha = -\frac{7\sqrt{53}}{53}$

Clave C

7. Por dato:

$\frac{4}{5}\text{sen}\alpha = \frac{1}{4} + \frac{1}{28} + \frac{1}{70} + \frac{1}{130}$

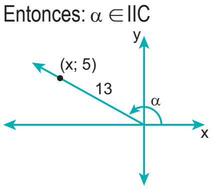
$\frac{4}{5}\text{sen}\alpha = \frac{4}{13} \Rightarrow \text{sen}\alpha = \frac{5}{13}$ ...(1)

Además: $\cos\alpha < 0$ ...(2)

De (1): $\alpha \in IC \vee \alpha \in IIC$

De (2): $\alpha \in IIC \vee \alpha \in IIIC$

Entonces: $\alpha \in IIC$



Por radio vector:

$x^2 + y^2 = r^2$

$x^2 + (5)^2 = (13)^2$

$x^2 = 144$

$\Rightarrow x = 12 \vee x = -12$

Como: $x < 0 \Rightarrow x = -12$

Piden:

$H = 2\text{sen}\alpha + 3\cos\alpha$

$H = 2\left(\frac{y}{r}\right) + 3\left(\frac{x}{r}\right) = 2\left(\frac{5}{13}\right) + 3\left(\frac{-12}{13}\right)$

$\Rightarrow H = \frac{10}{13} - \frac{36}{13} = -\frac{26}{13}$

$\therefore H = -2$

Clave D

8. Por dato:

$f(x) = \frac{\text{sen}2x + \text{sen}4x - \text{sen}6x}{\cos 2x + \cos 4x + \tan x - 4\sec 4x}$

Piden: $f\left(\frac{\pi}{4}\right)$

$f\left(\frac{\pi}{4}\right) = \frac{\text{sen}2\left(\frac{\pi}{4}\right) + \text{sen}4\left(\frac{\pi}{4}\right) - \text{sen}6\left(\frac{\pi}{4}\right)}{\cos 2\left(\frac{\pi}{4}\right) + \cos 4\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{4}\right) - 4\sec 4\left(\frac{\pi}{4}\right)}$

$f\left(\frac{\pi}{4}\right) = \frac{\text{sen}\frac{\pi}{2} + \text{sen}\pi - \text{sen}\frac{3\pi}{2}}{\cos\frac{\pi}{2} + \cos\pi + \tan\frac{\pi}{4} - 4\sec\pi}$

$f\left(\frac{\pi}{4}\right) = \frac{(1) + (0) - (-1)}{(0) + (-1) + (1) - 4(-1)} = \frac{2}{4} = \frac{1}{2}$

$\therefore f\left(\frac{\pi}{4}\right) = \frac{1}{2}$

Clave D

9. $E = \frac{(a+b)^2\text{sen}^3\frac{\pi}{2} + (a-b)^2\cos^3\pi}{a\text{sen}\frac{3\pi}{2} + b\cos^2\frac{\pi}{2}}$

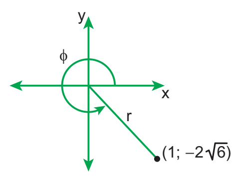
$E = \frac{(a+b)^2(1)^3 + (a-b)^2(-1)^3}{a(-1) + b(0)^2}$

$E = \frac{(a+b)^2 - (a-b)^2}{-a} = \frac{4ab}{-a}$

$\therefore E = -4b$

Clave E

10. $P(1; -2\sqrt{6}) \in IVC \Rightarrow \phi \in IVC$



Por radio vector:

$r^2 = 1^2 + (-2\sqrt{6})^2 = 25$

$\Rightarrow r = 5$

Piden:

$E = \text{sen}\phi - 3\sqrt{6}\cos\phi$

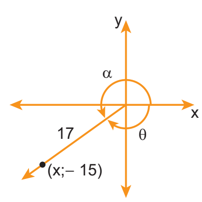
$E = \left(\frac{y}{r}\right) - 3\sqrt{6}\left(\frac{x}{r}\right)$

$E = \left(\frac{-2\sqrt{6}}{5}\right) - 3\sqrt{6}\left(\frac{1}{5}\right)$

$\therefore E = -\sqrt{6}$

Clave C

11.



Por radio vector:

$x^2 + y^2 = r^2$

$x^2 + (-15)^2 = 17^2 \Rightarrow x = 8 \vee x = -8$

Del gráfico: $x < 0 \Rightarrow x = -8$

Además: $\alpha - \theta = 360°$

Piden:

$B = \tan\alpha + \tan\theta + \tan(\alpha - \theta)$

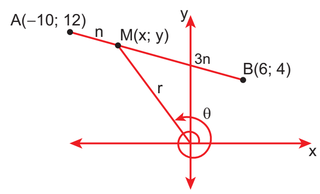
$B = \left(\frac{y}{x}\right) + \left(\frac{y}{x}\right) + \tan(360°)$

$\Rightarrow B = \left(\frac{-15}{-8}\right) + \left(\frac{-15}{-8}\right) + (0) = \frac{15}{4}$

$\therefore B = \frac{15}{4} = 3{,}75$

Clave C

12.



Por dato: $AB = 4AM$

$\Rightarrow AM + MB = 4AM$

$\Rightarrow MB = 3AM \Rightarrow \frac{AM}{MB} = \frac{1}{3}$

Por división de un segmento en una razón:

$x = \frac{n(6) + 3n(-10)}{n + 3n} = \frac{-24n}{4n} = -6$

$\Rightarrow x = -6$

$y = \frac{n(4) + 3n(12)}{n + 3n} = \frac{40n}{4n} = 10 \Rightarrow y = 10$

Por radio vector: $x^2 + y^2 = r^2 \Rightarrow r = 2\sqrt{34}$

Piden:

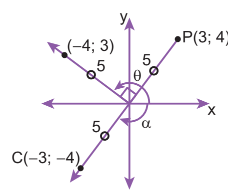
$E = 34\text{sen}\theta\cos\theta$

$E = 34\left(\frac{y}{r}\right)\left(\frac{x}{r}\right) = 34\left(\frac{10}{2\sqrt{34}}\right)\left(\frac{-6}{2\sqrt{34}}\right) = -\frac{60}{4}$

$\therefore E = -15$

Clave B

13.



Piden: $R = \cos\alpha(\sec\theta\tan\alpha - 2\csc\theta)$

$R = \left(-\frac{3}{5}\right)\left[\left(\frac{5}{-4}\right)\left(\frac{-4}{-3}\right) - 2\left(\frac{5}{3}\right)\right]$

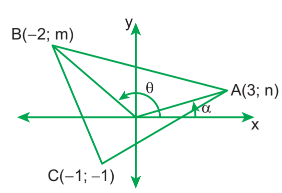
$R = -\frac{3}{5}\left(-\frac{5}{3} - \frac{10}{3}\right)$

$R = -\frac{3}{5}\left(-\frac{15}{3}\right) = \frac{15}{5} = 3$

$\therefore R = 3$

Clave B

14.



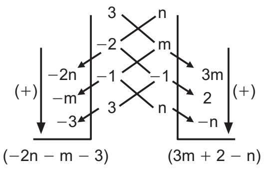
Piden:

$T = 3\tan\alpha - 8\tan\theta$

$T = 3\left(\frac{n}{3}\right) - 8\left(\frac{m}{-2}\right)$

$\Rightarrow T = n + 4m$ ...(1)

Por dato: $A_{\triangle ABC} = 10$



$\Rightarrow A_{\triangle ABC} = \frac{|(3m+2-n)-(-2n-m-3)|}{2}$

$10 = \frac{|4m+n+5|}{2}$

$20 = |4m + n + 5|$

Del gráfico: $m \wedge n$ son positivos.

$\Rightarrow 20 = (4m + n + 5)$

$15 = 4m + n$ ...(2)

Reemplazando (2) en (1):

$\Rightarrow T = n + 4m = 15$

$\therefore T = 15$

Clave D

PRACTIQUEMOS

Nivel 1 (página 34) Unidad 2

Comunicación matemática

1.

2.

3.

I. $\underbrace{\text{sen}127°}_{(+)} . \underbrace{\cos 135°}_{(-)} > 0$ (F)
< 0

II. $\underbrace{\text{sen}90°}_{(1)} . \underbrace{\text{sen}60°}_{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{2}$ (F)
$= \frac{\sqrt{3}}{2}$

III. $\underbrace{\text{sen}130°}_{(+)} . \underbrace{\cos 60°}_{(+)} < 0$ (F)
> 0

Clave B

Razonamiento y demostración

4. $M = \text{sen}\alpha . \cos\alpha$

$M = \frac{\sqrt{3}}{\sqrt{7}} \cdot \frac{2}{\sqrt{7}} = \frac{2\sqrt{3}}{7}$

Clave B

5. Del gráfico:

$P(-2; \sqrt{5})$

$x = -2 ; y = \sqrt{5}$

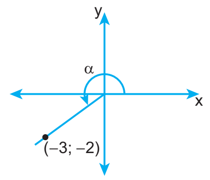
$r = \sqrt{(-2)^2 + (\sqrt{5})^2}$

$r = \sqrt{4+5} = 3$

$\sec\theta = \frac{r}{x} = \frac{3}{-2} = -\frac{3}{2}$

Clave C

6.



$\left.\begin{array}{l} x = -3 \\ y = -2 \end{array}\right\} r = \sqrt{13}$

Piden: $A = \sqrt{13}(\text{sen}\alpha - \cos\alpha)$

Pero:

$\text{sen}\alpha = \frac{y}{r} = \frac{-2}{\sqrt{13}} = -\frac{2}{\sqrt{13}}$

$\cos\alpha = \frac{x}{r} = \frac{-3}{\sqrt{13}} = -\frac{3}{\sqrt{13}}$

Reemplazando:

$A = \sqrt{13}\left[\left(-\frac{2}{\sqrt{13}}\right) - \left(-\frac{3}{\sqrt{13}}\right)\right]$

$A = -2 + 3 = 1$

Clave D

7. Por dato: $\tan\theta = \frac{5}{1} = \frac{y}{x}$; $\theta \in$ III

$\Rightarrow x = -1; y = -5$

Del gráfico:

$a - 3 = -1 \Rightarrow a = 2$

$b - 7 = -5 \Rightarrow b = 2$

Nos piden calcular: $a + b = 2 + 2 = 4$

Clave B

8. Para el punto P: $\tan\theta = \frac{3}{5a}$

Para el punto Q: $\tan\theta = \frac{a+1}{a}$

Igualando: $\tan\theta = \frac{3}{5a} = \frac{a+1}{a}$

$\Rightarrow \frac{3}{5} = a + 1 \Rightarrow a = -\frac{2}{5}$

Reemplazando:

$\tan\theta = \frac{3}{5\left(\frac{-2}{5}\right)} = -\frac{3}{2} \Rightarrow \cot\theta = -\frac{2}{3}$

Piden: $S = \tan\theta + \cot\theta$

$= \left(-\frac{3}{2}\right) + \left(-\frac{2}{3}\right) = -\frac{13}{6}$

$\therefore S = -\frac{13}{6}$

Clave C

Resolución de problemas

9. $\underbrace{\text{sen}\theta}_{(-)} \underbrace{\sqrt{\cos\theta}}_{(+)} < 0$

$\Rightarrow \cos\theta > 0, \theta \in \text{IC} \vee \text{IVC}$

$\text{sen}\theta < 0, \theta \in \text{IIIC} \vee \text{IVC}$

De ambas condiciones: $\theta \in$ IVC

$270° < \theta < 360°$

$135° < \frac{\theta}{2} < 180° \Rightarrow \left(\frac{\theta}{2}\right) \in \text{IIC}$

$90° < \frac{\theta}{3} < 120° \Rightarrow \left(\frac{\theta}{3}\right) \in \text{IIC}$

$108° < \frac{2\theta}{5} < 144° \Rightarrow \left(\frac{2\theta}{5}\right) \in \text{IIC}$

Piden los signos de:

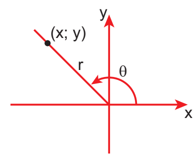
$C = \underbrace{\text{sen}\frac{\theta}{2}}_{(+)} \underbrace{\cos\frac{\theta}{3}}_{(-)} = (-)$

$L = \underbrace{\cos\frac{2\theta}{5}}_{(-)} - \underbrace{\cos\theta}_{(+)} = (-)$

Clave B

10. $\underbrace{\sqrt{\sqrt[3]{7}}}_{7^{\frac{1}{6}}} = \underbrace{\sqrt[5]{7^{\text{sen}\theta}}}_{7^{\frac{\text{sen}\theta}{5}}}$; $\cos\theta < 0$

$\Rightarrow \frac{1}{6} = \frac{\text{sen}\theta}{5} \Rightarrow \text{sen}\theta = \frac{5}{6}$



$\text{sen}\theta = \frac{5}{6} = \frac{y}{r}$

$y = 5; r = 6 \Rightarrow x = -\sqrt{11}$

$\cot\theta = \frac{x}{y} = \frac{-\sqrt{11}}{5} = -\frac{\sqrt{11}}{5}$

$\cos\theta = \frac{x}{r} = \frac{-\sqrt{11}}{6} = -\frac{\sqrt{11}}{6}$

$k = \left(-\frac{\sqrt{11}}{5}\right)\left(-\frac{\sqrt{11}}{6}\right) + \frac{5}{6}$

$k = \frac{11}{30} + \frac{5}{6} = \frac{36}{30} = \frac{6}{5}$

Clave B

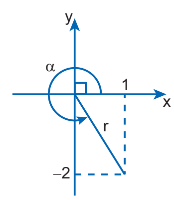
11. Se sabe:

$\cot\alpha = \frac{x}{y} = -0,5 = -\frac{5}{10} = -\frac{1}{2}$

Pero: $\alpha \in$ IVC

Entonces: $\cot\alpha = \frac{1}{-2} = \frac{x}{y} \wedge r^2 = x^2 + y^2$

$r = \sqrt{5}$



Piden:

$\csc\alpha = \frac{r}{y} = \frac{\sqrt{5}}{-2}$

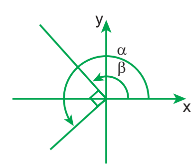
$= -1,118$

Clave B

Nivel 2 (página 34) Unidad 2

Comunicación matemática

12. $\tan\beta < 0; \alpha > \beta$



$\beta \in \text{IIC} \Rightarrow 90° < \beta < 180°$

$45° < \frac{\beta}{2} < 90° \Rightarrow \left(\frac{\beta}{2}\right) \in \text{IC}$

$180° < 2\beta < 360° \Rightarrow (2\beta) \in \text{IIIC} \vee \text{IVC}$

$\alpha \in \text{IIIC} \Rightarrow 180° < \alpha < 270°$

$90° < \dfrac{\alpha}{2} < 135°$

$\Rightarrow \left(\dfrac{\alpha}{2}\right) \in \text{IIC}$

$360° < 2\alpha < 540°$

$(2\alpha) \in \text{IC} \vee \text{IIC}$

Piden los signos de:

$M = \underbrace{\text{sen}\alpha}_{(-)} + \underbrace{\cos\alpha}_{(-)} = (-)$

$N = \underbrace{\cos\dfrac{\alpha}{2}}_{(-)} - \underbrace{\text{sen}\dfrac{\beta}{2}}_{(+)} = (-)$

$P = \underbrace{\text{sen}2\alpha}_{(+)} - \underbrace{\text{sen}2\beta}_{(-)} = (+)$

$\therefore (-); (-); (+)$

Clave A

13.

14. $\sqrt{1-\cos A} + \sqrt{\cos A - 1} = 1 + \text{sen}B$ ...(1)

$\sqrt{\csc B + 2} = |\tan C - 1|$ ...(2)

Recordando el siguiente teorema:

$\sqrt{a} \geq 0 \Leftrightarrow a \geq 0$

Analizamos la condición (1):

$1 - \cos A \geq 0 \ \wedge \ \cos A - 1 \geq 0$

$\cos A \leq 1 \ \wedge \ \cos A \geq 1$

$\Rightarrow \cos A = 1 \Rightarrow A = 360° \in \langle 0°; 360°]$

Reemplazamos $\cos A = 1$, en (1):

$\sqrt{1-1} + \sqrt{1-1} = 1 + \text{sen}B \Rightarrow \text{sen}B = -1$

$\Rightarrow B = 270° \in \langle 0°; 360°]$

Reemplazando $\csc B = -1$, en (2):

$\sqrt{-1+2} = |\tan C - 1| \Rightarrow |\tan C - 1| = 1$

Recordando el teorema:

$|a| = b;\ b > 0 \Rightarrow a = b \ \vee a = -b$

Luego:

$\tan C - 1 = 1 \quad \vee \quad \tan C - 1 = -1$

$\tan C = 2 \quad \vee \quad \tan C = 0$

Como A, B y C son cuadrantes diferentes:

$\tan C = 0 \Rightarrow C = 180°$

Piden:

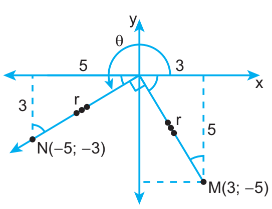
$A + B + C = 360° + 270° + 180°$

$\therefore \ A + B + C = 810°$

Clave C

Razonamiento y demostración

15.



Por radio vector:

$r^2 = (-5)^2 + (-3)^2 = 34 \Rightarrow r = \sqrt{34}$

Piden:

$T = 5\tan\theta + \sqrt{34}\cos\theta$

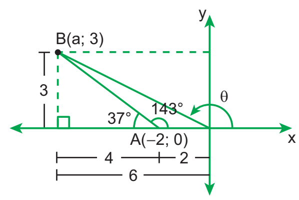
$T = 5\left(\dfrac{y}{x}\right) + \sqrt{34}\left(\dfrac{x}{r}\right)$

$T = 5\left(\dfrac{-3}{-5}\right) + \sqrt{34}\left(\dfrac{-5}{\sqrt{34}}\right)$

$T = 3 + (-5) = -2 \Rightarrow \therefore T = -2$

Clave C

16.



Piden:

$\tan\theta = \dfrac{y}{x} = \dfrac{3}{-6} = -\dfrac{1}{2} \Rightarrow \tan\theta = -\dfrac{1}{2}$

Clave C

17. Reemplazando los valores de las razones en la condición tenemos:

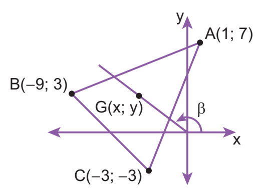
$$\frac{(\sqrt{2})^2(-1)\left(\frac{1}{2}\right) - (-1) + (\sqrt{3})^2}{x(-1) + (0)} = 3$$

$$-\frac{3}{x} = 3$$

$$\therefore x = -1$$

Clave A

18.



Por dato: G es baricentro del $\triangle$ABC.

$\Rightarrow x = \dfrac{1 + (-9) + (-3)}{3} = -\dfrac{11}{3}$

$\Rightarrow y = \dfrac{7 + 3 + (-3)}{3} = \dfrac{7}{3}$

Piden: $\tan\beta = \dfrac{y}{x} = \dfrac{\frac{7}{3}}{-\frac{11}{3}} = -\dfrac{7}{11}$

$\therefore \tan\beta = -\dfrac{7}{11}$

Clave B

Resolución de problemas

19. Sea el ángulo $\beta < 3000°$ ...(1)

β es coterminal con $\alpha \Rightarrow \beta - \alpha = (360°)n$

$\beta = \alpha + (360°)n \Rightarrow \beta = 20° + (360°)n$

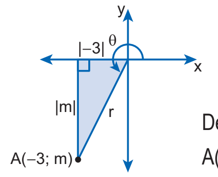
De (1): $20° + (360°)n < 3000° \Rightarrow n < 8{,}27$

Como n debe ser un número entero y β el mayor posible $\Rightarrow n = 8$

$\therefore \beta = 20° + 8(360°) = 2900°$

Clave A

20.



Del gráfico:

$A(-3; m) \in \text{IIIC} \Rightarrow m < 0$

Por dato: $A_{\text{somb.}} = 9$

$\dfrac{|m|.|-3|}{2} = 9 \Rightarrow |m| = 6 \Rightarrow m = -6$

Por radio vector:

$r^2 = (-3)^2 + m^2 = 9 + (-6)^2 \Rightarrow r = 3\sqrt{5}$

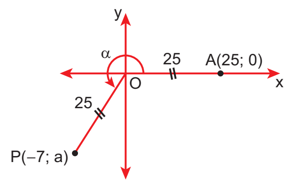
Piden: $P = \tan\theta\,\text{sen}\theta = \left(\dfrac{y}{x}\right)\left(\dfrac{y}{r}\right)$

$\Rightarrow P = \left(\dfrac{-6}{-3}\right)\left(\dfrac{-6}{3\sqrt{5}}\right) = -\dfrac{4}{\sqrt{5}}$

$\therefore P = -\dfrac{4\sqrt{5}}{5}$

Clave D

21. Por dato: AO = OP



Del gráfico: $P(-7; a) \in \text{IIIC} \Rightarrow a < 0$

Empleando el radio vector:

$(-7)^2 + a^2 = 25^2 \Rightarrow a = -24$

Piden:

$\text{sen}\alpha = \dfrac{y}{r} = \dfrac{a}{r} = -\dfrac{24}{25}$

$\therefore \text{sen}\alpha = -\dfrac{24}{25}$

Clave A

22. Por dato: α es un ángulo en posición normal.

Además:

$P(-k; 1-k)$ es un punto de su lado final.

$\Rightarrow \tan\alpha = \dfrac{y}{x} = \dfrac{1-k}{-k}$

$\Rightarrow \tan\alpha = -\dfrac{1-k}{k}$

También: $\tan\alpha = 4$

$\Rightarrow 4 = -\dfrac{1-k}{k}$

$4k = -1 + k$

$3k = -1$

$\therefore k = -\dfrac{1}{3}$

Clave C

Nivel 3 (página 35) Unidad 2

Comunicación matemática

23. Por dato: $\theta \in \text{IIIC}$ y es menor que una vuelta y positivo:

$\Rightarrow 180° < \theta < 270°$

Luego:

$90° < \frac{\theta}{2} < 135° \Rightarrow \frac{\theta}{2} \in \text{IIC}$

$60° < \frac{\theta}{3} < 90° \Rightarrow \frac{\theta}{3} \in \text{IC}$

$120° < \frac{2\theta}{3} < 180° \Rightarrow \frac{2\theta}{3} \in \text{IIC}$

$45° < \frac{\theta}{4} < 67{,}5° \Rightarrow \frac{\theta}{4} \in \text{IC}$

Piden, los signos de las expresiones:

$H = \tan\theta + \operatorname{sen}\frac{\theta}{2}$

$H = (+) + (+) = (+)$

$\Rightarrow H = (+)$

$I = \operatorname{sen}\theta\cos\frac{\theta}{2}\tan\frac{\theta}{3}$

$I = (-)(-)(+) = (+)$

$\Rightarrow I = (+)$

$J = \sec\frac{2\theta}{3} - \csc\frac{\theta}{4}$

$J = (-) - (+) = (-)$

$\Rightarrow J = (-)$

Por lo tanto, los signos son: (+); (+); (–)

Clave D

24.

25. Por dato:

$\theta \in \text{IIIC}$:

$180° < \theta < 270° \Rightarrow 90° < \frac{\theta}{2} < 135° \Rightarrow \frac{\theta}{2} \in \text{IIC}$

$\alpha \in \text{IVC}$

Piden el signo de:

$$A = \frac{\operatorname{sen}\theta.\cos\theta.\tan\alpha}{\csc\alpha + \cot\alpha} = \frac{(-).(-).(-)}{(-)+(-)} = \frac{(-)}{(-)}$$

$\Rightarrow A = (+)$

$$B = \frac{\sec\alpha - \operatorname{sen}\alpha}{\cot\frac{\theta}{2}} = \frac{(+)-(-)}{(-)} = \frac{(+)}{(-)}$$

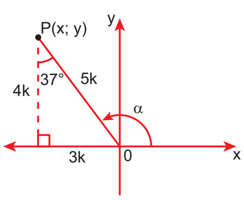
$\Rightarrow B = (-)$

Por lo tanto, los signos son: (+); (–)

Clave B

Razonamiento y demostración

26.



Del gráfico: $P(x; y) = P(-3k; 4k)$

Piden:

$E = (\operatorname{sen}\alpha + \cos\alpha)^{(-\tan 45°)}$

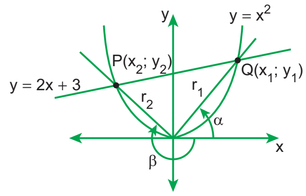
$E = \left(\frac{y}{r} + \frac{x}{r}\right)^{-1} = \left(\frac{4k}{5k} + \frac{-3k}{5k}\right)^{-1}$

$E = \left(\frac{4}{5} - \frac{3}{5}\right)^{-1} = \left(\frac{1}{5}\right)^{-1}$

$\therefore E = 5$

Clave E

27.



Calculando los puntos de intersección de ambas funciones: $y = x^2 = 2x + 3$

$\Rightarrow x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\Rightarrow x = 3 \vee x = -1$

Entonces: $x_1 = 3 \wedge x_2 = -1$

Luego: $y_1 = 3^2 = 9 \wedge y_2 = (-1)^2 = 1$

Los puntos serán: $Q(x_1; y_1) = Q(3; 9)$

$P(x_2; y_2) = P(-1; 1)$

Por radio vector:

$r_1^2 = 3^2 + 9^2 \Rightarrow r_1 = 3\sqrt{10}$

$r_2^2 = (-1)^2 + 1^2 \Rightarrow r_2 = \sqrt{2}$

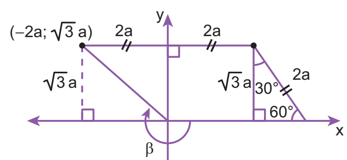
Piden: $L = \operatorname{sen}\alpha\cos\beta = \left(\frac{y_1}{r_1}\right)\left(\frac{x_2}{r_2}\right)$

$\Rightarrow L = \left(\frac{9}{3\sqrt{10}}\right)\left(\frac{-1}{\sqrt{2}}\right) = -\frac{3\sqrt{5}}{10}$

$\therefore L = -0{,}3\sqrt{5}$

Clave C

28.



Piden: $\cot\beta = \frac{x}{y} = \frac{-2a}{\sqrt{3}a}$

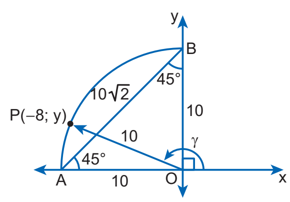
$\Rightarrow \cot\beta = \frac{-2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\therefore \cot\beta = -\frac{2\sqrt{3}}{3}$

Clave C

Resolución de problemas

29.



Por radio vector:

$(-8)^2 + y^2 = 10^2$

$y^2 = 36 \Rightarrow y = 6 \vee y = -6$

Del gráfico: $P(-8; y) \in \text{IIC} \Rightarrow y > 0 \Rightarrow y = 6$

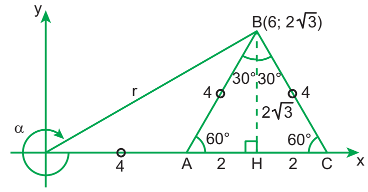
Piden: $H = \operatorname{sen}\gamma - \cos\gamma = \left(\frac{y}{r}\right) - \left(\frac{x}{r}\right)$

$\Rightarrow H = \left(\frac{6}{10}\right) - \left(\frac{-8}{10}\right) = \frac{14}{10}$

$\therefore H = \frac{7}{5}$

Clave C

30.



Del gráfico:

El punto $A(x_1; 0) = A(4; 0)$

El punto $C(x_2; 0) = C(8; 0)$

Por radio vector: $r = 4\sqrt{3}$

$\Rightarrow \csc\alpha = \frac{r}{y} = \frac{4\sqrt{3}}{2\sqrt{3}} = 2$

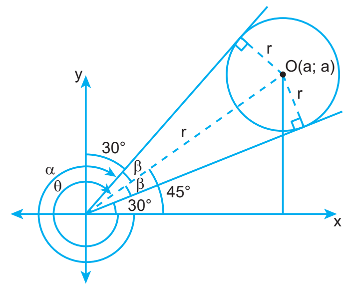
Piden:

$\csc\alpha + x_1 + x_2 = 2 + 4 + 8$

$\therefore \csc\alpha + x_1 + x_2 = 14$

Clave E

31.



Por dato: $\alpha = -300°$

Del gráfico: $30° + 2\beta + 30° = 90° \Rightarrow \beta = 15°$

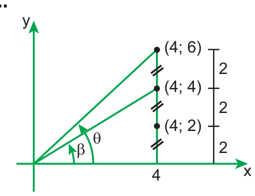
Por radio vector: $r = a\sqrt{2}$

Piden: $\csc\theta = \frac{r}{y} = \frac{a\sqrt{2}}{a} = \sqrt{2}$

$\therefore \csc\theta = \sqrt{2}$

Clave A

32.



Del gráfico:

$\tan\beta = \frac{y}{x} = \frac{4}{4} = 1$

$\tan\theta = \frac{y}{x} = \frac{6}{4} = \frac{3}{2}$

Piden: $E = \tan\theta - \tan\beta$

$E = \frac{3}{2} - 1 = \frac{1}{2}$

$\therefore E = \frac{1}{2}$

Clave C

APLICAMOS LO APRENDIDO (página 37) Unidad 2

1. $Q = \dfrac{\text{sen}250°\,\csc 290°\,\tan 300°}{\text{sen}840°\,\tan 3000°\,\cos 1200°}$

sen250° = sen(270° – 20°) = –cos20°
csc290° = csc(270° + 20°) = –sec20°
tan300° = tan(360°– 60°) = – tan60°

$840° = 360°\cdot 2 + 120°$; $1200° = 360°\cdot 3 + 120°$; $3000° = 360°\cdot 8 + 120°$

$Q = \dfrac{(-\cos 20°)(-\sec 20°)(-\tan 60°)}{(\text{sen}120°)(\tan 120°)(\cos 120°)}$

$Q = \dfrac{-\overbrace{\cos 20°.\sec 20°}^{1}.\tan 60°}{\text{sen}60°(-\tan 60°)(-\cos 60°)}$

$Q = \dfrac{-(1)(\sqrt{3})}{\left(\frac{\sqrt{3}}{2}\right)(\sqrt{3})\left(\frac{1}{2}\right)} = \dfrac{-4}{\sqrt{3}} = -\dfrac{4\sqrt{3}}{3}$

$\therefore Q = -\dfrac{4\sqrt{3}}{3} = \dfrac{-4(1,73)}{3} = -2,307$

Clave C

2. Piden:

E = tan(36 660°)sec(180 330°)

tan(36 660°) = tan(360° . 101 + 300°)
tan(36 660°) = tan300° = tan(360° – 60°)
tan(36 660°) = –tan60° = $-(\sqrt{3})$
$\Rightarrow$ tan(36 660°) = $-\sqrt{3}$

sec(180 330°) = sec(360° . 500 + 330°)
sec(180 330°) = sec330°
sec(180 330°) = sec(360° – 30°) = sec30°

$\Rightarrow$ sec(180 330°) = $\dfrac{2\sqrt{3}}{3}$

Reemplazamos los valores en la expresión E:

$E = (-\sqrt{3})\left(\dfrac{2\sqrt{3}}{3}\right) = -\dfrac{6}{3} = -2$

$\therefore E = -2$

Clave A

3. Por dato: 17x = 180°

Piden:

$M = \dfrac{\csc 13x}{\csc 4x} - \dfrac{\tan 16x}{\tan x}$

$M = \dfrac{\csc(17x-4x)}{\csc 4x} - \dfrac{\tan(17x-x)}{\tan x}$

$M = \dfrac{\csc(180°-4x)}{\csc 4x} - \dfrac{\tan(180°-x)}{\tan x}$

$\Rightarrow M = \dfrac{\csc 4x}{\csc 4x} - \dfrac{(-\tan x)}{\tan x}$

M = 1 – (–1) = 1 + 1 = 2
$\therefore$ M = 2

Clave C

4. Por dato: tan20° = a
Piden:

$A = \dfrac{\text{sen}160°\cos 250°}{\text{sen}340°\sec 110°}$

$A = \dfrac{\text{sen}(180°-20°)\cos(270°-20°)}{\text{sen}(360°-20°)\sec(90°+20°)}$

$A = \dfrac{(\text{sen}20°)(-\text{sen}20°)}{(-\text{sen}20°)(-\csc 20°)}$

$A = -\text{sen}^2 20° = -(1-\cos^2 20°)$

$A = \cos^2 20° - 1 = \dfrac{1}{\sec^2 20°} - 1$

$A = \dfrac{1-\sec^2 20°}{\sec^2 20°} = \dfrac{1-(1+\tan^2 20°)}{1+\tan^2 20°}$

$A = -\dfrac{\tan^2 20°}{1+\tan^2 20°} = -\dfrac{(a)^2}{1+(a)^2}$

$\therefore A = -\dfrac{a^2}{1+a^2}$

Clave A

5. En un triángulo los ángulos internos suman 180°.

Entonces: A + B + C = 180°

Piden:

$E = \dfrac{2\cos(A+B)}{\cos C} - 3\sec(A+B+C)$

$E = \dfrac{2\cos(180°-C)}{\cos C} - 3\sec(180°)$

$E = \dfrac{-2\cos C}{\cos C} - 3(-1)$

E = –2 + 3 = 1
$\therefore$ E = 1

Clave B

6. $x + y = 2\pi \Rightarrow y = 2\pi - x$

Piden:

$A = \text{sen}x + \tan\dfrac{x}{2} + \text{sen}y + \tan\dfrac{y}{2}$

$A = \text{sen}x + \tan\dfrac{x}{2} + \text{sen}(2\pi - x) + \tan\left(\pi - \dfrac{x}{2}\right)$

$A = \text{sen}x + \tan\dfrac{x}{2} + (-\text{sen}x) + \left(-\tan\dfrac{x}{2}\right)$

$A = \text{sen}x + \tan\dfrac{x}{2} - \text{sen}x - \tan\dfrac{x}{2} = 0$

$\therefore$ A = 0

Clave E

7. Por dato: $f(\theta) = \dfrac{\text{sen}2\theta + \cos 4\theta}{\tan 8\theta + \csc 6\theta}$

Para: $\theta = -\dfrac{\pi}{4}$

$f\left(-\dfrac{\pi}{4}\right) = \dfrac{\text{sen}2\left(-\frac{\pi}{4}\right) + \cos 4\left(-\frac{\pi}{4}\right)}{\tan 8\left(-\frac{\pi}{4}\right) + \csc 6\left(-\frac{\pi}{4}\right)}$

$f\left(-\dfrac{\pi}{4}\right) = \dfrac{\text{sen}\left(-\frac{\pi}{2}\right) + \cos(-\pi)}{\tan(-2\pi) + \csc\left(-\frac{3\pi}{2}\right)}$

$f\left(-\dfrac{\pi}{4}\right) = \dfrac{-\text{sen}\frac{\pi}{2} + \cos\pi}{-\tan 2\pi - \csc\frac{3\pi}{2}} = \dfrac{-(1)+(-1)}{-(0)-(-1)}$

$\Rightarrow f\left(\dfrac{\pi}{4}\right) = \dfrac{-2}{1} = -2$

Para: $\theta = \dfrac{\pi}{4}$

$f\left(\dfrac{\pi}{4}\right) = \dfrac{\text{sen}2\left(\frac{\pi}{4}\right) + \cos 4\left(\frac{\pi}{4}\right)}{\tan 8\left(\frac{\pi}{4}\right) + \csc 6\left(\frac{\pi}{4}\right)}$

$f\left(\dfrac{\pi}{4}\right) = \dfrac{\text{sen}\frac{\pi}{2} + \cos\pi}{\tan 2\pi + \csc\frac{3\pi}{2}}$

$f\left(\dfrac{\pi}{4}\right) = \dfrac{(1)+(-1)}{(0)+(-1)} = \dfrac{0}{-1}$

$f\left(\dfrac{\pi}{4}\right) = 0$

Piden:

$f\left(-\dfrac{\pi}{4}\right) + f\left(\dfrac{\pi}{4}\right) = -2 + 0 = -2$

$\therefore f\left(-\dfrac{\pi}{4}\right) + f\left(\dfrac{\pi}{4}\right) = -2$

Clave D

8. $f(x) = \dfrac{\text{sen}5x + \cos 8x}{\cos 2x + \text{sen}6x}$

$f\left(\dfrac{\pi}{2}\right) = \dfrac{\text{sen}\frac{5\pi}{2} + \cos 4\pi}{\cos\pi + \text{sen}3\pi} = \dfrac{(1)+(1)}{(-1)+(0)}$

$f\left(\dfrac{\pi}{2}\right) = \dfrac{2}{-1} = -2 \Rightarrow f\left(\dfrac{\pi}{2}\right) = -2$

$f(\pi) = \dfrac{\text{sen}5\pi + \cos 8\pi}{\cos 2\pi + \text{sen}6\pi} = \dfrac{(0)+(1)}{(1)+(0)}$

$f(\pi) = \dfrac{1}{1} = 1 \Rightarrow f(\pi) = 1$

$f\left(\dfrac{3\pi}{2}\right) = \dfrac{\text{sen}\frac{15\pi}{2} + \cos 12\pi}{\cos 3\pi + \text{sen}9\pi} = \dfrac{(-1)+(1)}{(-1)+(0)}$

$f\left(\dfrac{3\pi}{2}\right) = \dfrac{0}{-1} = 0 \Rightarrow f\left(\dfrac{3\pi}{2}\right) = 0$

Piden:

$f\left(\dfrac{\pi}{2}\right) + f(\pi) + f\left(\dfrac{3\pi}{2}\right) = (-2) + (1) + (0) = -1$

$\therefore f\left(\dfrac{\pi}{2}\right) + f(\pi) + f\left(\dfrac{3\pi}{2}\right) = -1$

Clave C

9. $N(1 - \tan 205° \cot 258°) = \dfrac{\text{sen}335°}{\text{sen}115°} + \dfrac{\cos 282°}{\text{sen}258°}$

$N(1 - \tan 25° \cot 78°) = \dfrac{-\text{sen}25°}{\text{sen}65°} + \dfrac{\cos 78°}{-\text{sen}78°}$

$N(1 - \tan 25° \cot 78°) = -\dfrac{\text{sen}25°}{\cos 25°} - \dfrac{\cos 78°}{\text{sen}78°}$

N(1 – tan25° cot78°) = –tan25° – cot78°
N(1 – tan25° tan12°) = –(tan25° + tan12°)

Observación:
por identidades trigonométricas para el ángulo compuesto se sabe:

$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$N = -\left[\dfrac{(\tan 25° + \tan 12°)}{1 - \tan 25°\tan 12°}\right]$

N = –[tan(25° + 12°)]
N = –tan37°
$\therefore N = -\dfrac{3}{4}$

Clave A

10.

$\text{sen}20° = n$

$C = \text{sen}200°\tan340°\cos160°$

$C = \text{sen}(180° + 20°)\tan(360° - 20°)\cos(180° - 20°)$

$C = -\text{sen}20° . -\tan20° . -\cos20°$

$C = -\text{sen}20° . \tan20° \cos 20°$

$C = -\text{sen}20° . \dfrac{\text{sen}20°}{\cos 20°} . \cos20°$

$\therefore C = -\text{sen}^2 20° = -n^2$

Clave B

11.

$J = \dfrac{\text{sen}(A+B)}{\text{sen}C} + \dfrac{\tan(B+C)}{\tan A} + \dfrac{\cos(A+C)}{\cos B}$

$J = \dfrac{\text{sen}(180° - C)}{\text{sen}C} + \dfrac{\tan(180° - A)}{\tan A} + \dfrac{\cos(180° - B)}{\cos B}$

$J = \dfrac{\text{sen}C}{\text{sen}C} - \dfrac{\tan A}{\tan A} - \dfrac{\cos B}{\cos B}$

$\therefore J = 1 - 1 - 1 = -1$

Clave C

12. Dato: $A + B = 90°$ y $B + C = 180°$

Sumando: $A + 2B + C = 270$

Restando: $C - A = 90°$

$M = \dfrac{\text{sen}(270° - B)}{\cos B} + \dfrac{\tan A}{\cot(90° + A)}$

$M = \dfrac{-\cos B}{\cos B} + \dfrac{\tan A}{-\tan A}$

$M = -1 - 1$

$M = -2$

Clave E

13. Dato: $x + y = 90°$

En (I): $\tan x + \cot x = \sqrt{a}$

$\sec x \csc x = \sqrt{a}$ ... (III)

En (II): $\sec x - \csc x = \sqrt{b}$

Elevando al cuadrado:

$\sec^2 x + \csc^2 x - 2\sec x \csc x = b$

$(\sec x . \csc x)^2 - 2\sec x \csc x = b$

$(\sqrt{a})^2 - 2\sqrt{a} = b$

$a - b = 2\sqrt{a}$

Clave C

14. $S = \dfrac{-\cos B}{\cos B} + \dfrac{\tan A}{-\tan A}$

$S = -1 + (-1)$

$\therefore S = -2$

Clave C

PRACTIQUEMOS

Nivel 1 (página 39) Unidad 2

Comunicación matemática

1.

2.

Razonamiento y demostración

3. Piden: $\text{sen}2580°$

$\text{sen}2580° = \text{sen}(7 \times 360° + 60°)$

$\Rightarrow \text{sen}2580° = \text{sen}60° = \dfrac{\sqrt{3}}{2}$

$\therefore \text{sen}2580° = \dfrac{\sqrt{3}}{2}$

Clave D

4. Piden: $\tan6173°$

$\tan6173° = \tan(17 \times 360° + 53°)$

$\Rightarrow \tan6173° = \tan53° = \dfrac{4}{3} \quad \therefore \tan6173° = \dfrac{4}{3}$

Clave B

5. Piden: $\tan5520°$

$\tan5520° = \tan(15 \times 360° + 120°)$

$\Rightarrow \tan5520° = \tan120° = \tan(180° - 60°)$

$\tan5520° = -\tan60° = -(\sqrt{3})$

$\therefore \tan5520° = -\sqrt{3}$

Clave B

6. $C = \text{sen}120°\cos225°$

$C = \text{sen}(180° - 60°) . \cos(180° + 45°)$

$C = (\text{sen}60°)(-\cos45°)$

$\Rightarrow C = \left(\dfrac{\sqrt{3}}{2}\right)\left(-\dfrac{\sqrt{2}}{2}\right) = -\dfrac{\sqrt{6}}{4}$

$\therefore C = -\dfrac{\sqrt{6}}{4}$

Clave D

7. $C = (\text{sen}330° + \cos240°)\tan210°$

Reducimos al IC:

$\text{sen}330° = \text{sen}(360° - 30°) = -\text{sen}30°$

$\cos240° = \cos(180° + 60°) = -\cos60°$

$\tan210° = \tan(180° + 30°) = \tan30°$

Reemplazamos en C:

$C = (-\text{sen}30° + (-\cos60°))\tan30°$

$\Rightarrow C = -(\text{sen}30° + \cos60°)\tan30°$

$C = -\left(\dfrac{1}{2} + \dfrac{1}{2}\right)\dfrac{\sqrt{3}}{3} = -\dfrac{\sqrt{3}}{3} \Rightarrow C = -\dfrac{\sqrt{3}}{3}$

Clave D

8. $N = \text{sen}(-240°)\cos(-120°)$

$N = (-\text{sen}240°)(\cos120°)$

$N = -\text{sen}(180° + 60°)\cos(180° - 60°)$

$N = -(-\text{sen}60°)(-\cos60°)$

$N = -\text{sen}60°\cos60°$

$\Rightarrow N = -\left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{2}\right) = -\dfrac{\sqrt{3}}{4}$

$\therefore N = -\dfrac{\sqrt{3}}{4}$

Clave D

9. $D = \dfrac{\text{sen}3015° \tan4290°}{\cos 2730°}$

Reducimos al IC:

$\text{sen}3015° = \text{sen}(8° \times 360° + 135°) = \text{sen}135°$

$\text{sen}3015° = \text{sen}(180° - 45°) = \text{sen}45°$

$\Rightarrow \text{sen}3015° = \dfrac{\sqrt{2}}{2}$

$\tan4290° = \tan(11 \times 360° + 330°) = \tan330°$

$\tan4290° = \tan(360° - 30°) = -\tan30°$

$\Rightarrow \tan4290° = -\dfrac{\sqrt{3}}{3}$

$\cos2730° = \cos(7 \times 360° + 210°) = \cos210°$

$\cos2730° = \cos(180° + 30°) = -\cos30°$

$\Rightarrow \cos2730° = -\dfrac{\sqrt{3}}{2}$

Reemplazamos en D:

$D = \dfrac{\left(\dfrac{\sqrt{2}}{2}\right)\left(-\dfrac{\sqrt{3}}{3}\right)}{\left(-\dfrac{\sqrt{3}}{2}\right)} = \dfrac{\sqrt{2}}{3} \qquad \therefore D = \dfrac{\sqrt{2}}{3}$

Clave A

10. $U = (\cos^2 135° - 3\tan127°)\text{sen}^2 240°$

Reducimos al IC:

$\cos135° = \cos(180° - 45°) = -\cos45°$

$\tan127° = \tan(180° - 53°) = -\tan53°$

$\text{sen}240° = \text{sen}(180° + 60°) = -\text{sen}60°$

Reemplazamos en U:

$U = [(-\cos45°)^2 - 3(-\tan53°)](-\text{sen}60°)^2$

$U = (\cos^2 45° + 3\tan53°)\text{sen}^2 60°$

$\Rightarrow U = \left[\left(\dfrac{\sqrt{2}}{2}\right)^2 + 3\left(\dfrac{4}{3}\right)\right]\left(\dfrac{\sqrt{3}}{2}\right)^2$

$U = \left(\dfrac{1}{2} + 4\right)\dfrac{3}{4} = \dfrac{27}{8} \qquad \therefore U = \dfrac{27}{8}$

Clave C

Nivel 2 (página 39) Unidad 2

Comunicación matemática

11.

12.

Razonamiento y demostración

13. $T = \dfrac{\text{sen}(-x) + \cos(-x)}{\text{sen}x - \cos x}$

$T = \dfrac{(-\text{sen}x) + (\cos x)}{\text{sen}x - \cos x}$

$T = \dfrac{-(\text{sen}x - \cos x)}{\text{sen}x - \cos x} = -1 \qquad \therefore T = -1$

Clave B

14. $R = \dfrac{\text{sen}(90° + x)}{\cos(180° - x)} + \dfrac{\tan(270° - x)}{\cot(-x)}$

$R = \dfrac{\cos x}{-\cos x} + \dfrac{\cot x}{-\cot x}$

$R = -1 + (-1) = -2$

$\therefore R = -2$

Clave E

15. $E=\dfrac{\text{sen}(180°+x)\cos(360°-x)}{\text{sen}(270°+x)}$

$E=\dfrac{(-\text{sen}x)(\cos x)}{-\cos x}=\text{sen}x$

$E=\text{sen}x$

Clave A

16. $A=\dfrac{\text{sen}(\pi+x)\tan\left(\frac{\pi}{2}+x\right)\text{sen}\left(\frac{3\pi}{2}-x\right)}{\cot(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$

$A=\dfrac{(-\text{sen}x)(-\cot x)(-\cos x)}{(-\cot x)(-\text{sen}x)}$

$\therefore A=-\cos x$

Clave D

17. $S=\dfrac{\text{sen}(x-\pi)\tan\left(x-\frac{\pi}{2}\right)}{\cos\left(x-\frac{3\pi}{2}\right)}$

$S=\dfrac{\text{sen}(-(\pi-x))\tan\left(-\left(\frac{\pi}{2}-x\right)\right)}{\cos\left(-\left(\frac{3\pi}{2}-x\right)\right)}$

$S=\dfrac{[-\text{sen}(\pi-x)]\left[-\tan\left(\frac{\pi}{2}-x\right)\right]}{\cos\left(\frac{3\pi}{2}-x\right)}$

$S=\dfrac{\text{sen}(\pi-x)\tan\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{3\pi}{2}-x\right)}=\dfrac{(\text{sen}x)(\cot x)}{-\text{sen}x}$

$\therefore S=-\cot x$

Clave B

18. $\left\{\dfrac{\text{sen}150°\cos225°}{\tan143°}\right\}^{\tan315°}$

Reducimos al IC:

$\text{sen}150°=\text{sen}(180°-30°)=\text{sen}30°$

$\cos225°=\cos(180°+45°)=-\cos45°$

$\tan143°=\tan(180°-37°)=-\tan37°$

$\tan315°=\tan(360°-45°)=-\tan45°$

Reemplazamos:

$A=\left\{\dfrac{\text{sen}30°(-\cos45°)}{-\tan37°}\right\}^{(-\tan45°)}$

$A=\left\{\dfrac{\left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{3}{4}\right)}\right\}^{-1}=\left(\dfrac{\sqrt{2}}{3}\right)^{-1}$

$\therefore A=\dfrac{3}{\sqrt{2}}$

Clave B

19. $T=\dfrac{\tan(123\pi+x)\text{sen}\left(\frac{135\pi}{2}+x\right)}{\cot\left(\frac{1533\pi}{2}-x\right)}$

$T=\dfrac{\tan(122\pi+\pi+x)\text{sen}\left(66\pi+\frac{3\pi}{2}+x\right)}{\cot\left(766\pi+\frac{\pi}{2}-x\right)}$

$T=\dfrac{\tan(\pi+x)\text{sen}\left(\frac{3\pi}{2}+x\right)}{\cot\left(\frac{\pi}{2}-x\right)}$

$T=\dfrac{(\tan x)(-\cos x)}{\tan x}=-\cos x$

$\therefore T=-\cos x$

Clave B

20. $E=\dfrac{\csc(-240°)+\sec(-150°)+\cos(-120°)}{\cot(-315°)+\text{sen}(-135°)-\cos(-225°)}$

$E=\dfrac{-\csc240°+\sec150°+\cos120°}{-\cot315°-\text{sen}135°-\cos225°}$

Reducimos cada término al IC y reemplazamos en E:

$E=\dfrac{-(-\csc60°)+(-\sec30°)+(-\cos60°)}{-(-\cot45°)-(\text{sen}45°)-(-\cos45°)}$

$E=\dfrac{\csc60°-\sec30°-\cos60°}{\cot45°-\text{sen}45°+\cos45°}$

$E=\dfrac{\left(\frac{2\sqrt{3}}{3}\right)-\left(\frac{2\sqrt{3}}{3}\right)-\left(\frac{1}{2}\right)}{(1)-\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)}=\dfrac{-\frac{1}{2}}{1}$

$\therefore E=-\dfrac{1}{2}$

Clave B

Nivel 3 (página 40) Unidad 2

Comunicación matemática

21.

22.

Razonamiento y demostración

23. Por dato: $x+y=180°$

$\Rightarrow \tan x=-\tan y \wedge \cos x=-\cos y$

Además:

$3\tan x+2\tan y=\cos x+\cos y+2$

$3\tan x+2(-\tan x)=\cos x+(-\cos x)+2$

$3\tan x-2\tan x=\cos x-\cos x+2$

$\Rightarrow \tan x=2$

Piden:

$V=2\tan x+3\tan y$

$V=2\tan x+3(-\tan x)$

$V=2\tan x-3\tan x=-\tan x$

$\therefore V=-2$

Clave D

24. $L=\cos10°+\cos20°+\cos30°+...+\cos180°$

$L=\cos10°+\cos20°+...+\cos160°+\cos170°+\cos180°$

Si: $x+y=180° \Rightarrow \cos x=-\cos y$

Entonces:

$L=\cos10°+\cos20°+...-\cos20°-\cos10°+\cos180°$

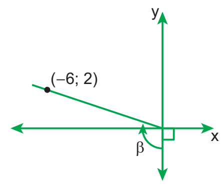
Luego, al simplificar los términos nos quedará solo el término central que es cos90°.

$\Rightarrow L=\cos90°+\cos180°=0+(-1)$

$\therefore L=-1$

Clave D

25.



Del gráfico:

$\cot(90°+\beta)=\dfrac{x}{y}=\dfrac{-6}{2}$

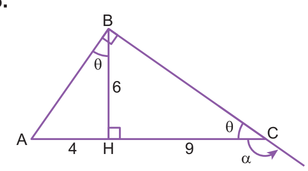
$\cot(90°+\beta)=-3$

$-\tan\beta=-3$

$\therefore \tan\beta=3$

Clave B

26.



Del gráfico:

$\tan\theta=\dfrac{BH}{9}=\dfrac{4}{BH}$

$\Rightarrow BH^2=36 \Rightarrow BH=6$

Además: $\theta+\alpha=180°$

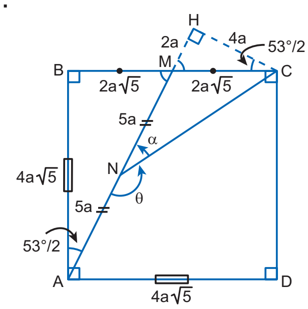
$\Rightarrow \tan\alpha=-\tan\theta$

$\tan\alpha=-\dfrac{BH}{9}=-\dfrac{6}{9}=-\dfrac{2}{3}$

$\therefore \tan\alpha=-\dfrac{2}{3}$

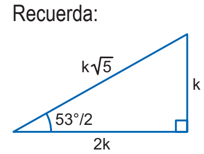
Clave C

27.



Sea: $AB=4a\sqrt{5}$

Recuerda:



Entonces:

$AM=10a$; $MH=2a$ y $HC=4a$

$\Rightarrow \tan\alpha=\dfrac{HC}{HN}=\dfrac{4a}{7a} \Rightarrow \tan\alpha=\dfrac{4}{7}$

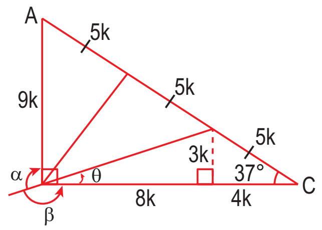
Además: $\theta+\alpha=180°$

$\Rightarrow \tan\theta=-\tan\alpha$

$\therefore \tan\theta=-\dfrac{4}{7}$

Clave B

28.



Del gráfico:

$\tan\theta = \dfrac{3k}{8k} \Rightarrow \tan\theta = \dfrac{3}{8}$

Además: $\beta + \theta = 180°$

$\Rightarrow \tan\beta = -\tan\theta = -\dfrac{3}{8}$

$\tan\beta = -\dfrac{3}{8}$

También: $\beta - \alpha = 270° \Rightarrow \beta = 270° + \alpha$

$\Rightarrow \tan\beta = \tan(270° + \alpha)$

$-\dfrac{3}{8} = -\cot\alpha$

$\cot\alpha = \dfrac{3}{8}$

$\tan\alpha = \dfrac{8}{3}$

Piden:

$\tan\alpha - \tan\beta = \dfrac{8}{3} - \left(-\dfrac{3}{8}\right)$

$\tan\alpha - \tan\beta = \dfrac{8}{3} + \dfrac{3}{8} = \dfrac{73}{24}$

$\therefore\ \tan\alpha - \tan\beta = \dfrac{73}{24}$

Clave C

29. $P = \sum\limits_{n=1}^{3}\left\{\operatorname{sen}\left(n\dfrac{\pi}{2} + x\right) + \cos(n\pi - x)\right\}$

Evaluando:

$n = 1$: $\operatorname{sen}\left(\dfrac{\pi}{2} + x\right) + \cos(\pi - x) = \cos x - \cos x$

$n = 2$: $\operatorname{sen}(\pi + x) + \cos(2\pi - x) = -\operatorname{sen}x + \cos x$

$n = 3$: $\operatorname{sen}\left(\dfrac{3\pi}{2} + x\right) + \cos(3\pi - x) = -\cos x - \cos x$

Entonces:

$P = (\cos x - \cos x) + (-\operatorname{sen}x + \cos x) + (-\cos x - \cos x)$

$\therefore\ P = -\operatorname{sen}x - \cos x$

Clave D

30. $\sum\limits_{n=1}^{3}\left\{\tan\left(n!\dfrac{\pi}{2} + \theta\right)\right\} = 0$

Evaluando:

$n = 1$: $\tan\left(1!\dfrac{\pi}{2} + \theta\right) = \tan\left(\dfrac{\pi}{2} + \theta\right) = -\cot\theta$

$n = 2$: $\tan\left(2!\dfrac{\pi}{2} + \theta\right) = \tan(\pi + \theta) = \tan\theta$

$n = 3$: $\tan\left(3!\dfrac{\pi}{2} + \theta\right) = \tan(3\pi + \theta) = \tan\theta$

Entonces:

$(-\cot\theta) + (\tan\theta) + (\tan\theta) = 0$

$2\tan\theta = \cot\theta$

$\dfrac{2}{\cot\theta} = \cot\theta$

$2 = \cot^2\theta$

$\Rightarrow |\cot\theta| = \sqrt{2}$

$\Rightarrow \cot\theta = \sqrt{2}\ \vee\ \cot\theta = -\sqrt{2}$

Por dato: $\theta \in IC \Rightarrow \cot\theta > 0$

$\therefore E = \cot\theta = \sqrt{2}$

Clave B

CIRCUNFERENCIA TRIGONOMÉTRICA

APLICAMOS LO APRENDIDO (página 41) Unidad 2

1. Tenemos: $\dfrac{\pi}{4} < \theta \le \dfrac{\pi}{3}$

$45° < \theta \le 60°$

$\dfrac{1}{2} \le \cos\theta < \dfrac{\sqrt{2}}{2}$ ... (I)

Reducimos la expresión:

$M = \cos^2\theta - 4\cos\theta + 3 + 1 - 1$

$M = \underbrace{\cos^2\theta - 4\cos\theta + 4} - 1$

$M = (\cos\theta - 2)^2 - 1$

En (I): $-\dfrac{3}{2} \le \cos\theta - 2 < \dfrac{\sqrt{2} - 4}{2}$

$\dfrac{9}{4} \ge (\cos\theta - 2)^2 > \dfrac{9}{2} - 2\sqrt{2}$

$\dfrac{5}{4} \ge (\cos\theta - 2)^2 - 1 \ge \dfrac{7}{2} - 2\sqrt{2}$

$\Rightarrow M \in \left[\dfrac{7}{2} - 2\sqrt{2};\ \dfrac{5}{4}\right]$

$M_{máx.} = \dfrac{5}{4}$

Clave C

2. Reducimos la expresión:

$\dfrac{2}{6+3\operatorname{sen}2x} = \dfrac{5a-4}{3} + \dfrac{3-3a}{2} = \dfrac{10a-8+9-9a}{6}$

$\dfrac{2}{6+3\operatorname{sen}2x} = \dfrac{a+1}{6}$

$a = \dfrac{4}{\operatorname{sen}2x + 2} - 1$

Sabemos: $-1 \le \operatorname{sen}2x \le 1$

$1 \le \operatorname{sen}2x + 2 \le 3$

$\dfrac{1}{3} \le \dfrac{1}{\operatorname{sen}2x + 2} \le 1 \Rightarrow \dfrac{4}{3} \le \dfrac{4}{\operatorname{sen}2x + 2} \le 4$

$\dfrac{1}{3} \le \dfrac{4}{\operatorname{sen}2x + 2} - 1 \le 3$

$\dfrac{1}{3} \le a \le 3$

$a \in \left[\dfrac{1}{3}; 3\right]$

Clave B

3. Reducimos la expresión:

$F = \dfrac{2 - 2\cos 2\theta - \cos^2 2\theta}{\cos 2\theta + 2}$

$F = \dfrac{2 - \cos 2\theta(\cos 2\theta + 2)}{\cos 2\theta + 2}$

$F = \dfrac{2}{\cos 2\theta + 2} - \dfrac{\cos 2\theta(\cos 2\theta + 2)}{\cos 2\theta + 2}$

$F = \dfrac{2}{\cos 2\theta + 2} - \cos 2\theta$

$F = \underbrace{\dfrac{2}{\cos 2\theta + 2}}_{A} \underbrace{- \cos 2\theta}_{B}$ $\quad F = A + B$

Sabemos:

$-1 \le \cos 2\theta \le 1\ ;\ -1 \le \cos 2\theta \le 1$

$1 \le \cos 2\theta + 2 \le 3\ ;\ -1 \le -\cos 2\theta \le 1$

$$\left.\begin{array}{l} 1 \ge \dfrac{1}{\cos 2\theta + 2} \ge \dfrac{1}{3}\ ;\ -1 \le B \le 1 \\ \dfrac{2}{3} \le A \le 2 \qquad \Rightarrow\ \dfrac{2}{3} \le A \le 2 \end{array}\right\} +$$

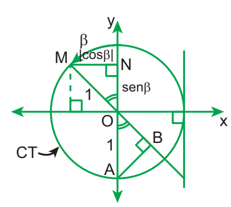
$-\dfrac{1}{3} \le A + B \le 3$

$-\dfrac{1}{3} \le F \le 3$

$F = \left[-\dfrac{1}{3}; 3\right]$

Clave E

4. Del gráfico, tenemos:



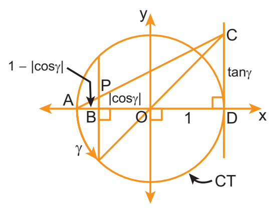
$\triangle MNO \cong \triangle ABO$

$\Rightarrow\ MN = AB$

$\therefore\ AB = |\cos\beta|$

Clave A

5.



Del gráfico, en el ⊿ ADC:

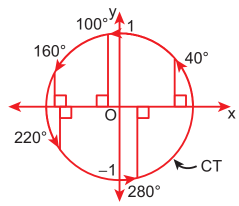
$$\frac{PB}{AB}=\frac{CD}{AD}$$

$$\frac{PB}{1-|\cos\gamma|}=\frac{\tan\gamma}{2}$$

$$\Rightarrow PB=\frac{\tan\gamma(1+\cos\gamma)}{2}$$

Clave E

6.

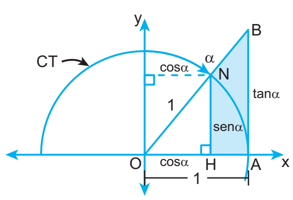


Del gráfico tenemos:

$\underbrace{\text{sen}100°}_{\text{mayor}} > \text{sen}40° > \text{sen}160° > \text{sen}220° > \text{sen}280°$

Clave B

7.



Del gráfico:

Como $\alpha \in$ IC, entonces sus RT son positivas.

Piden:

$A_{somb.} = A_{⊾OAB} - A_{⊾OHN}$

$$A_{somb.}=\frac{(1).(\tan\alpha)}{2}-\frac{(\cos\alpha)(\text{sen}\alpha)}{2}$$

$$\Rightarrow A_{somb.}=\frac{1}{2}(\tan\alpha-\text{sen}\alpha\cos\alpha)$$

Sabemos: $\tan\alpha=\dfrac{\text{sen}\alpha}{\cos\alpha}$

$$\Rightarrow A_{somb.}=\frac{1}{2}\left(\frac{\text{sen}\alpha}{\cos\alpha}-\text{sen}\alpha\cos\alpha\right)$$

$$A_{somb.}=\frac{\text{sen}\alpha}{2\cos\alpha}(1-\cos^2\alpha)$$

$$\Rightarrow A_{somb.}=\frac{\tan\alpha}{2}(1-\cos^2\alpha)$$

En el ⊾OHN por el teorema de Pitágoras:

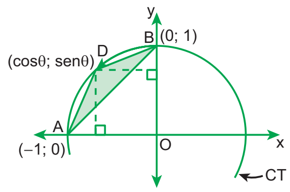
$\cos^2\alpha+\text{sen}^2\alpha=1^2 \Rightarrow \text{sen}^2\alpha=1-\cos^2\alpha$

$$\Rightarrow A_{somb.}=\frac{\tan\alpha}{2}(\text{sen}^2\alpha)$$

$$\therefore A_{somb.}=\frac{\tan\alpha\,\text{sen}^2\alpha}{2}$$

Clave B

8.



Sea S_x el área de la región sombreada.

Del gráfico se tiene:

$A_{AOBD} = S_x + A_{⊿AOB} = A_{\triangle ADO} + A_{\triangle DOB}$

Luego:

$$S_x+\frac{1.1}{2}=\frac{1.\text{sen}\theta}{2}+\frac{1.|\cos\theta|}{2}$$

$2S_x + 1 = \text{sen}\theta - \cos\theta$

$\therefore S_x = 0{,}5(\text{sen}\theta - \cos\theta - 1)$

Clave E

9. Tenemos: $R=\dfrac{2}{(\text{sen}\alpha+2)(\text{sen}\alpha+4)}$

$$R=\frac{2}{\text{sen}^2\alpha+6\text{sen}\alpha+8}$$

$$\Rightarrow R=\frac{2}{\text{sen}^2\alpha+6\text{sen}\alpha+8+1-1}$$

$$R=\frac{2}{(\text{sen}\alpha+3)^2-1}$$

Como $\alpha \in$ IVC, entonces:

$-1 < \text{sen}\alpha < 0$

$2 < \text{sen}\alpha + 3 < 3$

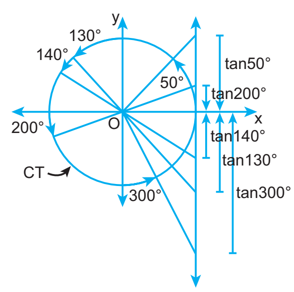
$4 < (\text{sen}\alpha+3)^2 < 9$

$3 < (\text{sen}\alpha+3)^2 - 1 < 8$

$$\frac{1}{4}<\frac{2}{(\text{sen}\alpha+3)^2-1}<\frac{2}{3}$$

Clave A

10. Representamos cada cantidad en la CT:



La cantidad menor es: tan300°

Clave E

11. Operamos la expresión:

$$F=\frac{3+\tan\theta}{2}-\frac{1+\tan\theta}{3}$$

$$F=\frac{9+3\tan\theta-2-2\tan\theta}{6}$$

$$F=\frac{7+\tan\theta}{6}$$

Si $\theta \in$ IIC, entonces:

$-\infty < \tan\theta < 0$

$-\infty < 7 + \tan\theta < 7$

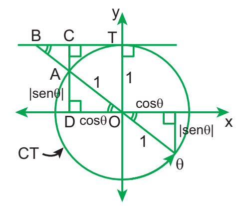
$$-\infty<\frac{7+\tan\theta}{6}<\frac{7}{6}$$

$-\infty < F < 7/6$

$\therefore F_{máx} = 1$

Clave A

12.



Del gráfico tenemos:

AC + AD = CD

AC = CD – AD = 1 – |senθ| ...(I)

$\cos^2\theta + \text{sen}^2\theta = 1$

$$\left(\frac{\sqrt{3}}{2}\right)^2+\text{sen}^2\theta=1 \Rightarrow \text{sen}^2\theta+\frac{3}{4}=1$$

$$\text{sen}^2\theta=\frac{1}{4}$$

|senθ| = 1/2 ...(II)

(II) en (I):

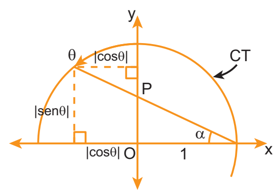
AC = 1 – |senθ|

AC = 1 – 1/2 = 1/2

∴ AC = 1/2

Clave B

13.



Del gráfico:

$$\tan\alpha=\frac{OP}{1}=\frac{|\text{sen}\theta|}{|\cos\theta|+1}$$

$$\Rightarrow OP=\frac{|\text{sen}\theta|}{1+|\cos\theta|}$$

Como: $\theta \in$ IIC $\Rightarrow \text{sen}\theta > 0 \Rightarrow |\text{sen}\theta| = \text{sen}\theta$

$\cos\theta < 0 \Rightarrow |\cos\theta| = -\cos\theta$

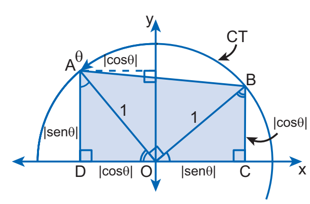
Reemplazando tenemos:

$$\Rightarrow OP=\frac{\text{sen}\theta}{1-\cos\theta}=\frac{\text{sen}\theta}{\text{vers}\theta}$$

$$\therefore OP=\frac{\text{sen}\theta}{\text{vers}\theta}$$

Clave C

14.



Del gráfico: $\triangle ADO \cong \triangle OCB$ (ALA)

$A_{somb.} = A_{\triangle ADO} + A_{\triangle AOB} + A_{\triangle OCB}$

$$A_{somb.} = \frac{|sen\theta||\cos\theta|}{2} + \frac{1.1}{2} + \frac{|sen\theta||\cos\theta|}{2}$$

$$\Rightarrow A_{somb.} = \frac{1}{2} + |sen\theta||\cos\theta|$$

$$\qquad\qquad (+) \qquad (-)$$

$$\Rightarrow A_{somb.} = \frac{1}{2} + (sen\theta)(-\cos\theta)$$

$$\therefore A_{somb.} = \frac{1}{2} - sen\theta\cos\theta$$

Clave D

PRACTIQUEMOS

Nivel 1 (página 43) Unidad 2

Comunicación matemática

1. QM : exsecθ

   QR : exsecα

   CB : covα

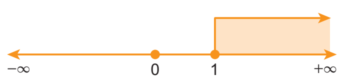
   AB : covθ

   ON : cosθ

   OC : senα

2. A)

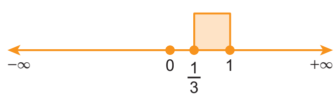
B) $1 \le \sec^2 x \le \infty^+$



C) $-1 \le \cos x \le 1$

$1 \le \cos x + 2 \le 3$

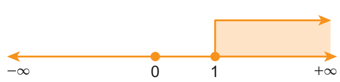
$$\frac{1}{3} \le \frac{1}{\cos x + 2} \le 1$$



D) $0 < \theta \le \pi/4$

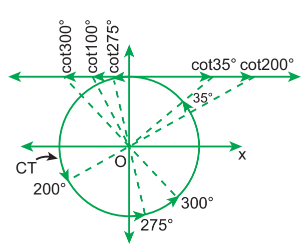
$0 < \tan\theta \le 1$

$$\frac{1}{\tan\theta} \ge 1$$



Razonamiento y demostración

3.



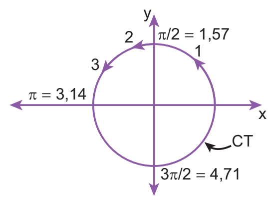
Ordenando de mayor a menor, tenemos:

cot200° > cot35° > cot275° > cot100° > cot300°

Por lo tanto, el menor valor es cot300°.

Clave C

4.



Del gráfico:

$1 \in$ IC; $2 \in$ IIC y $3 \in$ IIC

Entonces:

tan1 = (+); cot2 = (–) y tan3 = (–)

Piden el signo de:

P = tan1cot2tan3

$\Rightarrow$ tan1cot2tan3

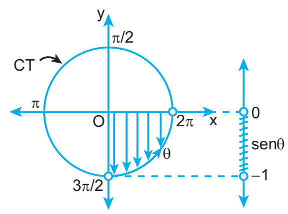
$\Rightarrow$ P = (+)(–)(–) = (+)

$\therefore$ P = (+)

Clave A

5. Por dato: $sen\theta = \frac{a-2}{5}$ y $\theta \in$ IVC

Analizando en la CT:



Se deduce: $-1 < sen\theta < 0$

$$-1 < \frac{a-2}{5} < 0$$

$$-5 < a - 2 < 0$$

$$-3 < a < 2$$

Valores enteros de a: {–2; –1; 0; 1}

Por lo tanto, a puede tomar cuatro valores enteros.

Clave B

6. $M = 2 - 3\tan^2 x$

Sabemos: $-\infty < \tan x < +\infty$

$0 \le \tan^2 x < +\infty$

$0 \le 3\tan^2 x < +\infty$

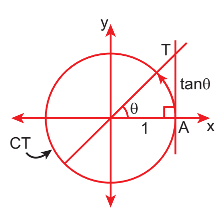
$-\infty < -3\tan^2 x \le 0$

$-\infty < 2 - 3\tan^2 x \le 2$

$-\infty < M \le 2$

Clave B

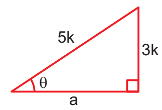
7.



Del gráfico: AT = tanθ

Del dato: $sen\theta = 0,6 = \frac{3}{5}$

Como $\theta \in$ IC, entonces:



Por el teorema de Pitágoras: a = 4k

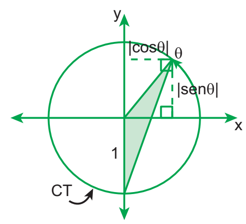
$$\Rightarrow \tan\theta = \frac{3k}{a} = \frac{3k}{4k}$$

$$\Rightarrow \tan\theta = \frac{3}{4}$$

$$\therefore AT = \frac{3}{4} = 0,75$$

Clave C

8.



Del gráfico:

$$A_{somb.} = \frac{(base)(altura)}{2} = \frac{(1)(|\cos\theta|)}{2}$$

$$\Rightarrow A_{somb.} = \frac{|\cos\theta|}{2}$$

Además: $\theta \in$ IC $\Rightarrow \cos\theta > 0$

$\Rightarrow |\cos\theta| = \cos\theta$

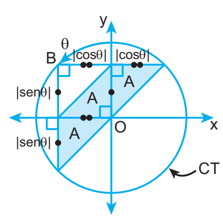
Entonces:

$$A_{somb.} = \frac{|\cos\theta|}{2} = \frac{\cos\theta}{2}$$

$$\therefore A_{somb.} = \frac{1}{2}\cos\theta$$

Clave B

9.



Del gráfico: los cuatro triángulos rectángulos son congruentes.

$$\Rightarrow A = \frac{|sen\theta||\cos\theta|}{2}$$

Como $\theta \in$ IIC $\Rightarrow sen\theta > 0 \wedge \cos\theta < 0$

$$\Rightarrow A = \frac{(sen\theta)(-\cos\theta)}{2} = -\frac{sen\theta\cos\theta}{2}$$

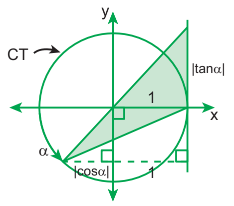
Piden:

$$A_{somb.} = 3A = 3\left(-\frac{sen\theta\cos\theta}{2}\right)$$

$$\therefore A_{somb.} = -\frac{3}{2}sen\theta\cos\theta$$

Clave C

10.



$$A_{somb.} = \frac{(\text{base})(\text{altura})}{2} = \frac{|\tan\alpha|(1+|\cos\alpha|)}{2}$$

Como $\alpha \in \text{IIIC} \Rightarrow \tan\alpha > 0 \wedge \cos\alpha < 0$

$$\Rightarrow A_{somb.} = \frac{(\tan\alpha)(1+(-\cos\alpha))}{2}$$

$$\therefore A_{somb.} = \frac{1}{2}\tan\alpha\,(1-\cos\alpha)$$

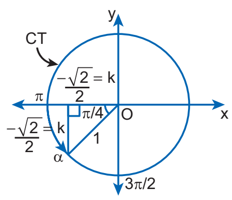
Clave D

Resolución de problemas

11. $\cos^2\alpha + \text{sen}^2\alpha = 1$

$$\left(\frac{\sqrt{2}}{2}\right)^2 + \text{sen}^2\alpha = 1 \Rightarrow \text{sen}^2\alpha = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\alpha \in \text{IIIC} \Rightarrow \text{sen}\alpha = -\frac{\sqrt{2}}{2}$$



$\therefore \alpha = \pi + \pi/4 = 5\pi/4$

$A \leq \cos^2x - 4\cos x - 4 \leq B$
$A \leq \cos^2x - 4\cos x + 4 - 4 - 4 \leq B$
$A \leq (\cos x - 2)^2 - 8 \leq B$

Sabemos:

$$\alpha \leq x \leq 4\pi/3$$
$$\frac{5\pi}{4} \leq x \leq 4\pi/3$$
$$-\frac{\sqrt{2}}{2} \leq \cos x \leq -1/2$$
$$-\frac{\sqrt{2}}{2} - 2 \leq \cos x - 2 \leq -1/2 - 2$$
$$\frac{18+8\sqrt{2}}{4} \geq (\cos x - 2)^2 \geq \frac{25}{4}$$
$$\underbrace{\frac{8\sqrt{2}-14}{4}}_{A} \geq (\cos x - 2)^2 - 8 \geq \underbrace{-\frac{7}{4}}_{B}$$
$$A + B = \frac{8\sqrt{2}-14}{4} - \frac{7}{4}$$
$$\therefore 4(A+B) = 8\sqrt{2} - 21$$

Clave C

12. Reducimos la igualdad:

$(2\text{sen}\theta - 1)(\text{sen}x - \cos x) = (\text{sen}x + \cos x)$

$(2\text{sen}\theta)(\text{sen}x - \cos x) - \text{sen}x + \cos x = \text{sen}x + \cos x$

$(2\text{sen}\theta)(\text{sen}x - \cos x) = 2\text{sen}x$

$$(\text{sen}\theta)\left(\frac{\text{sen}x}{\text{sen}x} - \frac{\cos x}{\text{sen}x}\right) = 1$$
$$\text{sen}\theta = \frac{1}{1-\cot x}$$

Sabemos:

$\theta \in \text{IC}$
$0 < \theta < \pi/2$
$0 < \text{sen}\theta < 1$
$0 < \frac{1}{1-\cot x} < 1$
$1 < 1 - \cot x < +\infty$
$0 < -\cot x < +\infty$
$-\infty < \cot x < 0$
$x \in \left\langle \frac{\pi}{2}; \pi \right\rangle \cup \left\langle \frac{3\pi}{2}; 2\pi \right\rangle$

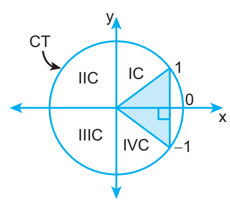
Clave B

Nivel 2 (página 44) Unidad 2

Comunicación matemática

13. Sabemos:

$\alpha \in \text{IIC}; \beta \in \text{IIC}$
$\pi/2 < \alpha < \pi;\ \pi/2 < \beta < \pi$
$\underbrace{0 < \text{sen}\alpha < 1;\ -1 < \cos\beta < 0}_{+}$
$-1 < \text{sen}\alpha + \cos\beta < 1$
$\Rightarrow -1 < \gamma < 1$



I. $\gamma \in$ IC o IVC (V)
II. $\gamma \in$ IIIC (F)
III. γ es cuadrantal (F)
IV. $\text{sen}\gamma \in [0; 1\rangle$ (F)
$\therefore$ VFFF

Clave A

14.
I. senx > cosx
¡No es necesario!

II. $x \in \left\langle \frac{2}{3}\pi; \frac{5\pi}{6} \right\rangle$

$\Rightarrow \frac{2}{3}\pi < x < \frac{5\pi}{6}$

$\frac{1}{2} < \text{sen}x < \frac{\sqrt{3}}{2}$ ... (a)

$-\frac{\sqrt{3}}{2} < \cos x < -1/2$

$\frac{1}{4} < \cos^2x < 3/4$ ... (b)

(a) + (b):

$\frac{3}{4} < \cos^2x + \text{sen}x < \frac{3}{4} + \frac{\sqrt{3}}{2}$

III. ¡No es necesario!

Clave D

Razonamiento y demostración

15. Por dato: $\text{sen}\alpha = \frac{k-1}{2}$

Sabemos: $-1 \leq \text{sen}\alpha \leq 1$

$-1 \leq \frac{k-1}{2} \leq 1$
$-2 \leq k - 1 \leq 2$
$-1 \leq k \leq 3$
$\therefore k \in [-1; 3]$

Clave C

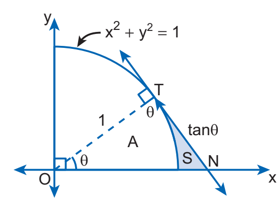
16. $E = 3 + 2\tan^2x$

Sabemos: $-\infty < \tan x < +\infty$
$0 \leq \tan^2x < +\infty$
$0 \leq 2\tan^2x < +\infty$
$3 \leq E < +\infty$

$\Rightarrow E \in [3; +\infty\rangle$
$\therefore E_{mín.} = 3$

Clave C

17.



Del gráfico:

$A_{\triangle OTN} = A + S$

$\Rightarrow \frac{1.\tan\theta}{2} = \frac{\theta.(1)^2}{2} + S$

$\Rightarrow 2S = \tan\theta - \theta$ ...(1)

Piden:
$M = (2S + \theta)\cot\theta$ ...(2)

Reemplazando (1) en (2):
$\Rightarrow M = (\tan\theta - \theta + \theta)\cot\theta$
$M = \tan\theta\cot\theta$

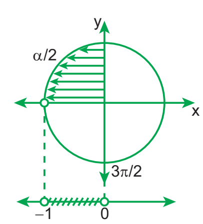
Por razones trigonométricas recíprocas:
$\tan\theta\cot\theta = 1$
$\therefore M = 1$

Clave D

18. Por dato: $\pi < \alpha < 2\pi$

$\Rightarrow \frac{\pi}{2} < \frac{\alpha}{2} < \pi$

Analizando en la CT:



Se deduce: $-1 < \cos\frac{\alpha}{2} < 0$

Piden la variación de:

$M = 3\cos\frac{\alpha}{2} - 1$

Como:

$-1 < \cos\frac{\alpha}{2} < 0$

$-3 < 3\cos\frac{\alpha}{2} < 0$

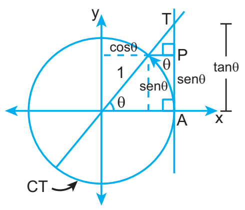
$-4 < 3\cos\frac{\alpha}{2} - 1 < -1$

$\Rightarrow -4 < M < -1$

$\therefore\ M \in \langle -4; -1 \rangle$

Clave D

19.



Como $\theta \in$ IC, entonces todas sus razones trigonométricas son positivas.

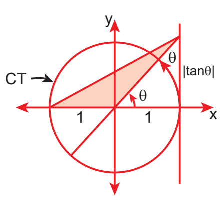
Luego: $AP + PT = AT$

$\Rightarrow \operatorname{sen}\theta + PT = \tan\theta$

$\therefore\ PT = \tan\theta - \operatorname{sen}\theta$

Clave C

20.



$A_{somb.} = \dfrac{(\text{base})(\text{altura})}{2} = \dfrac{(1)|\tan\theta|}{2}$

$\Rightarrow A_{somb.} = \dfrac{|\tan\theta|}{2}$

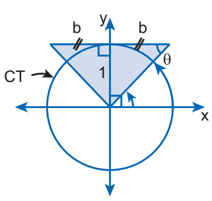
Como $\theta \in \text{IC} \Rightarrow \tan\theta > 0$

$\Rightarrow A_{somb.} = \dfrac{(\tan\theta)}{2}$

$\therefore\ A_{somb.} = \dfrac{1}{2}\tan\theta$

Clave A

21.



$A_{somb.} = \dfrac{(\text{base})(\text{altura})}{2} = \dfrac{(2b)(1)}{2}$

$\Rightarrow A_{somb.} = b$

Del gráfico: $b = \cot\theta$

$\Rightarrow A_{somb.} = b = \cot\theta$

$\therefore A_{somb.} = \cot\theta$

Clave A

Resolución de problemas

22. Simplificamos la expresión:

$T = \dfrac{4 - 4\cos\alpha - \operatorname{sen}^2\alpha}{\cos\alpha - \cot\frac{53°}{2}}$

$T = \dfrac{4 - 4\cos\alpha - (1 - \cos^2\alpha)}{\cos\alpha - 2}$

$T = \dfrac{\cos^2\alpha - 4\cos\alpha + 4 - 1}{\cos\alpha - 2}$

$T = \dfrac{(\cos\alpha - 2)^2 - 1}{\cos\alpha - 2}$

$T = \underbrace{(\cos\alpha - 2)}_{A} - \underbrace{\dfrac{1}{\cos\alpha - 2}}_{B}$

Sabemos: $\alpha \in$ IVC

$\dfrac{3\pi}{2} < \alpha < 2\pi$

$0 < \cos\alpha < 1$

$\underbrace{-2 < \cos\alpha - 2}_{A} < -1 \qquad -2 < \cos\alpha - 2 < -1$

$-1 < \dfrac{1}{\cos\alpha - 2} < \dfrac{-1}{2}$

$\dfrac{1}{2} < \underbrace{-\dfrac{1}{\cos\alpha - 2}}_{B} < 1$

$-2 < A < -1$

$\dfrac{1}{2} < B < 1$

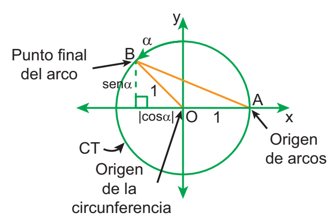
$-\dfrac{3}{2} < A + B < 0 \Rightarrow -\dfrac{3}{2} < T < 0$

$T_{entero} = \{-1\}$

$\therefore\ \Sigma_{val.} = -1$

Clave B

23.



Perímetro $\triangle$ BOA:

$p = BO + OA + AB$

$p = 1 + 1 + \sqrt{\operatorname{sen}^2\alpha + (|\cos\alpha| + 1)^2}$

$p = 2 + \sqrt{\operatorname{sen}^2\alpha + (1 - \cos\alpha)^2}$

$p = 2 + \sqrt{\operatorname{sen}^2\alpha + 1 - 2\cos\alpha + \cos^2\alpha}$

$p = 2 + \sqrt{2 - 2\cos\alpha}$

Clave A

Nivel 3 (página 44) Unidad 2

Comunicación matemática

24.

$M:\ (\operatorname{sen}\theta + \cos^2\theta)$

$-1 \le \operatorname{sen}\theta \le 1$

$0 \le \cos^2\theta \le 1$

$-1 \le \operatorname{sen}\theta + \cos^2\theta \le 2$

$-1 \le \dfrac{x-2}{3} \le 2$

$-3 \le x - 2 \le 6$

$-1 \le x \le 8$

$M = x_{mín.} = -1$

$N: 0 \le \operatorname{sen}^2\theta \le 1$

$-1 \le \cos\theta \le 1$

$-1 \le \cos\theta + \operatorname{sen}^2\theta \le 2$

$-1 \le \dfrac{k+3}{2} \le 2$

$-2 \le k + 3 \le 4$

$-5 \le k \le 1$

$N = k_{máx.} = 1$

$\therefore\ M + N = 0$

Clave B

25.

I. $0° < 90°$, pero $\cos 0° > \cos 90°$ (F)

II. $x_1 > x_2 \ \wedge\ x_1; x_2 \in$ IIIC

$\Rightarrow \tan x_1 > \tan x_2$ (F)

III. $x_1; x_2 \in$ IIIC $\wedge\ x_1 > x_2$

$\Rightarrow \operatorname{sen}x_1 < \operatorname{sen}x_2$ (F)

IV. $x_1; x_2 \in$ IC $\wedge\ x_1 < x_2$

$\Rightarrow \cot x_2 < \cot x_1$ (V)

$\therefore$ 3 falsas

Clave C

Razonamiento y demostración

26. Del ejercicio anterior:

Si: $\theta \in \text{IVC} \Rightarrow \operatorname{sen}\theta \in \langle -1; 0 \rangle$

Por dato: $\operatorname{sen}\theta = \dfrac{2n-5}{3}$

$\Rightarrow -1 < \operatorname{sen}\theta < 0$

$-1 < \dfrac{2n-5}{3} < 0$

$-3 < 2n - 5 < 0$

$2 < 2n < 5$

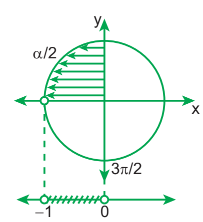
$1 < n < \dfrac{5}{2}$

$\therefore n \in \left\langle 1; \dfrac{5}{2} \right\rangle$

Clave A

27. Por dato: $\cos\theta = \dfrac{k-3}{5}$ y $\theta \in$ IIC

Analizando en la CT:



Se deduce: $-1 < \cos\theta < 0$

$-1 < \dfrac{k-3}{5} < 0$

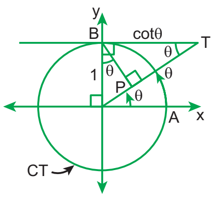
$-5 < k - 3 < 0$

$-2 < k < 3$

$\therefore\ k \in \langle -2; 3 \rangle$

Clave C

28.



Como $\theta \in$ IC, entonces todas sus razones trigonométricas son positivas.

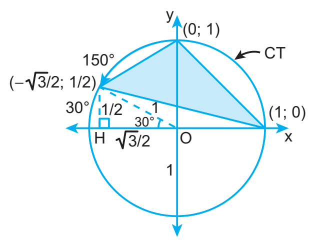
Luego, en el ⊿ BPT:

$PT = (\cot\theta)\cos\theta$

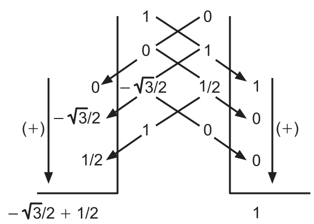
$\therefore PT = \cos\theta\cot\theta$

Clave D

29.



Piden: el área de la región sombreada.
Entonces:



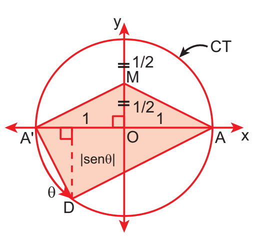
$$A_{somb.} = \frac{\left|1-\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}\right)\right|}{2}$$

$$A_{somb.} = \frac{\left|\frac{\sqrt{3}}{2}+\frac{1}{2}\right|}{2} = \frac{\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)}{2}$$

$$\therefore A_{somb.} = \frac{\sqrt{3}}{4}+\frac{1}{4}$$

Clave A

30.



Del gráfico:

$A_{somb.} = A_{\Delta AMA'} + A_{\Delta ADA'}$

$$A_{somb.} = \frac{(2)\left(\frac{1}{2}\right)}{2} + \frac{(2).|\operatorname{sen}\theta|}{2}$$

$$\Rightarrow A_{somb.} = \frac{1}{2} + |\operatorname{sen}\theta|$$

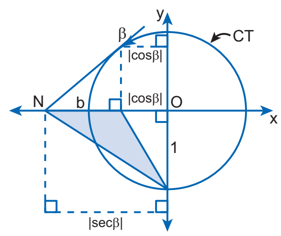
Como $\theta \in$ IIIC $\Rightarrow \operatorname{sen}\theta < 0$

$$\Rightarrow A_{somb.} = \frac{1}{2} + (-\operatorname{sen}\theta)$$

$$\therefore A_{somb.} = \frac{1}{2} - \operatorname{sen}\theta$$

Clave B

31.



Por dato: $A_{somb.} = 2$

$$\Rightarrow \frac{b\,.\,1}{2} = 2 \Rightarrow b = 4$$

Del gráfico:

$$b = \underset{(-)}{|\sec\beta|} - \underset{(-)}{|\cos\beta|}$$

$\Rightarrow b = -\sec\beta - (-\cos\beta)$

$\Rightarrow \cos\beta - \sec\beta = b = 4$ ...(1)

Piden:

$H = \operatorname{sen}^2\beta + \cos^2\beta$

Elevando (1) al cuadrado:

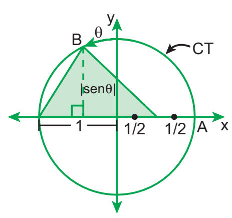
$$\cos^2\beta - \underbrace{2\cos\beta\sec\beta}_{1} + \sec^2\beta = 4^2$$

$\Rightarrow \cos^2\beta + \sec^2\beta = 16 + 2 = 18$

$\therefore H = 18$

Clave D

32.



$$A_{somb.} = \frac{(\text{base})(\text{altura})}{2} = \frac{\left(\frac{3}{2}\right)(|\operatorname{sen}\theta|)}{2}$$

$$\Rightarrow A_{somb.} = \frac{3}{4}|\operatorname{sen}\theta|$$

Como $\theta \in$ IIC $\Rightarrow \operatorname{sen}\theta > 0$

$\Rightarrow |\operatorname{sen}\theta| = \operatorname{sen}\theta$

Entonces:

$$A_{somb.} = \frac{3}{4}|\operatorname{sen}\theta| = \frac{3}{4}\operatorname{sen}\theta$$

$$\therefore A_{somb.} = \frac{3}{4}\operatorname{sen}\theta$$

Clave A

Resolución de problemas

33. $0 \le \alpha \le \pi/6$

$\pi/3 \le \alpha + \pi/3 \le \pi/2$

$$\frac{\sqrt{3}}{2} \le \operatorname{sen}(\alpha + \pi/3) \le 1$$

$$\frac{3}{4} \le \operatorname{sen}^2(\alpha + \pi/3) \le 1$$

$3 \le 4\operatorname{sen}^2(\alpha + \pi/3) \le 4$

$3 \le 1 - \sec\phi \le 4$

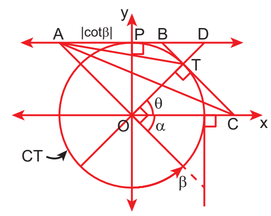
$2 \le -\sec\phi \le 3$

$-3 \le \sec\phi \le -2$

$\Rightarrow \sec\phi \in [-3; -2]$

Clave E

34.



Del gráfico:

$\alpha + \theta = 90°$

$PD = \cot\theta = \tan\alpha = |\tan\beta|$

$\Rightarrow OD^2 = PD^2 + PO^2$

$OD^2 = \tan^2\beta + 1^2$ $\quad\therefore OD = |\sec\beta|$

$\Rightarrow AO^2 = PO^2 + AP^2$

$AO^2 = 1^2 + \cot^2\beta$

$\therefore AO = |\csc\beta|$

En el Δ AOD:

$$\frac{OD}{AO} = \frac{DT}{BT}$$

$$BT = \frac{(|\sec\beta| - 1)(\csc\beta)}{|\sec\beta|}$$

$\overleftrightarrow{x}$ // Lcot $\wedge$ $\overline{AO}$ // $\overline{BC}$

$\Rightarrow BC = AO$

$BT + TC = AO$

$TC = AO - BT$

$$TC = |\csc\beta| - \frac{(|\sec\beta| - 1)|\csc\beta|}{|\sec\beta|}$$

$$TC = \frac{|\sec\beta||\csc\beta| - |\sec\beta||\csc\beta| + |\csc\beta|}{|\sec\beta|}$$

$$TC = \frac{|\csc\beta|}{|\sec\beta|}$$

$\beta \in$ IVC $\Rightarrow |\csc\beta| = -\csc\beta$

$|\sec\beta| = \sec\beta$

$$TC = \frac{-\csc\beta}{\sec\beta} = -\cot\beta$$

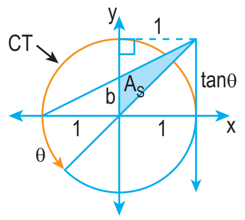
$$\text{Área}_{\Delta ATC} = \frac{TC \cdot TO}{2} = \frac{(-\cot\beta)\cdot(1)}{2}$$

$$\therefore \text{Área}_{\Delta ATC} = -\frac{\cot\beta}{2}$$

Clave D

MARATÓN MATEMÁTICA (página 41) Unidad 2

1.



$A_S = \dfrac{b\,h}{2}$

$\dfrac{b}{1} = \dfrac{\tan\theta}{2}$

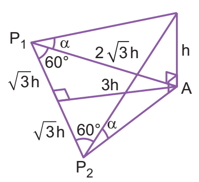
$\Rightarrow b = \dfrac{\tan\theta}{2}$

$A_S = \left(\dfrac{\tan\theta}{2}\right)\dfrac{1}{2}$

$\therefore\ A_S = \dfrac{\tan\theta}{4}$

Clave A

2.



Luego:

$\cot\alpha = \dfrac{2\sqrt{3}\,h}{h} = 2\sqrt{3}$

Clave E

3.

$180° < \theta < 270°$

Para A:

$90° < 2\theta - 270° < 270°$

$\Rightarrow \cos(2\theta - 270°) < 0$

$(-)$

$90° < \left(\dfrac{\theta}{2}\right) < 135°$

$\Rightarrow \tan\left(\dfrac{\theta}{2}\right) < 0$

$(-)$

$\therefore\ A = (-)\,(-) = (+)$

Para B:

$120° < \dfrac{\theta + 60°}{2} < 165°$

$\tan\left(\dfrac{\theta + 60°}{2}\right) < 0$

$\therefore\ B = (-)$

Clave B

4. De la condición tenemos:

$\tan\left((2k)\dfrac{\pi}{4} + \dfrac{\pi}{2} + \beta\right) = -\dfrac{3}{4}$

$\tan\left(k\pi + \dfrac{\pi}{2} + \beta\right) = -\dfrac{3}{4}$

$-\cot\beta = -\dfrac{3}{4} \Rightarrow \tan\beta = \dfrac{4}{3}$

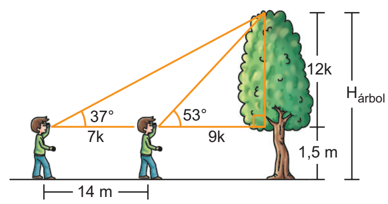
Nos piden:

$P = \operatorname{sen}\left(-\dfrac{3\pi}{2} + \beta\right) = \cos\beta = \dfrac{3}{5}$

$\therefore\ P = \dfrac{3}{5}$

Clave A

5.



- Del gráfico:

  $7k = 14$ m

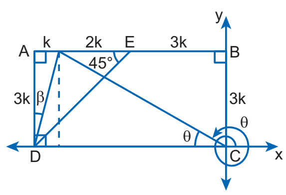
  $k = 2$ m $\Rightarrow H_{árbol} = 12k + 1{,}5$ m

  $H_{árbol} = 12(2\text{ m}) + 1{,}5$ m

  $\therefore\ H_{árbol} = 25{,}5$ m

Clave C

6.

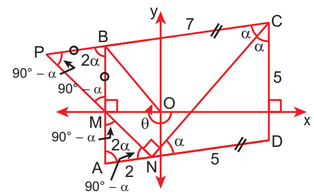


Del gráfico tenemos:

$\tan\theta = \dfrac{3k}{-5k} = -\dfrac{3}{5}$

Clave A

7.



Prolongamos $\overline{NM}$ y $\overline{CB}$, $\triangle PBM$ isósceles.

El $\triangle MAN$ es isósceles, entonces:

$\Rightarrow AN = AM = 2 = MO$

Luego: $AM + MB = 5$

$2 + MB = 5 \Rightarrow MB = 3$

Nos piden $\cot\theta$ ($\theta \in IIC$):

$\cot\theta = \dfrac{3}{-2} = -\dfrac{3}{2}$

Clave E

8.

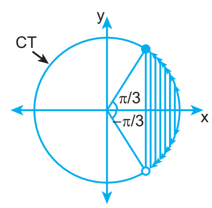
$-\dfrac{7\pi}{24} \le \theta < \dfrac{\pi}{24}$

$-\dfrac{7\pi}{12} \le 2\theta < \dfrac{\pi}{12}$

$-\dfrac{\pi}{12} \le -2\theta \le \dfrac{7\pi}{12}$

$-\dfrac{\pi}{3} < -\dfrac{\pi}{4} - 2\theta \le \dfrac{\pi}{3}$

Analizamos en la CT:



$\therefore\ \operatorname{sen}\left(-\dfrac{\pi}{4} - 2\theta\right) \in \left\langle -\dfrac{\sqrt{3}}{2}; \dfrac{\sqrt{3}}{2}\right]$

Clave C

APLICAMOS LO APRENDIDO (página 48) Unidad 3

1. Sabemos: $\text{sen}^4x + \cos^4x = 1 - 2\text{sen}^2x\cos^2x$

Reemplazamos en z:

$z = \text{sen}^4x + \cos^4x + 2\text{sen}^2x\cos^2x$

$z = 1 - 2\text{sen}^2x\cos^2x + 2\text{sen}^2x\cos^2x$

$\therefore\ z = 1$

Clave D

2. Sabemos: $\sec^2x = 1 + \tan^2x$

Entonces:

$(\sec x)^2 = (5 - \tan x)^2$

$\sec^2x = 25 - 10\tan x + \tan^2x$

$1 + \tan^2x = 25 - 10\tan x + \tan^2x$

$10\tan x = 24$

$\tan x = 12/5$

Clave E

3. Tenemos:

$$E = \frac{\text{sen}\phi}{1-\text{sen}\phi} + \frac{\sec\phi}{\sec\phi + \tan\phi} - \tan^2\phi$$

$$E = \frac{\text{sen}\phi}{1-\text{sen}\phi} + \frac{\frac{1}{\cos\phi}}{\frac{1}{\cos\phi} + \frac{\text{sen}\phi}{\cos\phi}} - \frac{\text{sen}^2\phi}{\cos^2\phi}$$

$$E = \frac{\text{sen}\phi}{1-\text{sen}\phi} + \frac{\frac{1}{\cos\phi}}{\frac{1+\text{sen}\phi}{\cos\phi}} - \frac{\text{sen}^2\phi}{\cos^2\phi}$$

$$E = \frac{\text{sen}\phi}{1-\text{sen}\phi} + \frac{1}{1+\text{sen}\phi} - \frac{\text{sen}^2\phi}{\cos^2\phi}$$

$$E = \frac{\text{sen}\phi + \text{sen}^2\phi + 1 - \text{sen}\phi}{1-\text{sen}^2\phi} - \frac{\text{sen}^2\phi}{\cos^2\phi}$$

$$E = \frac{\text{sen}^2\phi + 1}{\cos^2\phi} - \frac{\text{sen}^2\phi}{\cos^2\phi}$$

$$= \frac{\text{sen}^2\phi + 1 - \text{sen}^2\phi}{\cos^2\phi}$$

$\therefore\ E = \sec^2\phi$

Clave C

4. $(\text{sen}x + \cos x)^2 = (n)^2$

$\text{sen}^2x + 2\text{sen}x\cos x + \cos^2x = n^2$

$2\text{sen}x\cos x = n^2 - 1$

$\text{sen}x\cos x = \frac{n^2-1}{2}$ ...(1)

En D, tenemos:

$D = \sec x + \csc x$

$D = \frac{1}{\cos x} + \frac{1}{\text{sen}x}$

$D = \frac{\text{sen}x + \cos x}{\text{sen}x\cos x}$ ... (2)

Reemplazamos (1) en (2):

$$D = \frac{\text{sen}x + \cos x}{\text{sen}x\cos x} = \frac{n}{\frac{n^2-1}{2}} = \frac{2n}{n^2-1}$$

$\therefore\ D = \frac{2n}{n^2-1}$

Clave D

5. $M = \sec^2x\csc^2x - \frac{\cot^3x - \tan^3x}{\cot x - \tan x}$

Se tiene:

$$\frac{\cot^3x - \tan^3x}{\cot x - \tan x} = \frac{(\cot x - \tan x)(\cot^2x + \overbrace{\cot x\tan x}^{1} + \tan^2x)}{\cot x - \tan x}$$

$\frac{\cot^3x - \tan^3x}{\cot x - \tan x} = \cot^2x + \tan^2x + 1$ ...(1)

Además:

$\sec^2x\csc^2x = \sec^2x + \csc^2x$

$\sec^2x\csc^2x = (1 + \tan^2x) + (1 + \cot^2x)$

$\Rightarrow \sec^2x\csc^2x = 2 + \tan^2x + \cot^2x$ ...(2)

Reemplazando (2) y (1) en M:

$M = (2 + \tan^2x + \cot^2x) - (\cot^2x + \tan^2x + 1)$

$\Rightarrow M = 2 - 1 = 1$

$\therefore M = 1$

Clave B

6. Piden:

$C = \sec^2x + \csc^2x$

$C = (1 + \tan^2x) + (1 + \cot^2x)$

$C = \tan^2x + \cot^2x + 2$ ...(1)

Por dato:

$\tan x + \cot x = 3\sqrt{2}$

$(\tan x + \cot x)^2 = (3\sqrt{2})^2$

$\Rightarrow \tan^2x + 2\underbrace{\tan x\cot x}_{1} + \cot^2x = 18$

$\Rightarrow \tan^2x + \cot^2x = 16$ ...(2)

Reemplazando (2) en (1):

$\Rightarrow C = 16 + 2 = 18$

$\therefore C = 18$

Clave D

7. $C = \frac{\text{sen}x\tan x + \cos x}{\cos x\cot x + \text{sen}x}$

$$C = \frac{\text{sen}x\left(\frac{\text{sen}x}{\cos x}\right) + \cos x}{\cos x\left(\frac{\cos x}{\text{sen}x}\right) + \text{sen}x}$$

$$C = \frac{\frac{\text{sen}^2x + \cos^2x}{\cos x}}{\frac{\cos^2x + \text{sen}^2x}{\text{sen}x}} = \frac{\frac{1}{\cos x}}{\frac{1}{\text{sen}x}}$$

$C = \frac{\text{sen}x}{\cos x} = \tan x$

$\therefore\ C = \tan x$

Clave B

8. Por dato:

$\text{sen}^4x + \cos^4x = \frac{7}{9}$

$\Rightarrow 1 - 2\text{sen}^2x\cos^2x = \frac{7}{9}$

$1 - \frac{7}{9} = 2\text{sen}^2x\cos^2x$

$\frac{2}{9} = 2\text{sen}^2x\cos^2x$

$\Rightarrow \text{sen}^2x\cos^2x = \frac{1}{9}$

Piden:

$C = \text{sen}^6x + \cos^6x = 1 - 3\text{sen}^2x\cos^2x$

$\Rightarrow C = 1 - 3\left(\frac{1}{9}\right) = 1 - \frac{1}{3}$

$\therefore\ C = \frac{2}{3}$

Clave B

9. Por dato:

$\text{sen}^2\alpha - \cos^2\alpha = \frac{1}{2}$ ...(1)

Por identidad trigonométrica:

$\text{sen}^2\alpha + \cos^2\alpha = 1$ ...(2)

De (1) y (2):

$\text{sen}^2\alpha = \frac{3}{4}$ $(\alpha \in \text{IC}) \Rightarrow \text{sen}\alpha > 0 \wedge \cos\alpha > 0$

$\Rightarrow \text{sen}\alpha = \frac{\sqrt{3}}{2} \wedge \cos\alpha = \frac{1}{2}$

Piden:

$\tan\alpha + \cot\alpha = \sec\alpha\csc\alpha$

$$\Rightarrow \tan\alpha + \cot\alpha = \frac{1}{\cos\alpha\,\text{sen}\alpha} = \frac{1}{\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}$$

$\Rightarrow \tan\alpha + \cot\alpha = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\therefore \tan\alpha + \cot\alpha = \frac{4\sqrt{3}}{3}$

Clave B

10. $E = \left(\frac{\text{sen}x}{1+\cos x} + \frac{1+\cos x}{\text{sen}x}\right)^2 - 4\cot^2x$

Por propiedad:

$\frac{1-\cos x}{\text{sen}x} = \frac{\text{sen}x}{1+\cos x}$

Reemplazando en E:

$E = \left(\frac{1-\cos x}{\text{sen}x} + \frac{1+\cos x}{\text{sen}x}\right)^2 - 4\cot^2x$

$E = \left(\frac{2-\cos x+\cos x}{\text{sen}x}\right)^2 - 4\cot^2x$

$E = \left(\frac{2}{\text{sen}x}\right)^2 - 4\cot^2x$

$E = \frac{4}{\text{sen}^2x} - 4\cot^2x = 4\csc^2x - 4\cot^2x$

$E = 4\underbrace{(\csc^2x - \cot^2x)}_{1} = 4$

$\therefore\ E = 4$

Clave C

11. $\frac{\csc x - \cot x}{\csc x + \cot x} + \frac{\csc x + \cot x}{\csc x - \cot x} = M + 4\cot^Nx$

$$\frac{(\csc x - \cot x)^2 + (\csc x + \cot x)^2}{(\csc x + \cot x)(\csc x - \cot x)} = M + 4\cot^Nx$$

$$\frac{2(\csc^2x + \cot^2x)}{\csc^2x - \cot^2x} = M + 4\cot^Nx$$

Por identidad:

$$\csc^2x - \cot^2x = 1$$
$$\Rightarrow 2\csc^2x + 2\cot^2x = M + 4\cot^Nx$$
$$2(1+\cot^2x) + 2\cot^2x = M + 4\cot^Nx$$
$$2 + 4\cot^2x = M + 4\cot^Nx$$

Comparando: $M = 2 \wedge N = 2$

Piden:

$$M + N = 2 + 2 = 4$$
$$\therefore M + N = 4$$

Clave D

12. $T = \text{sen}^4\theta - \dfrac{\tan^2\theta}{(1+\tan^2\theta)+\csc^2\theta}$

$$T = \text{sen}^4\theta - \frac{\tan^2\theta}{(\sec^2\theta)+\csc^2\theta}$$
$$T = \text{sen}^4\theta - \frac{\tan^2\theta}{\sec^2\theta\csc^2\theta}$$
$$T = \text{sen}^4\theta - \cos^2\theta\,\text{sen}^2\theta\left(\frac{\text{sen}^2\theta}{\cos^2\theta}\right)$$

Luego:

$$T = \text{sen}^4\theta - \text{sen}^4\theta = 0$$
$$\therefore T = 0$$

Clave A

13. $S = (1+\cot^2\theta)\cos^2\theta - \csc^2\theta$

$$S = (\csc^2\theta)\cos^2\theta - \csc^2\theta$$
$$S = \frac{\cos^2\theta}{\text{sen}^2\theta} - \csc^2\theta$$
$$S = \cot^2\theta - \csc^2\theta = -(\underbrace{\csc^2\theta - \cot^2\theta}_{1})$$
$$\therefore S = -1$$

Clave C

14. Por dato: $\text{sen}x + \csc x = 3$

Piden:

$L = \text{sen}^2x + \csc^2x$

Luego:

$$(\text{sen}x + \csc x)^2 = (3)^2$$
$$\underbrace{\text{sen}^2x + \csc^2x}_{L} + 2\underbrace{\text{sen}x\csc x}_{1} = 9$$
$$\Rightarrow L + 2 = 9$$
$$\therefore L = 7$$

Clave C

PRACTIQUEMOS

Nivel 1 (página 50) Unidad 3

Comunicación matemática

1.

I. $\dfrac{\dfrac{1}{\text{sen}x}}{\dfrac{\cos x}{\text{sen}x} + \dfrac{\text{sen}x}{\cos x}} = \dfrac{\cos x\,\text{sen}x}{\text{sen}x\underbrace{(\text{sen}^2x + \cos^2x)}_{1}}$

$$\therefore \boxed{\cos x} = \cos x$$

II. $\dfrac{1}{\cos x} + \dfrac{\text{sen}x}{\cos x} = \dfrac{\cos x}{1 - \boxed{}}$

$$\frac{1+\text{sen}x}{\cos x} = \frac{\cos x}{1-\text{sen}x} = \frac{\cos x}{1-\boxed{}}$$
$$\therefore \boxed{\text{sen}x} = \text{sen}x$$

III. $\dfrac{1}{\csc^2x} + \dfrac{1}{\boxed{}} = 1$

$$\text{sen}^2x + \frac{1}{\boxed{}} = 1$$
$$\frac{1}{\boxed{}} = 1 - \text{sen}^2x = \cos^2x$$
$$\frac{1}{\boxed{}} = \frac{1}{\sec^2x}$$
$$\therefore \boxed{} = \sec^2x$$

IV. $\boxed{}\,.\,\cos^2x = 1 - \cos^2x$

$$\boxed{}\,.\,\cos^2x = \text{sen}^2x$$
$$\therefore \boxed{} = \tan^2x$$

V. $\cot x + \dfrac{\boxed{}}{1+\cos x} = \csc x$

$$\frac{\cos x}{\text{sen}x} + \frac{\boxed{}}{(1+\cos x)} = \frac{1}{\text{sen}x}$$
$$\frac{\cos x + \cos^2x + \text{sen}x\boxed{}}{\text{sen}x(1+\cos x)} = \frac{1}{\text{sen}x}$$
$$\cos x + \cos^2x + \text{sen}x\,.\,\boxed{} = 1 + \cos x$$
$$\text{sen}x\,.\,\boxed{} = 1 - \cos^2x$$
$$\text{sen}x\,.\,\boxed{} = \text{sen}^2x$$
$$\therefore \boxed{} = \text{sen}x$$

2. a. $\text{sen}^4x - \cos^4x = \overbrace{(\text{sen}^2x + \cos^2x)}^{1}(\text{sen}^2x - \cos^2x)$

$$\therefore \text{sen}^4x - \cos^4x = \text{sen}^2x - \cos^2x \Rightarrow \boxed{V}$$

b. $\tan x\,\text{sen}x + \cos x = \csc x$

$$\frac{\text{sen}x\,\text{sen}x}{\cos x} + \cos x =$$
$$\frac{\overbrace{\text{sen}^2x + \cos^2x}^{1}}{\cos x} = \sec x \neq \csc x \quad \boxed{F}$$

c. $\cot^2x\,\text{sen}^2x = 1 - \text{sen}^2x$

$$\frac{\cos^2x}{\text{sen}^2x}(\text{sen}^2x) = 1 - \text{sen}^2x$$
$$1 - \text{sen}^2x = 1 - \text{sen}^2x \Rightarrow \quad \boxed{V}$$

d. $\dfrac{1+\cos x}{1-\cos x} = \dfrac{\sec x - 1}{\sec x + 1}$

$$\frac{1 + \dfrac{1}{\sec x}}{1 - \dfrac{1}{\sec x}} =$$
$$\frac{\sec x + 1}{\sec x - 1} \neq \frac{\sec x - 1}{\sec x + 1} \Rightarrow \quad \boxed{F}$$

$\therefore$ 2 son verdaderas.

Clave C

Razonamiento y demostración

3. $A = \dfrac{\sec x + \cos x}{1+\cos^2x} = \dfrac{\dfrac{1}{\cos x} + \cos x}{1+\cos^2x}$

$$A = \frac{\dfrac{(1+\cos^2x)}{\cos x}}{\dfrac{(1+\cos^2x)}{1}} = \frac{1}{\cos x} = \sec x$$
$$\therefore A = \sec x$$

Clave E

4. $U = (\sec x\csc x - \tan x)\text{sen}x$

$$U = (\tan x + \cot x - \tan x)\text{sen}x$$
$$U = (\cot x)\text{sen}x = \left(\frac{\cos x}{\text{sen}x}\right)\text{sen}x$$
$$\therefore U = \cos x$$

Clave B

5. $A = (3\text{sen}x + 2\cos x)^2 + (2\text{sen}x - 3\cos x)^2$

$$(3\text{sen}x + 2\cos x)^2 = 9\text{sen}^2x + 12\text{sen}x\cos x + 4\cos^2x \quad (+)$$
$$(2\text{sen}x - 3\cos x)^2 = 4\text{sen}^2x - 12\text{sen}x\cos x + 9\cos^2x$$
$$A = 13\text{sen}^2x + 13\cos^2x$$
$$\Rightarrow A = 13(\text{sen}^2x + \cos^2x) = 13(1)$$
$$\therefore A = 13$$

Clave E

6. $C = \text{sen}x\cot x + \cos x$

$$C = \text{sen}x\left(\frac{\cos x}{\text{sen}x}\right) + \cos x$$
$$\Rightarrow C = \cos x + \cos x$$
$$\therefore C = 2\cos x$$

Clave B

7. Por dato: $\text{sen}x - \cos x = n$

Piden:

$H = \text{sen}x\cos x$

Luego:

$$(\text{sen}x - \cos x)^2 = n^2$$
$$\underbrace{\text{sen}^2x + \cos^2x}_{1} - 2\underbrace{\text{sen}x\cos x}_{H} = n^2$$
$$\Rightarrow 1 - 2H = n^2$$
$$\therefore H = \frac{1-n^2}{2}$$

Clave C

8. Piden: $L = \text{sen}x\cos x$

Por dato:

$\text{sen}x - \cos x = \dfrac{1}{3}$

Elevando al cuadrado:

$$(\text{sen}x - \cos x)^2 = \left(\frac{1}{3}\right)^2$$
$$\underbrace{\text{sen}^2x + \cos^2x}_{1} - 2\underbrace{\text{sen}x\cos x}_{L} = \frac{1}{9}$$
$$\Rightarrow 1 - \frac{1}{9} = 2L$$
$$\frac{8}{9} = 2L$$
$$\therefore L = \frac{4}{9}$$

Clave C

9. Por dato: secx = 1 + senx

Piden simplificar:

$$\frac{1-\text{senx}}{\cos^3 x}=\frac{\text{secx}}{\text{secx}}\cdot\frac{(1-\text{senx})}{\cos^3 x}$$

$$\frac{1-\text{senx}}{\cos^3 x}=\frac{(1+\text{senx})(1-\text{senx})}{\cos^2 x}$$

$$\frac{1-\text{senx}}{\cos^3 x}=\frac{(1-\text{sen}^2 x)}{\cos^2 x}$$

$$\Rightarrow \frac{1-\text{senx}}{\cos^3 x}=\frac{(\cos^2 x)}{\cos^2 x}=1$$

$$\therefore\ \frac{1-\text{senx}}{\cos^3 x}=1$$

Clave B

10. Por dato: tanx + cotx = 3

Luego:

tanx + cotx = 3

$$\text{secxcscx}=3 \Rightarrow \frac{1}{\cos x\,\text{senx}}=3$$

$$\Rightarrow \text{senxcosx}=\frac{1}{3}$$

Piden:

$$C=\text{sen}^4 x+\cos^4 x$$
$$C=1-2\text{sen}^2 x\cos^2 x$$
$$C=1-2(\text{senxcosx})^2$$
$$C=1-2\left(\frac{1}{3}\right)^2$$
$$C=1-\frac{2}{9}=\frac{7}{9}$$
$$\therefore\ C=\frac{7}{9}$$

Clave D

11. Piden: csca

Por dato:

$$x\cos^2\alpha-y\,\text{sen}\alpha=y\,\text{sen}^2\alpha$$
$$x(1-\text{sen}^2\alpha)-y\,\text{sen}\alpha=y\,\text{sen}^2\alpha$$
$$x(1-\text{sen}^2\alpha)=y\,\text{sen}^2\alpha+y\,\text{sen}\alpha$$
$$x(1+\text{sen}\alpha)(1-\text{sen}\alpha)=y\,\text{sen}\alpha(\text{sen}\alpha+1)$$
$$x-x\,\text{sen}\alpha=y\,\text{sen}\alpha$$
$$x=(y+x)\text{sen}\alpha$$
$$\Rightarrow \text{sen}\alpha=\frac{x}{y+x}$$
$$\therefore\ \csc\alpha=\frac{x+y}{x}$$

Clave C

Resolución de problemas

12.
$$\left.\begin{aligned} f(\text{sen}\beta)&=\text{sen}^2\beta+1\\ f(\cos\beta)&=\cos^2\beta+1\end{aligned}\right\}+$$

$$[f(\text{sen}\beta)+f(\cos\beta)]=\underbrace{\text{sen}^2\beta+\cos^2\beta}_{1}+2$$

$$\left.\begin{aligned} f(\tan\beta)&=\tan^2\beta+1=\sec^2\beta\\ f(\cot\beta)&=\cot^2\beta+1=\csc^2\beta\end{aligned}\right\}(+)$$

$$f(\tan\beta)+f(\cot\beta)=\frac{1}{\text{sen}^2\beta\cos^2\beta}$$

Reemplazamos en la expresión:

$$3\left(\frac{1}{\text{sen}^2\beta\cos^2\beta}\right)=3\sec^2\beta\csc^2\beta$$

Clave D

13. Resolvemos la ecuación:

$$2\tan^2 x-3\tan x+1=0$$
$$\begin{array}{ll} 2\text{tanx} & -1\\ \text{tanx} & -1\end{array}$$

$$\Rightarrow 2\text{tanx}-1=0\ ;\ \text{tanx}-1=0$$
$$\text{tanx}=1/2\ ;\ \text{tanx}=1$$

Sabemos: $-\frac{\pi}{4}<x<\frac{\pi}{4}$

$$-1<\text{tanx}<1$$
$$\therefore\ \text{tanx}=1/2$$

Hallamos el valor de M:

$$M=\sec^6 x-3\sec^4 x+3\text{secx}-1+1$$
$$M=(\sec^2 x-1)^3+1$$
$$M=(\tan^2 x)^3+1$$
$$M=\left(\frac{1}{2}\right)^6+1=\frac{1}{64}+1=\frac{65}{64}$$

Clave A

Nivel 2 (página 50) Unidad 3

Comunicación matemática

14.

- $$\frac{\text{senx}}{1+\text{cosx}}+\frac{\text{cosx}}{\text{senx}}$$
$$=\frac{\text{sen}^2 x+\text{cosx}+\cos^2 x}{(1+\text{cosx})\text{senx}}$$
$$=\frac{(1+\text{cosx})}{(1+\text{cosx})\text{senx}}=\text{cscx}\neq\text{secx}$$
$\boxed{\text{F}}$

- $$\frac{\frac{1}{\text{cosx}}-\text{cosx}}{\frac{1}{\text{senx}}-\text{senx}}=\frac{\frac{1-\cos^2 x}{\text{cosx}}}{\frac{1-\text{sen}^2 x}{\text{senx}}}$$
$$=\frac{\text{sen}^3 x}{\cos^3 x}=\tan^3 x$$
$\boxed{\text{V}}$

- $$\frac{\text{senx}}{\cos x}\left(\frac{1}{\text{senx}}-\text{senx}\right)$$
$$=\frac{\text{senx}}{\cos x}\left(\frac{\cos^2 x}{\text{senx}}\right)=\cos x$$
$\boxed{\text{V}}$

- $$\text{cosx}\,\frac{\text{senx}}{\text{cosx}}-\text{senx}$$
$$=\text{senx}-\text{senx}=0\neq 1$$
$\boxed{\text{F}}$

- $$\text{sen}^2 x+\cos^2 x+2\text{senx}\,.\,\text{cosx}+\text{sen}^2 x+\cos^2 x-2\text{senx}\,.\,\text{cosx}$$
$$1+1=2\neq 0$$
$\boxed{\text{F}}$

15. Simplificamos la expresión:

$$p=\frac{1}{\cos^2 x}+\frac{1}{\text{sen}^2 x}=\frac{\text{sen}^2 x+\cos^2 x}{\text{sen}^2 x\cos^2 x}$$
$$p=\left(\frac{1}{\text{senx}\,.\,\cos x}\right)^2=\left(\frac{\text{sen}^2 x+\cos^2 x}{\text{senx}\cos x}\right)^2$$
$$p=\left(\frac{\text{senx}}{\cos x}+\frac{\cos x}{\text{senx}}\right)^2=(\tan x+\cot x)^2$$

Utilizamos el dato II:

$p=(2)^2=4$

Clave C

Razonamiento y demostración

16.
$$U=\frac{\sec^2 x\csc^2 x-\csc^2 x}{\tan^2 x}$$
$$U=\frac{\csc^2 x(\sec^2 x-1)}{\tan^2 x}$$
$$U=\frac{\csc^2 x(\tan^2 x)}{\tan^2 x}=\csc^2 x$$
$$\therefore\ U=\csc^2 x$$

Clave B

17.
$$D=(\text{secxcscx}-\text{cotx})\text{cosx}$$
$$D=(\text{tanx}+\text{cotx}-\text{cotx})\text{cosx}$$
$$D=(\text{tanx})\text{cosx}=\left(\frac{\text{senx}}{\cos x}\right)\text{cosx}$$
$$\therefore\ D=\text{senx}$$

Clave A

18.
$$L=(\text{tanx senx}+\text{cosx})(\text{cotx cosx}+\text{senx})$$
$$L=\left(\frac{\text{senx}}{\cos x}\text{senx}+\cos x\right)\left(\frac{\cos x}{\text{senx}}\cos x+\text{senx}\right)$$
$$L=\left(\frac{\text{sen}^2 x}{\cos x}+\cos x\right)\left(\frac{\cos^2 x}{\text{senx}}+\text{senx}\right)$$
$$L=\left(\frac{\text{sen}^2 x+\cos^2 x}{\text{cosx}}\right)\left(\frac{\cos^2 x+\text{sen}^2 x}{\text{senx}}\right)$$
$$L=\left(\frac{1}{\text{cosx}}\right)\left(\frac{1}{\text{senx}}\right)=\text{secxcscx}$$
$$\therefore\ L=\text{secxcscx}$$

Clave E

19.
$$L=\frac{\sec^2 x\csc^2 x-\sec^2 x}{\cot^2 x}$$
$$L=\frac{\sec^2 x(\csc^2 x-1)}{\cot^2 x}$$
$$L=\frac{\sec^2 x(\cot^2 x)}{\cot^2 x}=\sec^2 x$$
$$\therefore\ L=\sec^2 x$$

Clave A

20.
$$R=\frac{\text{senx}+\cos x}{\sec x+\csc x}=\frac{\text{senx}+\cos x}{\frac{1}{\cos x}+\frac{1}{\text{senx}}}$$
$$R=\frac{\frac{(\text{senx}+\cos x)}{1}}{\frac{(\text{senx}+\cos x)}{\text{senx}\cos x}}=\text{senxcosx}$$
$$\therefore\ R=\text{senxcosx}$$

Clave C

21. Por dato:

tanx + cotx = 4

$\Rightarrow$ secxcscx = 4 ...(1)

Piden:

L = secx + cscx

Elevando al cuadrado:

$$L^2=(\text{secx}+\text{cscx})^2$$
$$L^2=\underbrace{\sec^2 x+\csc^2 x}+2\text{secxcscx}$$
$$L^2=\sec^2 x\csc^2 x+2\text{secxcscx}$$
$$\Rightarrow L^2=(\text{secxcscx})^2+2\text{secxcscx}$$

Reemplazando (1) en la expresión:

$\Rightarrow L^2 = (4)^2 + 2(4) = 24$

$\Rightarrow L^2 = 24$

$\therefore\ L = 2\sqrt{6}$

Clave B

22. Piden: $A = \tan x - 2\cot x$

Por dato:

$\tan^2 x - 3\tan x = 2$

Dividiendo entre tanx:

$$\frac{\tan^2 x}{\tan x} - \frac{3\tan x}{\tan x} = \frac{2}{\tan x}$$

$$\tan x - 3 = 2\cot x$$

$\Rightarrow \tan x - 2\cot x = 3$

$\therefore\ A = 3$

Clave C

23. Piden: $\tan x + \cot x$

Por identidad: $\tan x + \cot x = \sec x\csc x$

Por dato:

$\operatorname{sen}x + \cos x = \sqrt{15}\operatorname{sen}x\cos x \wedge x \in \text{IIIC}$

Elevando al cuadrado:

$\underbrace{\operatorname{sen}^2 x + \cos^2 x}_{1} + 2\operatorname{sen}x\cos x = 15\operatorname{sen}^2 x\cos^2 x$

$\Rightarrow 1 + 2\operatorname{sen}x\cos x = 15\operatorname{sen}^2 x\cos^2 x$

Sea: $\operatorname{sen}x\cos x = a$

$\Rightarrow 1 + 2a = 15a^2$

$0 = 15a^2 - 2a - 1$

$3a \quad -1$

$5a \quad 1$

$\Rightarrow (3a - 1)(5a + 1) = 0$

$\Rightarrow a = \frac{1}{3} \ \vee\ a = -\frac{1}{5}$

$\operatorname{sen}x\cos x = \frac{1}{3} \vee \operatorname{sen}x\cos x = -\frac{1}{5}$

Como $x \in$ IIIC: $\operatorname{sen}x < 0 \wedge \cos x < 0$

$\Rightarrow \operatorname{sen}x\cos x > 0$

Entonces:

$\operatorname{sen}x\cos x = \frac{1}{3}$

$\sec x\csc x = 3$

$\therefore\ \tan x + \cot x = 3$

Clave B

24. $E = m(\operatorname{sen}^4\theta + \cos^4\theta) + 2(\operatorname{sen}^6\theta + \cos^6\theta)$

$E = m(1 - 2\operatorname{sen}^2\theta\cos^2\theta) + 2(1 - 3\operatorname{sen}^2\theta\cos^2\theta)$

$E = m + 2 - \operatorname{sen}2\cos^2\theta(2m + 6)$

Entonces:

$E = m + 2 - (2m + 6)f(\theta)$

Luego para que E sea independiente de θ, el coeficiente que acompaña a $f(\theta)$ debe ser cero.

$\Rightarrow 2m + 6 = 0$

$m = -\frac{6}{2}$

$\therefore\ m = -3$

Clave B

25. Por dato: $\tan x + \cot x = \sqrt{5}$

Luego:

$\underbrace{\tan x + \cot x}_{\sec x\csc x} = \sqrt{5}$

$\sec x\csc x = \sqrt{5} \Rightarrow \frac{1}{\cos x\operatorname{sen}x} = \sqrt{5}$

$\Rightarrow \operatorname{sen}x\cos x = \frac{1}{\sqrt{5}}$

Piden:

$M = \operatorname{sen}^6 x + \cos^6 x$

$M = 1 - 3\operatorname{sen}^2 x\cos^2 x$

$M = 1 - 3(\operatorname{sen}x\cos x)^2$

$M = 1 - 3\left(\frac{1}{\sqrt{5}}\right)^2$

$M = 1 - \frac{3}{5} = \frac{2}{5}$

$\therefore\ M = \frac{2}{5} = 0{,}4$

Clave D

Resolución de problemas

26. Tenemos la ecuación: $ax^2 + bx + c = 0$

Raíces: $\operatorname{sen}\theta$ y $\cos\theta$

Suma de raíces $= -\frac{b}{a}$

Producto de raíces $= \frac{c}{a}$

Entonces:

$(\operatorname{sen}\theta + \cos\theta)^2 = \left(\frac{-b}{a}\right)^2$

$\operatorname{sen}^2\theta + 2\operatorname{sen}\theta\cos\theta + \cos^2\theta = \frac{b^2}{a^2}$

$1 + 2\frac{c}{a} = \frac{b^2}{a^2}$

$a^2 + 2ac = b^2$

Clave C

27. $R(4) = \operatorname{sen}^4\alpha + \cos^4\alpha = 1 - 2\operatorname{sen}^2\alpha\cos^2\alpha$

$R(6) = \operatorname{sen}^6\alpha + \cos^6\alpha = 1 - 3\operatorname{sen}^2\alpha\cos^2\alpha \quad (-)$

$R(4) - R(6) = \operatorname{sen}^2\alpha\cos^2\alpha$

$R(2) = \operatorname{sen}^2\alpha + \cos^2\alpha = 1$

$R(-2) = \frac{1}{\operatorname{sen}^2\alpha} + \frac{1}{\cos^2\alpha} = \frac{1}{\operatorname{sen}^2\alpha\cos^2\alpha}$

Reemplazamos en P:

$P = (\operatorname{sen}^2\alpha\cos^2\alpha) \times 1 \times \frac{1}{\operatorname{sen}^2\alpha\cos^2\alpha}$

$P = 1$

Clave E

Nivel 3 (página 51) Unidad 3

Comunicación matemática

28. $M = 4\operatorname{sen}^2 x + 4\operatorname{sen}x\cos x + \cos^2 x + \operatorname{sen}^2 x - 4\operatorname{sen}x\cos x + 4\cos^2 x$

$M = 4(\operatorname{sen}^2 x + \cos^2 x) + \operatorname{sen}^2 x + \cos^2 x$

$M = 4 + 1 = 5$

$N = \operatorname{sen}^2 x + \frac{1}{\operatorname{sen}^2 x} = \operatorname{sen}^2 x + 2(\operatorname{sen}x)\left(\frac{1}{\operatorname{sen}x}\right) + \frac{1}{\operatorname{sen}^2 x} - 2$

$N = \left(\operatorname{sen}x + \frac{1}{\operatorname{sen}x}\right)^2 - 2 = 7 - 2 = 5$

$M - N = 5 - 5 = 0$

Clave B

29. En la sucesión, tenemos:

$\boxed{k};\ 1;\ 2;\ \boxed{3+k};\ 4+3\operatorname{sen}^2 x;\ \ldots$

$1-k \quad 1 \quad 1+k \quad 1+2k$

$k \quad k \quad k$

Hallamos el primer término:

$4 + 3\operatorname{sen}^2 x - (1 + 2k) - (1 + k) = 2$

$4 + 3\operatorname{sen}^2 x - 2 - 3k = 2$

$3\operatorname{sen}^2 x = 3k$

$k = \operatorname{sen}^2 x$

$\Rightarrow t_1 = \operatorname{sen}^2 x$

$t_4 = 3 + \operatorname{sen}^2 x$

$N = 3 + \operatorname{sen}^2 x - \operatorname{sen}^2 x$

$\therefore\ N = 3$

Clave B

Razonamiento y demostración

30. $M = \dfrac{\sec^4 x - \sec^2 x}{\csc^4 x - \csc^2 x}$

$M = \dfrac{\sec^2 x(\sec^2 x - 1)}{\csc^2 x(\csc^2 x - 1)}$

$M = \dfrac{\sec^2 x(\tan^2 x)}{\csc^2 x(\cot^2 x)}$

$M = \left(\dfrac{\operatorname{sen}^2 x}{\cos^2 x}\right)\tan^2 x(\tan^2 x)$

$\Rightarrow M = (\tan^2 x)\tan^4 x$

$\therefore\ M = \tan^6 x$

Clave C

31. Por dato: $\tan x - \cot x = 2$

Piden:

$E = \tan^2 x + \cot^2 x$

Luego:

$(\tan x - \cot x)^2 = 2^2$

$\underbrace{\tan^2 x + \cot^2 x}_{E} - 2\underbrace{\tan x\cot x}_{1} = 4$

$\Rightarrow E - 2 = 4$

$\therefore\ E = 6$

Clave C

32. $A = \dfrac{\sec x - \tan x - 2}{\csc x - 2\cot x - 1} = \dfrac{\frac{1}{\cos x} - \frac{\operatorname{sen}x}{\cos x} - 2}{\frac{1}{\operatorname{sen}x} - \frac{2\cos x}{\operatorname{sen}x} - 1}$

$A = \dfrac{\frac{1 - \operatorname{sen}x - 2\cos x}{\cos x}}{\frac{1 - 2\cos x - \operatorname{sen}x}{\operatorname{sen}x}}$

$A = \dfrac{\operatorname{sen}x(1 - \operatorname{sen}x - 2\cos x)}{\cos x(1 - \operatorname{sen}x - 2\cos x)}$

$A = \dfrac{\operatorname{sen}x}{\cos x} = \tan x$

$\therefore\ A = \tan x$

Clave A

33. Por dato:

$$\tan^2x - \text{sen}^2x = n\text{sen}^2x$$

$$\frac{\text{sen}^2x}{\cos^2x} - \text{sen}^2x = n\text{sen}^2x$$

$$\Rightarrow \text{sen}^2x\frac{(1-\cos^2x)}{\cos^2x} = n\text{sen}^2x$$

$$\frac{(\text{sen}^2x)}{\cos^2x} = n$$

$$\Rightarrow \tan^2x = n$$

$$\therefore\ n = \tan^2x$$

Clave C

34. $E = \dfrac{(\cos x\tan x - \text{sen}x\cot x)^2 - 1}{2\cos x}$

$$E = \frac{\left(\cos x\dfrac{\text{sen}x}{\cos x} - \text{sen}x\dfrac{\cos x}{\text{sen}x}\right)^2 - 1}{2\cos x}$$

$$E = \frac{(\text{sen}x - \cos x)^2 - 1}{2\cos x}$$

$$E = \frac{\overbrace{\text{sen}^2x + \cos^2x}^{1} - 2\text{sen}x\cos x - 1}{2\cos x}$$

$$E = \frac{1 - 2\text{sen}x\cos x - 1}{2\cos x} = \frac{-2\text{sen}x\cos x}{2\cos x}$$

$$\therefore\ E = -\text{sen}x$$

Clave B

35. Por dato:

$\tan x + \cot x = a$

$(\tan x + \cot x)^2 = a^2$

$\tan^2x + \cot^2x + 2\underbrace{\tan x\cot x}_{1} = a^2$

$\Rightarrow \tan^2x + \cot^2x = a^2 - 2 \qquad ...(1)$

Además:

$\tan x - \cot x = b$

$(\tan x - \cot x)^2 = b^2$

$\tan^2x + \cot^2x - 2\underbrace{\tan x\cot x}_{1} = b^2$

$\Rightarrow \tan^2x + \cot^2x = b^2 + 2 \qquad ...(2)$

De (1) y (2):

$a^2 - 2 = b^2 + 2$

$\therefore\ a^2 - b^2 = 4$

Clave C

36. Por dato:

$\cot^2x = \csc x$

$\Rightarrow \dfrac{\cos^2x}{\text{sen}^2x} = \dfrac{1}{\text{sen}x} \Rightarrow \cos^2x = \text{sen}x \quad ...(1)$

Piden:

$E = \cos^4x + \cos^2x$

$E = (\cos^2x)^2 + \cos^2x \qquad ...(2)$

Reemplazando (1) en (2):

$\Rightarrow E = (\text{sen}x)^2 + \cos^2x$

$E = \text{sen}^2x + \cos^2x = 1$

$\therefore\ E = 1$

Clave A

Resolución de problemas

37. $M = \sqrt{1 - 2\cos\theta\cos\beta + \cos^2\theta\cos^2\beta\ \overline{(\cos^2\theta - 2\cos\theta\cos\beta + \cos^2\beta)}}$

$M = \sqrt{1 + \cos^2\theta\cos^2\beta - \cos^2\theta - \cos^2\beta}$

$M = \sqrt{1 - \cos^2\beta + \cos^2\theta(\cos^2\beta - 1)}$

$M = \sqrt{\text{sen}^2\beta + \cos^2\theta(-\text{sen}^2\beta)}$

$M = \sqrt{\text{sen}^2\beta(1 - \cos^2\theta)}$

$M = \sqrt{\text{sen}^2\beta\,.\,\text{sen}^2\theta} = \text{sen}\beta\text{sen}\theta$

$\theta \in \text{IC} \Rightarrow |\text{sen}\beta| = \text{sen}\beta$

$\beta \in \text{IC} \Rightarrow |\text{sen}\theta| = \text{sen}\theta$

$\Rightarrow M = \text{sen}\beta\text{sen}\theta$

$\therefore\ \Sigma\text{fact} = \text{sen}\beta + \text{sen}\theta$

Clave D

38. $N = \sqrt{1 - 2\text{sen}\theta\text{sen}\beta + \text{sen}^2\beta\text{sen}^2\theta\ \overline{-(\text{sen}\theta^2 - 2\text{sensen}\beta + \text{sen}^2\beta)}}$

$N = \sqrt{1 + \text{sen}^2\theta\text{sen}^2\beta - \text{sen}^2\theta - \text{sen}^2\beta}$

$N = \sqrt{1 - \text{sen}^2\beta + \text{sen}^2\theta(\text{sen}^2\beta - 1)}$

$N = \sqrt{\cos^2\beta + \text{sen}^2\theta(-\cos^2\beta)}$

$N = \sqrt{\cos^2\beta(1 - \text{sen}^2\theta)}$

$N = \sqrt{\cos^2\beta\cos^2\theta} = |\cos\beta||\cos\theta|$

$\beta \in \text{IIIC} \Rightarrow |\cos\beta| = -\cos\beta$

$\theta \in \text{IVC} \Rightarrow (\cos\theta) = \cos\theta$

$\Rightarrow N = -\cos\beta\cos\theta$

$\therefore\ \Sigma\text{fact} = \cos\theta - \cos\beta$

Clave E

39. $\tan x + \cot x = 3$

$(\tan x + \cot x)^2 = 9$

$\tan^2x + \cot^2x + 2\underbrace{\tan x\cot x}_{1} = 9$

$\tan^2x + \cot^2x = 7$

Luego:

$(\tan^2x + \cot^2x)^3 = 343$

$\underbrace{\tan^6x + \cot^6x}_{B} + 3\underbrace{\tan^2x\cot^2x}_{1}\underbrace{(\tan^2x + \cot^2x)}_{7} = 343$

$\Rightarrow B + 3(1)(7) = 343$

$B = 322$

Clave B

APLICAMOS LO APRENDIDO (página 52) Unidad 3

1. $C = \dfrac{\text{sen}(\alpha+\beta) - \text{sen}\beta\cos\alpha}{\cos(\alpha-\beta) - \text{sen}\alpha\,\text{sen}\beta}$

$C = \dfrac{\text{sen}\alpha\cos\beta + \cos\alpha\,\text{sen}\beta - \text{sen}\beta\cos\alpha}{\cos\alpha\cos\beta + \text{sen}\alpha\,\text{sen}\beta - \text{sen}\alpha\,\text{sen}\beta}$

$C = \dfrac{\text{sen}\alpha\,\cos\beta}{\cos\alpha\,\cos\beta} = \dfrac{\text{sen}\alpha}{\cos\alpha} = \tan\alpha$

$\therefore C = \tan\alpha$

Clave B

2. Por dato: $\tan x = 5 \wedge \tan\beta = 3$

Piden:

$\tan(x+\beta) = \dfrac{\tan x + \tan\beta}{1 - \tan x\tan\beta}$

$\tan(x+\beta) = \dfrac{5+3}{1-5\cdot 3} = \dfrac{8}{-14}$

$\therefore \tan(x+\beta) = -\dfrac{4}{7}$

Clave D

3. Por dato:

$\text{sen}(40° + x) + \text{sen}(40° - x) = \text{sen}40°$

Desarrollando por partes:

$\text{sen}(40° + x) = \text{sen}40°\cos x + \cos40°\text{sen}x$ (+)

$\text{sen}(40° - x) = \text{sen}40°\cos x - \cos40°\text{sen}x$

$\text{sen}40° = 2\text{sen}40°\cos x$

$1 = 2\cos x$

$\Rightarrow \cos x = \dfrac{1}{2}$

$\Rightarrow x = 60° \quad \vee \quad x = 300°$

Piden: el ángulo x agudo.

$\therefore x = 60°$

Clave E

4. $A = (\cos x + \cos y)^2 + (\text{sen}x - \text{sen}y)^2$

Efectuando por partes:

$(\cos x + \cos y)^2 = \cos^2x + 2\cos x\cos y + \cos^2y$

$(\text{sen}x - \text{sen}y)^2 = \text{sen}^2x - 2\text{sen}x\,\text{sen}y + \text{sen}^2y$ (+)

$A = 1 + 2\underbrace{(\cos x\cos y - \text{sen}x\,\text{sen}y)}_{\cos(x+y)} + 1$

$\Rightarrow A = 2 + 2\cos(x+y)$

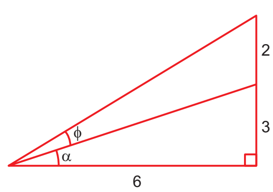
Por dato: $x + y = \dfrac{\pi}{4}$

$\Rightarrow A = 2 + 2\cos\dfrac{\pi}{4} = 2 + 2\left(\dfrac{\sqrt{2}}{2}\right)$

$\therefore A = 2 + \sqrt{2}$

Clave D

5.



Del gráfico:

$\tan\alpha = \dfrac{3}{6} = \dfrac{1}{2} \quad \wedge \quad \tan(\phi+\alpha) = \dfrac{5}{6}$

Entonces: $\dfrac{\tan\phi + \tan\alpha}{1 - \tan\phi\tan\alpha} = \dfrac{5}{6}$

$\dfrac{\tan\phi + \left(\frac{1}{2}\right)}{1 - \tan\phi\left(\frac{1}{2}\right)} = \dfrac{5}{6}$

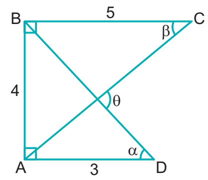
$\Rightarrow 6\tan\phi + 3 = 5 - \dfrac{5\tan\phi}{2}$

$\dfrac{17\tan\phi}{2} = 2$

$\therefore \tan\phi = \dfrac{4}{17}$

Clave C

6.



Del gráfico: $\overline{AD} \,//\, \overline{BC} \Rightarrow \theta = \alpha + \beta$

Además: $\tan\beta = \dfrac{4}{5} \;\wedge\; \tan\alpha = \dfrac{4}{3}$

Entonces: $\tan\theta = \tan(\alpha+\beta)$

$\tan\theta = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\,\tan\beta}$

$\tan\theta = \dfrac{\frac{4}{3} + \frac{4}{5}}{1 - \left(\frac{4}{3}\right)\left(\frac{4}{5}\right)} = \dfrac{\frac{32}{15}}{-\frac{1}{15}}$

$\therefore \tan\theta = -32$

Clave D

7. Por dato: $\tan x = 5 \wedge \tan y = 3$

Además: $x + y + z = 180° = \pi$ rad

Entonces, se cumple:

$\tan x + \tan y + \tan z = \tan x\tan y\tan z$

$\Rightarrow (5) + (3) + \tan z = (5)(3)\tan z$

$8 + \tan z = 15\tan z$

$8 = 14\tan z$

$\therefore \tan z = \dfrac{8}{14} = \dfrac{4}{7}$

Clave D

8. En un $\triangle ABC$, se cumple:

$A + B + C = 180° = \pi$ rad

Entonces:

$\cot A\cot B + \cot A\cot C + \cot B\cot C = 1$

Por dato:

$\dfrac{\cot A}{3} = \dfrac{\cot B}{5} = \dfrac{\cot C}{6} = k$

$\Rightarrow \cot A = 3k;\ \cot B = 5k;\ \cot C = 6k$

Luego:

$(3k)(5k) + (3k)(6k) + (5k)(6k) = 1$

$\Rightarrow 63k^2 = 1$

$\Rightarrow k = \dfrac{1}{3\sqrt{7}}$

Piden:

$\cot C = 6k = 6\left(\dfrac{1}{3\sqrt{7}}\right) = \dfrac{2}{\sqrt{7}}$

$\therefore \cot C = \dfrac{2}{\sqrt{7}}$

Clave E

Piden:

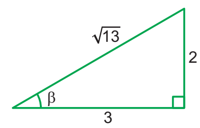
$A = \text{sen}(45° + \beta)$

$A = \text{sen}45°\cos\beta + \cos45°\text{sen}\beta$

$A = \left(\dfrac{1}{\sqrt{2}}\right)\cos\beta + \left(\dfrac{1}{\sqrt{2}}\right)\text{sen}\beta$

$\Rightarrow A = \dfrac{1}{\sqrt{2}}(\cos\beta + \text{sen}\beta) \quad ...(1)$

Entonces, del dato:



$\text{sen}\beta = \dfrac{2}{\sqrt{13}}$

$\cos\beta = \dfrac{3}{\sqrt{13}}$

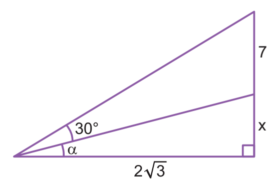
Reemplazando en (1):

$\Rightarrow A = \dfrac{1}{\sqrt{2}}\left(\dfrac{3}{\sqrt{13}} + \dfrac{2}{\sqrt{13}}\right) = \dfrac{1}{\sqrt{2}}\left(\dfrac{5}{\sqrt{13}}\right)$

$\therefore A = \dfrac{5}{\sqrt{26}}$

Clave C

9.



Del gráfico: $\tan\alpha = \dfrac{x}{2\sqrt{3}} \wedge x > 0$

Además: $\tan(30° + \alpha) = \dfrac{7+x}{2\sqrt{3}}$

$\dfrac{\tan30° + \tan\alpha}{1 - \tan30°\,\tan\alpha} = \dfrac{7+x}{2\sqrt{3}}$

$\dfrac{\left(\frac{1}{\sqrt{3}}\right) + \left(\frac{x}{2\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)\left(\frac{x}{2\sqrt{3}}\right)} = \dfrac{7+x}{2\sqrt{3}}$

$\dfrac{\left(\frac{2+x}{2\sqrt{3}}\right)}{\left(\frac{6-x}{6}\right)} = \dfrac{7+x}{2\sqrt{3}}$

Luego:

$(2 + x)6 = (6 - x)(7 + x)$

$\Rightarrow x^2 + 7x - 30 = 0$

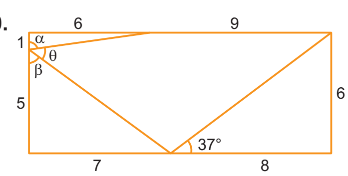
$(x - 3)(x + 10) = 0$

$\Rightarrow x = 3 \vee x = -10$

$\therefore x = 3$

Clave A

10.



$\alpha + \beta + \theta = 180°$

Entonces: $\tan\alpha + \tan\beta + \tan\theta = \tan\alpha\tan\beta\tan\theta$

$$6 + \frac{7}{5} + \tan\theta = 6 \cdot \frac{7}{5} \cdot \tan\theta$$

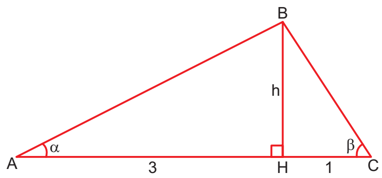
$$\frac{37}{5} + \tan\theta = \frac{42}{5}\tan\theta$$

$$\frac{37}{5} = \frac{37}{5}\tan\theta$$

$$\tan\theta = 1$$

Clave A

11.



$$\alpha + \beta + 135 = 180 \longrightarrow \alpha + \beta = 45°$$

$$\tan(\alpha + \beta) = \tan 45°$$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 1$$

$$\frac{\frac{h}{3} + h}{1 - \left(\frac{h}{3}\right)(h)} = 1$$

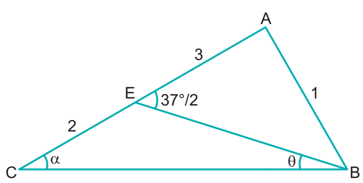
$$\frac{4}{3}h = \frac{3 - h^2}{3}$$

$$h^2 + 4h - 3 = 0$$

$$\Rightarrow \quad h = \sqrt{7} - 2$$

Clave E

12.



$$\alpha + \theta = \frac{37°}{2}$$

$$\tan(\alpha + \theta) = \tan\frac{37°}{2}$$

$$\frac{\tan\alpha + \tan\theta}{1 - \tan\alpha\tan\theta} = \frac{1}{3}$$

$$\frac{\frac{1}{5} + \tan\theta}{1 - \frac{1}{5}\tan} = \frac{1}{3}$$

$$\frac{16}{15} = \tan\theta = \frac{2}{15}$$

$$\tan\theta = \frac{1}{8}$$

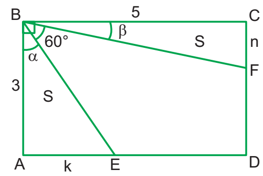
Luego: $\tan(\theta - \alpha) = \dfrac{\tan\theta - \tan\alpha}{1 + \tan\theta\tan\alpha}$

$$= \frac{\frac{1}{8} - \frac{1}{15}}{1 + \left(\frac{1}{8}\right)\left(-\frac{1}{15}\right)}$$

$$= -\frac{3}{41}$$

Clave D

13.



$\triangle$ BAE: $S = \dfrac{3k}{2}\text{sen}90° = \dfrac{3k}{2}$

$\triangle$ BCF: $S = \dfrac{5 \cdot n}{2}\text{sen}90° = \dfrac{5n}{2}$

$$\Rightarrow n = \frac{3k}{5}$$

Luego: $\alpha + \beta + 60° = 90° \longrightarrow \alpha + \beta = 30°$

$$\tan(\alpha + \beta) = \tan 30°$$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{\sqrt{3}}{3}$$

$$\frac{\frac{k}{3} + \frac{3k}{25}}{1 - \frac{k}{3} \cdot \frac{3k}{25}} = \frac{\sqrt{3}}{3}$$

$$\frac{\frac{34k}{75}}{\frac{25 - k^2}{25}} = \frac{\sqrt{3}}{3}$$

$$\frac{34k}{3(25 - k^2)} = \frac{\sqrt{3}}{3}$$

$$\frac{34k}{\sqrt{3}} = 25 - k^2$$

$$k^2 + \frac{34k}{\sqrt{3}} = 25$$

$$E = 25$$

Clave C

PRACTIQUEMOS
Nivel 1 (página 54) Unidad 3

Comunicación matemática

1.

2.

Razonamiento y demostración

3. $J = \text{sen}(30° + x) + \text{sen}(30° - x)$

Desarrollando cada término:

$\text{sen}(30° + x) = \text{sen}30°\cos x + \cos30°\text{sen}x$

$\text{sen}(30° + x) = \dfrac{1}{2}\cos x + \dfrac{\sqrt{3}}{2}\text{sen}x$ ...(I)

$\text{sen}(30° - x) = \text{sen}30°\cos x - \cos30°\text{sen}x$

$\text{sen}(30° - x) = \dfrac{1}{2}\cos x - \dfrac{\sqrt{3}}{2}\text{sen}x$ ...(II)

Sumando (I) y (II):

$$\underbrace{\text{sen}(30° + x) + \text{sen}(30° - x)}_{J} = \frac{1}{2}\cos x + \frac{1}{2}\cos x$$

$\therefore J = \cos x$

Clave B

4. $J = \cos(45° + x) + \cos(45° - x)$

Desarrollando cada término:

$\cos(45° + x) = \cos45°\cos x - \text{sen}45°\text{sen}x$

$\cos(45° + x) = \dfrac{\sqrt{2}}{2}\cos x - \dfrac{\sqrt{2}}{2}\text{sen}x$ ...(I)

$\cos(45° - x) = \cos45°\cos x + \text{sen}45°\text{sen}x$

$\cos(45° - x) = \dfrac{\sqrt{2}}{2}\cos x + \dfrac{\sqrt{2}}{2}\text{sen}x$ ...(II)

Sumando (I) y (II):

$$\underbrace{\cos(45° + x) + \cos(45° - x)}_{J} = 2\left(\frac{\sqrt{2}}{2}\cos x\right)$$

$\therefore J = \sqrt{2}\cos x$

Clave C

5. Piden: sen7°

$\text{sen}7° = \text{sen}(37° - 30°)$

$\text{sen}7° = \text{sen}37°\cos30° - \cos37°\text{sen}30°$

$\text{sen}7° = \left(\dfrac{3}{5}\right)\left(\dfrac{\sqrt{3}}{2}\right) - \left(\dfrac{4}{5}\right)\left(\dfrac{1}{2}\right)$

$\Rightarrow \text{sen}7° = \dfrac{3\sqrt{3}}{10} - \dfrac{4}{10} = \dfrac{3\sqrt{3} - 4}{10}$

$\therefore \text{sen}7° = \dfrac{3\sqrt{3} - 4}{10}$

Clave A

6. Piden: tan8°

$\tan8° = \tan(45° - 37°)$

$\tan8° = \dfrac{\tan45° - \tan37°}{1 + \tan45°\tan37°}$

$\tan8° = \dfrac{(1) - \left(\frac{3}{4}\right)}{1 + (1)\left(\frac{3}{4}\right)} = \dfrac{\frac{1}{4}}{\frac{7}{4}} = \dfrac{1}{7}$

$\therefore \tan8° = \dfrac{1}{7}$

Clave C

7. $E = \sqrt{2}\cos(45° + x) - \cos x$

$E = \sqrt{2}(\cos45°\cos x - \text{sen}45°\text{sen}x) - \cos x$

$E = \sqrt{2}\left(\dfrac{1}{\sqrt{2}}\cos x - \dfrac{1}{\sqrt{2}}\text{sen}x\right) - \cos x$

$E = \cos x - \text{sen}x - \cos x$

$\therefore E = -\text{sen}x$

Clave B

8. Por dato:

$\tan\alpha = \dfrac{1}{3} \quad \wedge \quad \tan\beta = \dfrac{2}{5}$

Piden:

$\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\tan(\alpha - \beta) = \dfrac{\left(\frac{1}{3}\right) - \left(\frac{2}{5}\right)}{1 + \left(\frac{1}{3}\right)\left(\frac{2}{5}\right)} = \dfrac{\left(-\frac{1}{15}\right)}{\left(\frac{17}{15}\right)}$

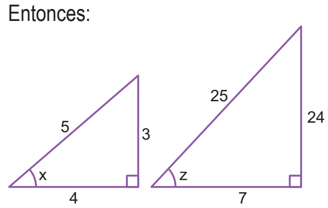
$\therefore \tan(\alpha - \beta) = -\dfrac{1}{17}$

Clave D

9. Por dato:

$\text{sen}x = \frac{3}{5} \wedge \text{sen}z = \frac{24}{25}$; (x ; z son agudos)

Entonces:



Piden:

$\text{sen}(x+z) = \text{sen}x\cos z + \cos x\,\text{sen}z$

$$\text{sen}(x+z) = \left(\frac{3}{5}\right)\left(\frac{7}{25}\right) + \left(\frac{4}{5}\right)\left(\frac{24}{25}\right)$$

$$\Rightarrow \text{sen}(x+z) = \frac{21}{125} + \frac{96}{125} = \frac{117}{125}$$

$$\therefore \text{sen}(x+z) = \frac{117}{125}$$

Clave D

10. Por dato: $\tan B = \frac{1}{3}$

Además: $\tan(A-B) = 2$

$$\frac{\tan A - \tan B}{1 + \tan A \tan B} = 2$$

$$\frac{\tan A - \frac{1}{3}}{1 + \tan A\left(\frac{1}{3}\right)} = 2$$

$$\tan A - \frac{1}{3} = 2 + \frac{2}{3}\tan A$$

$$\frac{\tan A}{3} = \frac{7}{3}$$

$$\therefore \tan A = 7$$

Clave B

Nivel 2 (página 54) Unidad 3

Comunicación matemática

11.

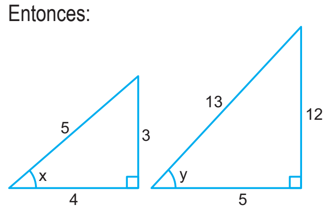
12.

Razonamiento y demostración

13. Por dato:

$\tan x = \frac{3}{4}$; $\sec y = \frac{13}{5}$

Entonces:



Piden:

$\text{sen}(x+y) = \text{sen}x\cos y + \cos x\,\text{sen}y$

$$\text{sen}(x+y) = \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$

$$\text{sen}(x+y) = \frac{15}{65} + \frac{48}{65} = \frac{63}{65}$$

$$\therefore \text{sen}(x+y) = \frac{63}{65}$$

Clave C

14. Por dato: $\tan y = \frac{1}{3}$

Además: $\tan(x-y) = 2$

$$\frac{\tan x - \tan y}{1 + \tan x \tan y} = 2$$

$$\frac{\tan x - \frac{1}{3}}{1 + \tan x\left(\frac{1}{3}\right)} = 2$$

$$\tan x - \frac{1}{3} = 2 + \frac{2}{3}\tan x$$

$$\frac{\tan x}{3} = \frac{7}{3}$$

$$\tan x = 7$$

$$\therefore \cot x = \frac{1}{7}$$

Clave B

15. $E = \cos 10° - \sqrt{3}\,\text{sen}10°$

$$E = 2\left(\frac{1}{2}.\cos 10° - \frac{\sqrt{3}}{2}.\text{sen}10°\right)$$

$$E = 2(\text{sen}30°.\cos 10° - \cos 30°.\text{sen}10°)$$

$$E = 2\text{sen}(30° - 10°)$$

$$\therefore E = 2\text{sen}20°$$

Clave A

16. Por dato:

$\tan x \tan y = \frac{1}{5} \wedge \text{sen}x\,\text{sen}y = \frac{\sqrt{3}}{12}$

Luego:

$$\frac{\text{sen}x\,\text{sen}y}{\cos x \cos y} = \frac{1}{5} \Rightarrow \frac{\left(\frac{\sqrt{3}}{12}\right)}{\cos x \cos y} = \frac{1}{5}$$

$$\Rightarrow \cos x \cos y = \frac{5\sqrt{3}}{12}$$

Piden:

$\cos(x-y) = \cos x\cos y + \text{sen}x\,\text{sen}y$

$$\cos(x-y) = \left(\frac{5\sqrt{3}}{12}\right) + \left(\frac{\sqrt{3}}{12}\right) = \frac{6\sqrt{3}}{12}$$

$$\therefore \cos(x-y) = \frac{\sqrt{3}}{2}$$

Clave D

17. Piden:

$E = (\text{sen}17° + \cos 13°)^2 + (\text{sen}13° + \cos 17°)^2$

Efectuando por partes:

$$\begin{aligned} (\text{sen}17° + \cos 13°)^2 &= \text{sen}^2 17° + \cos^2 13° \\ &\quad + 2\text{sen}17°\cos 13° \\ (\text{sen}13° + \cos 17°)^2 &= \cos^2 17° + \text{sen}^2 13° \quad (+) \\ &\quad + 2\cos 17°\,\text{sen}13° \end{aligned}$$

$$E = 1 + 2(\text{sen}17°\cos 13° + \cos 17°\text{sen}13°) + 1$$

$$\Rightarrow E = 2 + 2\text{sen}(17° + 13°)$$

$$E = 2 + 2\text{sen}30°$$

$$E = 2 + 2\left(\frac{1}{2}\right) = 2 + 1 = 3$$

$$\therefore E = 3$$

Clave C

18. Piden el valor agudo de x.

$$\underbrace{\cos 4x\cos x - \text{sen}4x\,\text{sen}x}_{\cos(4x+x)} = \frac{1}{2}$$

$$\Rightarrow \cos 5x = \frac{1}{2}$$

Sabemos: $\cos 60° = \frac{1}{2}$

$$\Rightarrow 5x = 60°$$

$$\therefore x = 12°$$

Clave B

19. Piden: un valor agudo de x.

$\text{sen}4x\cos x - \text{sen}x\cos 4x = 0{,}5$

$$\underbrace{\text{sen}4x\cos x - \cos 4x\,\text{sen}x}_{\text{sen}(4x-x)} = \frac{1}{2}$$

$$\Rightarrow \text{sen}3x = \frac{1}{2}$$

Sabemos: $\text{sen}30° = \frac{1}{2}$

$$\Rightarrow 3x = 30°$$

$$\therefore x = 10°$$

Clave B

20. $$M = \frac{\cos(30° - x) + \cos(30° + x)}{\text{sen}(30° - x) + \text{sen}(30° + x)} = \frac{N}{D}$$

Para el numerador (N):

$$\begin{aligned} \cos(30° - x) &= \cos 30°\cos x + \text{sen}30°\text{sen}x \\ \cos(30° + x) &= \cos 30°\cos x - \text{sen}30°\text{sen}x \quad (+) \\ \hline \Rightarrow N &= 2\cos 30°\cos x \end{aligned}$$

Para el denominador (D):

$$\begin{aligned} \text{sen}(30° - x) &= \text{sen}30°\cos x - \cos 30°\text{sen}x \\ \text{sen}(30° + x) &= \text{sen}30°\cos x + \cos 30°\text{sen}x \quad (+) \\ \hline \Rightarrow D &= 2\text{sen}30°\cos x \end{aligned}$$

Luego:

$$M = \frac{N}{D} = \frac{2\cos 30°\cos x}{2\text{sen}30°\cos x} = \cot 30°$$

$$M = \cot 30° = \sqrt{3}$$

$$\therefore M = \sqrt{3}$$

Clave C

Nivel 3 (página 55) Unidad 3

Comunicación matemática

21.

22.

Razonamiento y demostración

23. $E = \tan 27° + \tan 18° + \tan 27°\tan 18°$

Observamos: $27° + 18° = 45°$

$$\Rightarrow \tan(27° + 18°) = \tan 45°$$

Luego:

$$\frac{\tan 27° + \tan 18°}{1 - \tan 27°\tan 18°} = 1$$

$$\Rightarrow \tan 27° + \tan 18° = 1 - \tan 27°\tan 18°$$

Reemplazando en la expresión E:

$E = (1 - \tan 27°\tan 18°) + \tan 27°\tan 18°$

$$\therefore E = 1$$

Clave A

24. $E = \sqrt{3}\tan 80°(\tan 50° - \tan 40°)$

Por propiedad:

$\tan 50° - \tan 40° - \tan(10°)\tan 50°\tan 40° = \tan(10°)$

$\tan 50° - \tan 40° - \tan 10°\underbrace{\tan 50°\cot 50°}_{1} = \tan 10°$

$\Rightarrow \tan 50° - \tan 40° = 2\tan 10°$

Reemplazando en la expresión E:

$E = \sqrt{3}\tan 80°(2\tan 10°)$

$E = 2\sqrt{3}\tan 80°\tan 10°$

$E = 2\sqrt{3}\underbrace{\tan 80°\cot 80°}_{1}$

$\therefore E = 2\sqrt{3}$

Clave D

25. Por dato:

$\tan(\alpha + \beta) = \frac{1}{3} \wedge \tan\alpha + \tan\beta = 1$

$\Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{1}{3}$

$\frac{1}{1 - \tan\alpha\tan\beta} = \frac{1}{3}$

$\Rightarrow \tan\alpha\tan\beta = -2$

Luego:

$\tan\alpha + \tan\beta = 1$ ...(I)

$\tan\alpha\tan\beta = -2$ ...(II)

De (I) y (II):

$\tan\alpha = 2; \tan\beta = -1 \vee \tan\alpha = -1; \tan\beta = 2$

Como $\alpha \in$ IC

$\Rightarrow \tan\alpha = 2 \wedge \tan\beta = -1$

Piden:

$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\tan(\alpha - \beta) = \frac{(2) - (-1)}{1 + (2)(-1)} = \frac{3}{-1}$

$\therefore \tan(\alpha - \beta) = -3$

Clave E

26. Por dato:

$\tan x + \tan y = a \ \wedge \ \cot x + \cot y = b$

$\Rightarrow \frac{1}{\tan x} + \frac{1}{\tan y} = b$

$\frac{\tan x + \tan y}{\tan x \tan y} = b$

$\frac{a}{\tan x \tan y} = b$

$\Rightarrow \tan x \tan y = \frac{a}{b}$

Piden:

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan(x + y) = \frac{a}{1 - \left(\frac{a}{b}\right)} = \frac{a}{\frac{b - a}{b}}$

$\therefore \tan(x + y) = \frac{ab}{b - a}$

Clave D

27. $E = \frac{\operatorname{sen}(x + y)}{\cos(x - y) - \operatorname{sen}x\operatorname{sen}y} - \tan y$

$E = \frac{\operatorname{sen}(x + y)}{\cos x\cos y + \operatorname{sen}x\operatorname{sen}y - \operatorname{sen}x\operatorname{sen}y} - \tan y$

$E = \frac{\operatorname{sen}(x + y)}{\cos x\cos y} - \tan y$

Por propiedad:

$\frac{\operatorname{sen}(x + y)}{\cos x \cos y} = \tan x + \tan y$

$\Rightarrow E = (\tan x + \tan y) - \tan y$

$\therefore E = \tan x$

Clave B

28. Piden:

$E = \frac{\tan 18°}{\tan 54° - \tan 36°}$

Por propiedad:

$\tan x - \tan y - \tan(x - y)\tan x\tan y = \tan(x - y)$

Entonces:

$\tan 54° - \tan 36° - \tan(18°)\tan 54°\tan 36° = \tan(18°)$

$\tan 54° - \tan 36° - \tan 18°\underbrace{\tan 54°\cot 54°}_{1} = \tan 18°$

$\Rightarrow \tan 54° - \tan 36° = 2\tan 18°$

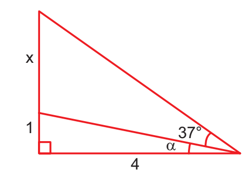
Reemplazando en la expresión E:

$E = \frac{\tan 18°}{2\tan 18°} = \frac{1}{2}$

$\therefore E = \frac{1}{2}$

Clave C

29. Piden: x



Del gráfico: $\tan\alpha = \frac{1}{4}$

Además: $\tan(\alpha + 37°) = \frac{x + 1}{4}$

$\Rightarrow \frac{\tan\alpha + \tan 37°}{1 - \tan\alpha\tan 37°} = \frac{x + 1}{4}$

$\frac{\frac{1}{4} + \frac{3}{4}}{1 - \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)} = \frac{x + 1}{4}$

$\frac{16}{13} = \frac{x + 1}{4}$

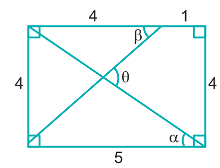
$64 = 13x + 13$

$51 = 13x$

$\therefore x = \frac{51}{13}$

Clave C

30. Piden: $\tan\theta$



Del gráfico:

$\tan\alpha = \frac{4}{5} \wedge \tan\beta = \frac{4}{4} = 1$

Además: $\theta = \alpha + \beta$

$\Rightarrow \tan\theta = \tan(\alpha + \beta)$

$\tan\theta = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\tan\theta = \frac{\frac{4}{5} + 1}{1 - \left(\frac{4}{5}\right)(1)} = \frac{\frac{9}{5}}{\frac{1}{5}} = 9$

$\therefore \tan\theta = 9$

Clave E

APLICAMOS LO APRENDIDO (página 56) Unidad 3

1. $\tan(45°\ell - x) = 4$

Sea: $45°\ell - x = a$

Entonces: $\tan\alpha = 4$

Luego:
$2\alpha = 90°\ell - 2x$
Entonces:
$\tan2\alpha = \tan(90°\ell - 2x)$

$\dfrac{2\tan\alpha}{1-\tan^2\alpha} = \cot2x$

$\dfrac{2(4)}{1-(4)^2} = \cot2x$

$\Rightarrow \cot2x = -\dfrac{8}{15}$

$\therefore \tan2x = -\dfrac{15}{8}$

Clave C

2. $K = (2 + 2\cos35°)(1 - \cos35°) + 2\text{sen}10°\cos10°$
$K = 2(1 + \cos35°)(1 - \cos35°) + \text{sen}20°$
$K = 2(1 - \cos^2 35°) + \text{sen}20°$
$K = 2 - 2\cos^2 35° + \text{sen}20°$
$K = 2 - (1 + \cos70°) + \text{sen}20°$
$K = 2 - 1 - \cos70° + \text{sen}20°$
$K = 1 - (\text{sen}20°) + \text{sen}20° = 1$
$\therefore K = 1$

Clave B

3. $E = \dfrac{1}{6\text{sen}18°\cos36°}$

$E = \dfrac{\cos18°}{3(2\text{sen}18°\cos18°)\cos36°}$

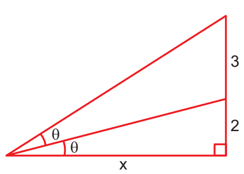
$E = \dfrac{\cos18°}{3(\text{sen}36°)\cos36°} = \dfrac{2\cos18°}{3(2\text{sen}36°\cos36°)}$

$E = \dfrac{2\cos18°}{3(\text{sen}72°)} = \dfrac{2\cos18°}{3(\cos18°)} = \dfrac{2}{3}$

$\therefore E = \dfrac{2}{3}$

Clave A

4.



Del gráfico:
$\tan\theta = \dfrac{2}{x}$

$\tan2\theta = \dfrac{5}{x}$

Luego:

$\tan2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} \Rightarrow \dfrac{5}{x} = \dfrac{2\left(\frac{2}{x}\right)}{1-\left(\frac{2}{x}\right)^2}$

$\dfrac{4x}{x^2-4} = \dfrac{5}{x}$

$4x^2 = 5x^2 - 20$
$x^2 = 20$
$\therefore\ x = 2\sqrt{5}$

Clave C

5. Nos piden:
$F = \sec76° - \tan76°$
$F = \sec(90° - 14°) - \tan(90° - 14°)$
$F = \underbrace{\csc14° - \cot14°}_{\tan\frac{14°}{2}}$

$\therefore F = \tan7°$

Clave E

6. $E = \dfrac{\cot\frac{x}{4} - \tan\frac{x}{4}}{\csc x + \cot x}$

$E = \dfrac{\left(\csc\frac{x}{2} + \cot\frac{x}{2}\right) - \left(\csc\frac{x}{2} - \cot\frac{x}{2}\right)}{\csc x + \cot x}$

$E = \dfrac{\csc\frac{x}{2} + \cot\frac{x}{2} - \csc\frac{x}{2} + \cot\frac{x}{2}}{\csc x + \cot x}$

$E = \dfrac{2\cot\frac{x}{2}}{\cot\frac{x}{2}} = 2$

$\therefore E = 2$

Clave B

7. $3\tan x = 2\cos x$

$3\left(\dfrac{\text{sen}x}{\cos x}\right) = 2\cos x$

$3\text{sen}x = 2\cos^2 x$

$3\text{sen}x = 2(1 - \text{sen}^2 x)$

Luego: $2\text{sen}^2 x + 3\text{sen}x - 2 = 0$
$2\text{sen}x \quad -1$
$\text{sen}x \quad 2$

$(2\text{sen}x - 1)(\text{sen}x + 2) = 0$

$\Rightarrow \text{sen}x = \dfrac{1}{2} \quad \vee \quad \text{sen}x = -2 \quad (F)$

Entonces: $\text{sen}x = \dfrac{1}{2}$

Piden:
$\text{sen}3x = 3\text{sen}x - 4\text{sen}^3 x$

$\text{sen}3x = 3\left(\dfrac{1}{2}\right) - 4\left(\dfrac{1}{2}\right)^3 = \dfrac{3}{2} - \dfrac{4}{8} = 1$

$\therefore \text{sen}3x = 1$

Clave A

8. $\dfrac{3\text{sen}3\theta}{\text{sen}\theta} + \dfrac{7\cos3\theta}{\cos\theta} = 1$

$\dfrac{3\text{sen}\theta(2\cos2\theta + 1)}{\text{sen}\theta} + \dfrac{7\cos\theta(2\cos2\theta - 1)}{\cos\theta} = 1$

$3(2\cos2\theta + 1) + 7(2\cos2\theta - 1) = 1$

$6\cos2\theta + 3 + 14\cos2\theta - 7 = 1$

$20\cos2\theta = 5$

$\Rightarrow \cos2\theta = \dfrac{1}{4}$

Piden:
$\cos6\theta = 4\cos^3 2\theta - 3\cos2\theta$

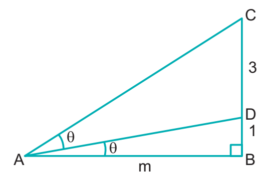
$\cos6\theta = 4\left(\dfrac{1}{4}\right)^3 - 3\left(\dfrac{1}{4}\right)$

$\cos6\theta = \dfrac{1}{16} - \dfrac{3}{4} = -\dfrac{11}{16}$

$\therefore \cos6\theta = -\dfrac{11}{16}$

Clave E

9.



Del gráfico: $\tan\theta = \dfrac{1}{m}$

$\tan2\theta = \dfrac{4}{m}$

$\tan2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} \Rightarrow \dfrac{4}{m} = \dfrac{2\left(\frac{1}{m}\right)}{1-\left(\frac{1}{m}\right)^2}$

$\dfrac{2m}{m^2-1} = \dfrac{4}{m}$

$2m^2 = 4 \quad \Rightarrow \quad m = \sqrt{2}$

Piden: $\tan\theta$

$\tan\theta = \dfrac{1}{m} = \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\therefore \tan\theta = \dfrac{\sqrt{2}}{2}$

Clave B

10. Por dato:

$2\text{sen}2\theta = 3\text{sen}\theta \quad \wedge \quad \dfrac{3\pi}{2} < \theta < 2\pi$

Entoces:
$2(2\text{sen}\theta\cos\theta) = 3\text{sen}\theta$
$4\cos\theta = 3$
$\Rightarrow \cos\theta = \dfrac{3}{4}$

Luego: $\dfrac{3\pi}{4} < \dfrac{\theta}{2} < \pi \quad \Rightarrow \quad \dfrac{\theta}{2} \in \text{ IIC}$

$\text{sen}\dfrac{\theta}{2} = +\sqrt{\dfrac{1-\cos\theta}{2}} = \sqrt{\dfrac{1-\left(\frac{3}{4}\right)}{2}} = \dfrac{\sqrt{2}}{4}$

$\cos\dfrac{\theta}{2} = -\sqrt{\dfrac{1+\cos\theta}{2}} = -\sqrt{\dfrac{1+\left(\frac{3}{4}\right)}{2}} = -\dfrac{\sqrt{14}}{4}$

Piden:
$2\left(\text{sen}\dfrac{\theta}{2} + \sqrt{7}\cos\dfrac{\theta}{2}\right) = 2\left(\dfrac{\sqrt{2}}{4} + \sqrt{7}\left(-\dfrac{\sqrt{14}}{4}\right)\right)$

$2\left(\text{sen}\dfrac{\theta}{2} + \sqrt{7}\cos\dfrac{\theta}{2}\right) = 2\left(\dfrac{\sqrt{2}}{4} - \dfrac{7\sqrt{2}}{4}\right) = 2\left(\dfrac{-6\sqrt{2}}{4}\right)$

$\therefore 2\left(\text{sen}\dfrac{\theta}{2} + \sqrt{7}\cos\dfrac{\theta}{2}\right) = -3\sqrt{2}$

Clave C

11. $M = 2\text{sen}^2\frac{\theta}{2}\cot\theta + \tan\frac{\theta}{2}$

$M = (1-\cos\theta)\cot\theta + \tan\frac{\theta}{2}$

$M = \cot\theta - \cos\theta\cot\theta + (\csc\theta - \cot\theta)$

$M = \csc\theta - \cos\theta\cot\theta$

$M = \frac{1}{\text{sen}\theta} - \cos\theta\left(\frac{\cos\theta}{\text{sen}\theta}\right)$

$M = \frac{1}{\text{sen}\theta} - \frac{\cos^2\theta}{\text{sen}\theta} = \frac{(1-\cos^2\theta)}{\text{sen}\theta}$

$\Rightarrow M = \frac{(\text{sen}^2\theta)}{\text{sen}\theta} = \text{sen}\theta$

$\therefore M = \text{sen}\theta$

Clave E

12. Piden: $\tan\theta$

Por dato:

$\tan\theta\sec2x - \tan2x\tan x - 1 = 0$

$\tan\theta\sec2x = 1 + \tan2x\tan x$

$\tan\theta\sec2x = 1 + \tan2x(\csc2x - \cot2x)$

$\tan\theta\sec2x = 1 + \tan2x\csc2x - \tan2x\cot2x$

$\tan\theta\sec2x = 1 + \left(\frac{\text{sen}2x}{\cos2x}\right)\csc2x - 1$

$\tan\theta\sec2x = \frac{\text{sen}2x\csc2x}{\cos2x} = \frac{1}{\cos2x} = \sec2x$

$\Rightarrow \tan\theta\sec2x = \sec2x$

$\tan\theta = \frac{\sec2x}{\sec2x} = 1$

$\therefore \tan\theta = 1$

Clave B

13. $M = \frac{12(4\cos^2 16° - 3)}{5\text{sen}21°\cos21°}$

$M = \frac{12(4\cos^2 16° - 3)}{5\text{sen}21°\cos21°}\cdot\frac{(2\cos16°)}{(2\cos16°)}$

$M = \frac{24(4\cos^3 16° - 3\cos16°)}{5\cos16°(2\text{sen}21°\cos21°)}$ ...(1)

Por ángulo doble:

$2\text{sen}21°\cos21° = \text{sen}2(21°) = \text{sen}42°$

$\Rightarrow 2\text{sen}21°\cos21° = \text{sen}42°$

Por ángulo triple:

$4\cos^3 16° - 3\cos16° = \cos3(16°) = \cos48°$

$\Rightarrow 4\cos^3 16° - 3\cos16° = \cos48°$

Reemplazando en (1):

$M = \frac{24(\cos48°)}{5\cos16°(\text{sen}42°)} = \frac{24\cos(90° - 42°)}{5\cos16°\text{sen}42°}$

$M = \frac{24\text{sen}42°}{5\cos16°\text{sen}42°} = \frac{24}{5\cos16°}$

$\Rightarrow M = \frac{24}{5\left(\frac{24}{25}\right)} = \frac{1}{\left(\frac{1}{5}\right)} = 5$

$\therefore M = 5$

Clave C

14. Por dato: $\text{sen}2\theta = \frac{1}{3}$

Piden: $\frac{\sec^3\theta - \csc^3\theta}{(\sec\theta - \csc\theta)\sec^2\theta\csc^2\theta}$

Sea:

$H = \frac{\sec^3\theta - \csc^3\theta}{(\sec\theta - \csc\theta)\sec^2\theta\csc^2\theta}$

$H = \frac{(\sec\theta - \csc\theta)(\sec^2\theta + \sec\theta\csc\theta + \csc^2\theta)}{(\sec\theta - \csc\theta)\sec^2\theta\csc^2\theta}$

$H = \frac{(\sec^2\theta + \csc^2\theta) + \sec\theta\csc\theta}{\sec^2\theta\csc^2\theta}$

$H = \frac{(\sec^2\theta\csc^2\theta) + \sec\theta\csc\theta}{\sec^2\theta\csc^2\theta}$

$H = \frac{\sec^2\theta\csc^2\theta}{\sec^2\theta\csc^2\theta} + \frac{\sec\theta\csc\theta}{\sec^2\theta\csc^2\theta}$

$H = 1 + \frac{1}{\sec\theta\csc\theta} = 1 + \text{sen}\theta\cos\theta$

$H = 1 + \frac{2\text{sen}\theta\cos\theta}{2} = 1 + \frac{\text{sen}2\theta}{2}$

$\Rightarrow H = 1 + \frac{\left(\frac{1}{3}\right)}{2} = 1 + \frac{1}{6} = \frac{7}{6}$

$\therefore H = \frac{7}{6}$

Clave C

PRACTIQUEMOS

Nivel 1 (página 58) Unidad 3

Comunicación matemática

1.

2.

Razonamiento y demostración

3. $\tan54° + \tan36° = (\cot36°) + \tan36°$

Sabemos: $\cot\theta + \tan\theta = 2\csc2\theta$

$\Rightarrow \tan54° + \tan36° = 2\csc2(36°)$

$\tan54° + \tan36° = 2\csc72°$

$\tan54° + \tan36° = 2\csc(90° - 18°)$

$\therefore \tan54° + \tan36° = 2\sec18°$

Clave A

4. Piden: $\tan7°30'$

$\tan7°30' = \tan\frac{15°}{2}$

$\tan7°30' = \csc15° - \cot15°$

$\tan7°30' = (\sqrt{6} + \sqrt{2}) - (2 + \sqrt{3})$

$\tan7°30' = \sqrt{6} + \sqrt{2} - 2 - \sqrt{3}$

$\therefore \tan7°30' = \sqrt{6} - \sqrt{4} - \sqrt{3} + \sqrt{2}$

Clave C

5. Por dato:

$\tan^2x - \tan x - 1 = 0$

$\Rightarrow 1 - \tan^2x = -\tan x$

$(-2)1 = \frac{-\tan x}{1 - \tan^2x}(-2)$

$-2 = \underbrace{\frac{2\tan x}{1 - \tan^2x}}_{(\tan2x)}$

$\Rightarrow \tan2x = -2$

Piden:

$M = \tan^2 2x - \tan2x - 1$

$M = (-2)^2 - (-2) - 1 = 4 + 2 - 1$

$\therefore M = 5$

Clave B

6. $E = \tan\frac{\pi}{8} - \cot\frac{\pi}{8}$

Por identidad: $2\cot2\theta = \cot\theta - \tan\theta$

$\Rightarrow \tan\theta - \cot\theta = -2\cot2\theta$

Entonces:

$E = -2\cot2\left(\frac{\pi}{8}\right) = -2\cot\frac{\pi}{4}$

$\Rightarrow E = -2\cot45° = -2(1)$

$\therefore E = -2$

Clave A

7. $P = \sqrt{\frac{1 - \text{sen}40°}{1 + \text{sen}40°}}$

$P = \sqrt{\frac{1 - \text{sen}(90° - 50°)}{1 + \text{sen}(90° - 50°)}}$

$P = \sqrt{\frac{1 - \cos50°}{1 + \cos50°}}$

$P = \left|\tan\frac{50°}{2}\right| = \underbrace{|\tan25°|}_{(+)}$

$\therefore P = \tan25°$

Clave C

8. $E = \sqrt{\frac{1 - \cos200°}{1 + \cos200°}} = \left|\tan\frac{200°}{2}\right|$

$\Rightarrow E = \underbrace{|\tan100°|}_{(-)} = -(\tan100°)$

$\therefore E = -\tan100°$

Clave A

Resolución de problemas

9. Dato: $\cos\theta = \frac{3}{5} \Rightarrow \text{sen}\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$

$\text{sen}\frac{\theta}{2} = \pm\sqrt{\frac{1 - 3/5}{2}}$

$\therefore \text{sen}\frac{\theta}{2} = \frac{\sqrt{5}}{5}$

Clave A

10. Dato: $\cos\theta = \frac{2}{3} \Rightarrow \cos\frac{\theta}{2} = \pm\sqrt{\frac{\cos\theta + 1}{2}}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{2/3 + 1}{2}}$

$\therefore \cos\frac{\theta}{2} = +\sqrt{\frac{5}{6}}$

Clave E

Nivel 2 (página 58) Unidad 3

Comunicación matemática

11.

12.

Razonamiento y demostración

13. $\dfrac{\text{sen}2\alpha+\text{sen}\alpha}{1+\cos2\alpha+\cos\alpha}=\dfrac{(2\text{sen}\alpha\cos\alpha)+\text{sen}\alpha}{(2\cos^2\alpha)+\cos\alpha}$

$\dfrac{\text{sen}2\alpha+\text{sen}\alpha}{1+\cos2\alpha+\cos\alpha}=\dfrac{\text{sen}\alpha(2\cos\alpha+1)}{\cos\alpha(2\cos\alpha+1)}$

$\therefore\ \dfrac{\text{sen}2\alpha+\text{sen}\alpha}{1+\cos2\alpha+\cos\alpha}=\dfrac{\text{sen}\alpha}{\cos\alpha}=\tan\alpha$

Clave A

14. Piden: tan2x

Por dato: $0° < x < 45°$

$\Rightarrow 0° < 2x < 90° \Rightarrow (2x)$ es agudo

Además: $\text{sen}x-\cos x=\dfrac{1}{5}$

$(\text{sen}x-\cos x)^2=\left(\dfrac{1}{5}\right)^2$

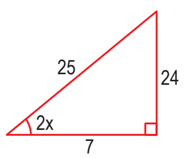
Resolviendo:

$\underbrace{\text{sen}^2x+\cos^2x}_{(1)}-2\text{sen}x\cos x=\dfrac{1}{25}$

$\Rightarrow 2\text{sen}x\cos x=1-\dfrac{1}{25}$

$\text{sen}2x=\dfrac{24}{25}$

Luego:



$\therefore\ \tan2x=\dfrac{24}{7}$

Clave A

15. $E=\sqrt{1-\text{sen}20°}+\text{sen}10°$

$E=\sqrt{1-2\text{sen}10°\cos10°}+\text{sen}10°$

$E=\sqrt{(\text{sen}10°-\cos10°)^2}+\text{sen}10°$

$\Rightarrow E=|\text{sen}10°-\cos10°|+\text{sen}10°$

Luego: $10° \in$ IC, y analizando en la CT obtenemos que: $\cos10° > \text{sen}10°$.

$\Rightarrow E=\underbrace{|\text{sen}10°-\cos10°|}+\text{sen}10°$

$E=-(\text{sen}10°-\cos10°)+\text{sen}10°$

$E=\cos10°-\text{sen}10°+\text{sen}10°$

$\therefore E=\cos10°$

Clave A

16. Por dato: $\tan\alpha=3$

Piden: $\cos4\alpha$

$\cos4\alpha=\dfrac{1-\tan^22\alpha}{1+\tan^22\alpha}$ ...(1)

Luego:

$\tan2\alpha=\dfrac{2\tan\alpha}{1-\tan^2\alpha}=\dfrac{2(3)}{1-(3)^2}$

$\Rightarrow \tan2\alpha=-\dfrac{3}{4}$

Reemplazando en (1):

$\cos4\alpha=\dfrac{1-\left(-\frac{3}{4}\right)^2}{1+\left(-\frac{3}{4}\right)^2}=\dfrac{7}{25}$ $\quad\therefore\ \cos4\alpha=\dfrac{7}{25}$

Clave B

17. Sea:

$H=\dfrac{\text{sen}\theta\cot\left(\frac{\theta}{2}\right)-1}{\text{sen}\theta\tan\left(\frac{\theta}{2}\right)+\cos\theta}$

$H=\dfrac{\text{sen}\theta(\csc\theta+\cot\theta)-1}{\text{sen}\theta(\csc\theta-\cot\theta)+\cos\theta}$

$H=\dfrac{\overbrace{\text{sen}\theta\csc\theta}^{1}+\text{sen}\theta\cot\theta-1}{\underbrace{\text{sen}\theta\csc\theta}_{1}-\text{sen}\theta\cot\theta+\cos\theta}$

$H=\dfrac{\text{sen}\theta\left(\frac{\cos\theta}{\text{sen}\theta}\right)}{1-\text{sen}\theta\left(\frac{\cos\theta}{\text{sen}\theta}\right)+\cos\theta}$

$\Rightarrow H=\dfrac{\cos\theta}{1-\cos\theta+\cos\theta}=\cos\theta$

$\therefore\ H=\cos\theta$

Clave A

18. Por dato:

$\dfrac{\cos4\theta}{\cos2\theta+\text{sen}2\theta}+\dfrac{\text{sen}4\theta}{2\cos2\theta}=\dfrac{\csc\theta}{5}$

Sabemos:

$\cos4\theta=\cos^22\theta-\text{sen}^22\theta$

$\cos4\theta=(\cos2\theta+\text{sen}2\theta)(\cos2\theta-\text{sen}2\theta)$

$\Rightarrow \dfrac{\cos4\theta}{\cos2\theta+\text{sen}2\theta}=\cos2\theta-\text{sen}2\theta$

Reemplazando tenemos:

$(\cos2\theta-\text{sen}2\theta)+\dfrac{2\text{sen}2\theta\cos2\theta}{2\cos2\theta}=\dfrac{\csc\theta}{5}$

$\cos2\theta-\text{sen}2\theta+\text{sen}2\theta=\dfrac{\csc\theta}{5}$

$\Rightarrow \cos2\theta=\dfrac{1}{5\text{sen}\theta} \Rightarrow \text{sen}\theta\cos2\theta=\dfrac{1}{5}$

Piden:

$\dfrac{\text{sen}4\theta}{\cos\theta}=\dfrac{2\text{sen}2\theta\cos2\theta}{\cos\theta}$

$\dfrac{\text{sen}4\theta}{\cos\theta}=\dfrac{2(2\text{sen}\theta\cos\theta)\cos2\theta}{\cos\theta}$

$\dfrac{\text{sen}4\theta}{\cos\theta}=4\text{sen}\theta\cos2\theta=4\left(\dfrac{1}{5}\right)$

$\therefore\ \dfrac{\text{sen}4\theta}{\cos\theta}=\dfrac{4}{5}$

Clave C

Resolución de problemas

19. Dato: $\tan\theta=2 \Rightarrow \text{sen}2\theta=\dfrac{2\tan\theta}{1+\tan^2\theta}$

$\text{sen}2\theta=\dfrac{2(2)}{1+2^2}$

$\therefore\ \text{sen}2\theta=\dfrac{4}{5}$

Clave A

20. Dato: $\tan\theta=3 \Rightarrow \cos2\theta=\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$\cos2\theta=\dfrac{1-3^2}{1+3^2}$

$\therefore\ \cos2\theta=-\dfrac{4}{5}$

Clave E

Nivel 3 (página 59) Unidad 3

Comunicación matemática

21.

22.

Razonamiento y demostración

23. Por dato:

$\dfrac{\pi}{2}<\theta<\pi \wedge \cos\theta=-\dfrac{3}{4}$

$\Rightarrow \dfrac{\pi}{4}<\dfrac{\theta}{2}<\dfrac{\pi}{2} \Rightarrow \dfrac{\theta}{2}\in$ IC

Luego:

$\text{sen}\dfrac{\theta}{2}=+\sqrt{\dfrac{1-\cos\theta}{2}}=\sqrt{\dfrac{1-\left(-\frac{3}{4}\right)}{2}}$

$\Rightarrow \text{sen}\dfrac{\theta}{2}=\sqrt{\dfrac{7}{8}}$

$\cos\dfrac{\theta}{2}=+\sqrt{\dfrac{1+\cos\theta}{2}}=\sqrt{\dfrac{1+\left(-\frac{3}{4}\right)}{2}}$

$\Rightarrow \cos\dfrac{\theta}{2}=+\sqrt{\dfrac{1}{8}}$

Piden:

$F=\sqrt{7}\,\text{sen}\dfrac{\theta}{2}+\cos\dfrac{\theta}{2}$

$F=\sqrt{7}\left(\sqrt{\dfrac{7}{8}}\right)+\left(\sqrt{\dfrac{1}{8}}\right)$

$\Rightarrow F=\dfrac{7}{\sqrt{8}}+\dfrac{1}{\sqrt{8}}=\dfrac{8}{\sqrt{8}}=\dfrac{8}{2\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}$

$\therefore F=2\sqrt{2}$

Clave E

24. $M=\dfrac{1}{\text{sen}x}+\dfrac{1}{\text{sen}2x}+\dfrac{1}{\text{sen}4x}+\dfrac{\cos^22x-\text{sen}^22x}{\text{sen}4x}$

$M=\dfrac{1}{\text{sen}x}+\dfrac{1}{\text{sen}2x}+\dfrac{1}{\text{sen}4x}+\dfrac{\cos4x}{\text{sen}4x}$

$M=\dfrac{1}{\text{sen}x}+\dfrac{1}{\text{sen}2x}+\dfrac{1+\cos4x}{\text{sen}4x}$

$M=\dfrac{1}{\text{sen}x}+\dfrac{1}{\text{sen}2x}+\dfrac{2\cos^22x}{2\text{sen}2x\cos2x}$

$M=\dfrac{1}{\text{sen}x}+\dfrac{1}{\text{sen}2x}+\dfrac{\cos2x}{\text{sen}2x}$

$M=\dfrac{1}{\text{sen}x}+\dfrac{1+\cos2x}{\text{sen}2x}$

$M=\dfrac{1}{\text{sen}x}+\dfrac{2\cos^2x}{2\text{sen}x\cos x}=\dfrac{1}{\text{sen}x}+\dfrac{\cos x}{\text{sen}x}$

$\Rightarrow M=\dfrac{1+\cos x}{\text{sen}x}=\dfrac{2\cos^2\frac{x}{2}}{2\text{sen}\frac{x}{2}\cos\frac{x}{2}}=\dfrac{\cos\frac{x}{2}}{\text{sen}\frac{x}{2}}$

$\therefore\ M=\cot\dfrac{x}{2}$

Clave C

25. Por dato:

$$\frac{\cos\theta}{a} = \frac{\text{sen}\theta}{b}$$

$$\Rightarrow \frac{\text{sen}\theta}{\cos\theta} = \frac{b}{a} \Rightarrow \tan\theta = \frac{b}{a}$$

Piden:

$E = a\cos2\theta + b\,\text{sen}2\theta$

$$E = a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$E = a\left[\frac{1-\left(\frac{b}{a}\right)^2}{1+\left(\frac{b}{a}\right)^2}\right] + b\left[\frac{2\left(\frac{b}{a}\right)}{1+\left(\frac{b}{a}\right)^2}\right]$$

$$E = \frac{a(a^2-b^2)}{a^2+b^2} + \frac{2ab^2}{a^2+b^2}$$

$$E = \frac{a(a^2-b^2+2b^2)}{a^2+b^2} = \frac{a(a^2+b^2)}{a^2+b^2}$$

$\therefore E = a$

Clave C

26. Por dato:

$$\underbrace{\cot x - \tan x}_{(2\cot 2x)} = k$$

$$\Rightarrow \cot2x = \frac{k}{2} \;\wedge\; \tan2x = \frac{2}{k}$$

Piden: tan4x

$$\tan4x = \frac{2\tan2x}{1-\tan^2 2x}$$

$$\tan4x = \frac{2\left(\frac{2}{k}\right)}{1-\left(\frac{2}{k}\right)^2}$$

$$\therefore \tan4x = \frac{4k}{k^2-4}$$

Clave A

27. Por dato: $\text{sen}\alpha = \dfrac{a-b}{a+b}$

Piden:

$$k = \tan\left(\frac{\pi}{4} - \frac{\alpha}{2}\right) = \frac{\tan\frac{\pi}{4} - \tan\frac{\alpha}{2}}{1+\tan\frac{\pi}{4}\tan\frac{\alpha}{2}}$$

$$k = \frac{1-\dfrac{\text{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}}{1+\dfrac{\text{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}} = \frac{\cos\frac{\alpha}{2} - \text{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2} + \text{sen}\frac{\alpha}{2}}$$

Luego multiplicamos al numerador y denominador por $\left(\cos\frac{\alpha}{2} + \text{sen}\frac{\alpha}{2}\right)$:

$$k = \frac{\cos^2\frac{\alpha}{2} - \text{sen}^2\frac{\alpha}{2}}{1+2\text{sen}\frac{\alpha}{2}\cos\frac{\alpha}{2}} = \frac{\cos\alpha}{1+\text{sen}\alpha}$$

Elevando al cuadrado:

$$k^2 = \frac{\cos^2\alpha}{(1+\text{sen}\alpha)^2} = \frac{1-\text{sen}^2\alpha}{(1+\text{sen}\alpha)^2}$$

$$k^2 = \frac{(1+\text{sen}\alpha)(1-\text{sen}\alpha)}{(1+\text{sen}\alpha)^2} = \frac{1-\text{sen}\alpha}{1+\text{sen}\alpha}$$

$$k^2 = \frac{1-\left(\frac{a-b}{a+b}\right)}{1+\left(\frac{a-b}{a+b}\right)} = \frac{\frac{2b}{a+b}}{\frac{2a}{a+b}} = \frac{b}{a}$$

$$\therefore k = \pm\sqrt{\frac{b}{a}}$$

Clave B

28. Sea:

$$S = \tan x + \frac{1}{2}\tan\frac{x}{2} + \frac{1}{4}\tan\frac{x}{4} + \ldots + \frac{1}{2^n}\tan\frac{x}{2^n}$$

Por identidades: $2\cot2\theta = \cot\theta - \tan\theta$

$\Rightarrow \tan\theta = \cot\theta - 2\cot2\theta$

$\Rightarrow \cot2\theta = \dfrac{1}{2}\cot\theta - \dfrac{1}{2}\tan\theta$

Para 2 términos:

$$S_2 = \tan x + \frac{1}{2}\tan\frac{x}{2} = \cot x - 2\cot2x + \frac{1}{2}\tan\frac{x}{2}$$

$$S_2 = \left(\frac{1}{2}\cot\frac{x}{2} - \frac{1}{2}\tan\frac{x}{2}\right) + \frac{1}{2}\tan\frac{x}{2} - 2\cot2x$$

$$\Rightarrow S_2 = \frac{1}{2}\cot\frac{x}{2} - 2\cot2x$$

Para 3 términos:

$$S_3 = \tan x + \frac{1}{2}\tan\frac{x}{2} + \frac{1}{4}\tan\frac{x}{4}$$

$$S_3 = \left(\frac{1}{2}\cot\frac{x}{2} - 2\cot2x\right) + \frac{1}{4}\tan\frac{x}{4}$$

$$S_3 = \frac{1}{2}\left(\frac{1}{2}\cot\frac{x}{4} - \frac{1}{2}\tan\frac{x}{4}\right) + \frac{1}{4}\tan\frac{x}{4} - 2\cot2x$$

$$S_3 = \frac{1}{4}\cot\frac{x}{4} - 2\cot2x$$

$$\Rightarrow S_3 = \frac{1}{2^2}\cot\frac{x}{2^2} - 2\cot2x$$

Para 4 términos, se obtiene:

$$\Rightarrow S_4 = \frac{1}{2^3}\cot\frac{x}{2^3} - 2\cot2x$$

Como la serie original tiene (n + 1) términos:

$$\therefore S = \frac{1}{2^n}\cot\frac{x}{2^n} - 2\cot2x$$

Clave D

Resolución de problemas

29. Dato: $\cot\theta = 2 \Rightarrow \tan\theta = \dfrac{1}{2}$

$$\tan3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan\theta^2}$$

$$\tan3\theta = \frac{3\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^3}{1-3\left(\frac{1}{2}\right)^2}$$

$$\therefore \tan3\theta = \frac{11}{2}$$

Clave D

30. Dato: $\sec\theta = 3 \Rightarrow \cos\theta = \dfrac{1}{3}$

$$\cos3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\cos3\theta = 4\left(\frac{1}{3}\right)^3 - 3\left(\frac{1}{3}\right)$$

$$\therefore \cos3\theta = -\frac{23}{27}$$

Clave B

APLICAMOS LO APRENDIDO (página 60) Unidad 3

1. $P = \frac{64}{2}\left(2\text{sen}\frac{5A}{4}\text{sen}\frac{3A}{4}\right)$

$P = 32\left(\cos\frac{A}{2} - \cos 2A\right)$ ... (1)

Sabemos:

$\cos 2A = 2\cos^2 A - 1$

$\cos A = 2\cos^2\frac{A}{2} - 1$

$\therefore \cos 2A = 2\left(2\cos^2\frac{A}{2} - 1\right)^2 - 1$

$\cos 2A = 2\left(2\left(\frac{1}{4}\right)^2 - 1\right)^2 - 1 = 2\left(\frac{-7}{8}\right)^2 - 1$

$\cos 2A = \frac{17}{32}$

En (1):

$P = 32\left(\frac{1}{4} - \frac{17}{32}\right) = 32\left(\frac{-9}{32}\right) = -9$

$\therefore P = -9$

Clave B

2. $A = 3 + \sqrt{3} = 2\sqrt{3}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)$

$A = 2\sqrt{3}\ (\text{sen}60° + \text{sen}30°)$

$A = 2\sqrt{3}\left(2\text{sen}\left(\frac{60° + 30°}{2}\right)\cos\left(\frac{60° - 30°}{2}\right)\right)$

$A = 2\sqrt{3}\ (2\text{sen}45°\cos 15°)$

$A = 4\sqrt{3}\ .\ \frac{\sqrt{2}}{2}\cos 15° = 2\sqrt{6}\cos 15°$

$\therefore A = 2\sqrt{6}\cos 15°$

Clave C

3.

$y = \frac{3}{\text{sen}^2 x} - 4 = \frac{3 - 4\text{sen}^2 x}{\text{sen}^2 x}\ .\ \frac{\text{sen}x}{\text{sen}x}$

$y = \frac{3\text{sen}x - 4\text{sen}^3 x}{\text{sen}^3 x} \Rightarrow y = \frac{\text{sen}3x}{\text{sen}^3 x}$

$\therefore y = \text{sen}3x\ \text{sen}^{-3}x$

Clave C

4. $N = [(1 + \cos 2\alpha) + 2\cos\alpha] / \cos^2\frac{\alpha}{2}$

$N = [2\cos^2\alpha + 2\cos\alpha] / \cos^2\frac{\alpha}{2}$

$N = [2\cos\alpha\ (\cos\alpha + 1)] / \cos^2\frac{\alpha}{2}$

$N = \frac{\left[2\cos\alpha\left(2\cos^2\frac{\alpha}{2}\right)\right]}{\cos^2\frac{\alpha}{2}} = 4\cos\alpha$

$\therefore N = 4\cos\alpha$

Clave A

5. $R = 2\cos 2x\cos 3x - \cos x$

$R = \cos(2x + 3x) + \cos(2x - 3x) - \cos x$

$R = \cos 5x + \underbrace{\cos(-x)} - \cos x$

$R = \cos 5x + \cos x - \cos x = \cos 5x$

$\therefore R = \cos 5x$

Clave C

6. $R = \text{sen}3x\text{sen}7x + \cos 2x\cos 8x$

$2R = 2\text{sen}7x\text{sen}3x + 2\cos 8x\cos 2x$

$2R = (\cos 4x - \cos 10x) + (\cos 10x + \cos 6x)$

$2R = \cos 4x - \cos 10x + \cos 10x + \cos 6x$

$2R = \cos 6x + \cos 4x$

$2R = 2\cos\left(\frac{6x + 4x}{2}\right)\cos\left(\frac{6x - 4x}{2}\right)$

$\Rightarrow 2R = 2\cos 5x\cos x$

$\therefore R = \cos 5x\cos x$

Clave D

7. $B = \frac{\text{sen}5x + \text{sen}2x - \text{sen}x}{\text{sen}2x}$

$B = \frac{(\text{sen}5x - \text{sen}x) + \text{sen}2x}{\text{sen}2x}$

$B = \frac{2\text{sen}\left(\frac{5x - x}{2}\right)\cos\left(\frac{5x + x}{2}\right) + \text{sen}2x}{\text{sen}2x}$

$B = \frac{2\text{sen}2x\cos 3x + \text{sen}2x}{\text{sen}2x}$

$B = \frac{2\text{sen}2x\cos 3x}{\text{sen}2x} + \frac{\text{sen}2x}{\text{sen}2x}$

$\therefore\ = 2\cos 3x + 1$

Clave C

8. $P = \frac{\text{sen}x + \text{sen}3x + \text{sen}5x + \text{sen}7x}{\cos x + \cos 3x + \cos 5x + \cos 7x}$

Empleando las series trigonométricas:
P: primer ángulo = x
U: último ángulo = 7x
n: n.° de términos = 4
r: razón = 2x
En el numerador (N):

$N = \frac{\text{sen}\frac{nr}{2}}{\text{sen}\frac{r}{2}}\ .\ \text{sen}\left(\frac{P + U}{2}\right)$

$N = \frac{\text{sen}\frac{4(2x)}{2}}{\text{sen}\frac{2x}{2}}\ .\ \text{sen}\left(\frac{x + 7x}{2}\right)$

$\Rightarrow N = \frac{\text{sen}4x}{\text{sen}x}\ .\ \text{sen}4x$

En el denominador (D):

$D = \frac{\text{sen}\frac{nr}{2}}{\text{sen}\frac{r}{2}}\ .\ \cos\left(\frac{P + U}{2}\right)$

$D = \frac{\text{sen}\frac{4(2x)}{2}}{\text{sen}\frac{2x}{2}}\ .\ \cos\left(\frac{x + 7x}{2}\right)$

$\Rightarrow D = \frac{\text{sen}4x}{\text{sen}x}\ .\ \cos 4x$

Entonces:

$P = \frac{N}{D} = \frac{\frac{\text{sen}4x}{\text{sen}x}\cdot\text{sen}4x}{\frac{\text{sen}4x}{\text{sen}x}\cdot\cos 4x}$

$P = \frac{\text{sen}4x}{\cos 4x} = \tan 4x$

$\therefore P = \tan 4x$

Clave E

9.

$F = \frac{\text{sen}2\alpha + \text{sen}\alpha}{\cos\frac{\alpha}{2}}$

$F = \frac{2\text{sen}\left(\frac{2\alpha + \alpha}{2}\right)\cos\left(\frac{2\alpha - \alpha}{2}\right)}{\cos\frac{\alpha}{2}}$

$F = \frac{2\text{sen}\frac{3\alpha}{2}\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = 2\text{sen}\frac{3\alpha}{2}$

$F = 2\left[3\text{sen}\frac{\alpha}{2} - 4\text{sen}^3\frac{\alpha}{2}\right]$

$F = 2\left[3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3\right]$

$F = 2\left[\frac{3}{2} - 4\left(\frac{1}{8}\right)\right] = 2\left(\frac{3}{2} - \frac{1}{2}\right)$

$F = 2(1) = 2$

Clave B

10. $K = \frac{2\text{sen}40°}{\text{sen}60° + \text{sen}20°}$

$K = \frac{2\text{sen}40°}{2\text{sen}\left(\frac{60° + 20°}{2}\right)\cos\left(\frac{60° - 20°}{2}\right)}$

$K = \frac{\text{sen}40°}{\text{sen}40°\cos 20°} = \frac{1}{\cos 20°}$

$\therefore K = \sec 20°$

Clave C

11.

$A = \text{sen}1° + \text{sen}(1° + 1°) + \text{sen}(1° + 2(1°)) + ... + \text{sen}(1° + 179(1°))$

Primer ángulo : $P = 1°$
Último ángulo : $U = 180°$
n.° de términos : $n = 180°$
Razón : $r = 1$

$A = \text{sen}1° + \text{sen}2° + \text{sen}3° + ... + \text{sen}180°$

$$A=\frac{\text{sen}\left(\frac{180^\circ(1)}{2}\right)}{\text{sen}\left(\frac{1^\circ}{2}\right)}\,\text{sen}\left(\frac{1^\circ+180^\circ}{2}\right)$$

$$A=\frac{\text{sen}90^\circ}{\text{sen}\left(\frac{1^\circ}{2}\right)}\,\text{sen}\,\frac{181^\circ}{2}$$

$$A=\frac{\text{sen}\,\frac{181^\circ}{2}}{\text{sen}\left(\frac{1^\circ}{2}\right)}=\frac{\text{sen}\ 90{,}5^\circ}{\text{sen}\ 0{,}5^\circ}$$

Clave D

12.

Por dato: A, B y C son los ángulos internos de un triángulo.

$\Rightarrow A+B+C=180^\circ$ ...(1)

Además: $\text{senAsenB}=\cos C$

$\Rightarrow 2\text{senAsenB}=2\cos C$

$\cos(A-B)-\cos(A+B)=2\cos C$

$\cos(A-B)-\cos(180^\circ-C)=2\cos C$

$\cos(A-B)-(-\cos C)=2\cos C$

$\cos(A-B)+\cos C=2\cos C$

$\cos(A-B)=\cos C$

$\Rightarrow \cos(A-B)=\cos C$

$\Rightarrow A-B=C\Rightarrow A=B+C$ ...(2)

Reemplazando (2) en (1):

$A+(A)=180^\circ$

$2A=180^\circ$

$\Rightarrow A=90^\circ$

Entonces uno de los ángulos internos del triángulo mide 90°. Por lo tanto, el triángulo es rectángulo.

Clave C

13. $S=\text{sen}(x+30^\circ)\cos x$

$2S=2\text{sen}(x+30^\circ)\cos x$

$2S=\text{sen}(x+30^\circ+x)+\text{sen}(x+30^\circ-x)$

$2S=\text{sen}(2x+30^\circ)+\text{sen}30^\circ$

$2S=\text{sen}(2x+30^\circ)+\frac{1}{2}$

$\Rightarrow S=\frac{\text{sen}(2x+30^\circ)}{2}+\frac{1}{4}$

Sabemos:

$-1\le \text{sen}(2x+30^\circ)\le 1$

$-\frac{1}{2}\le\frac{\text{sen}(2x+30^\circ)}{2}\le\frac{1}{2}$

$$-\frac{1}{2}+\frac{1}{4}\le\underbrace{\frac{\text{sen}(2x+30^\circ)}{2}+\frac{1}{4}}\le\frac{1}{2}+\frac{1}{4}$$

$$-\frac{1}{4}\le S\le\frac{3}{4}$$

$\therefore S_{\text{máx.}}=\frac{3}{4}$

Clave A

14. Por dato:

$\text{sen}8x+\text{sen}4x=\text{AsenBxcosCx}$

$2\text{sen}\left(\frac{8x+4x}{2}\right)\cos\left(\frac{8x-4x}{2}\right)=\text{AsenBxcosCx}$

$\Rightarrow 2\text{sen}6x\cos 2x=\text{AsenBxcosCx}$

Comparando: $A=2$; $B=6$; $C=2$

Piden:

$A+B+C=2+6+2=10$

$\therefore A+B+C=10$

Clave D

PRACTIQUEMOS

Nivel 1 (página 62) Unidad 3

Comunicación matemática

1.

- $\text{sen}52^\circ\ \text{sen}88^\circ=\frac{1}{2}(\cos36^\circ-\cos140^\circ)$
- $\text{sen}\theta\cos3\theta=\frac{1}{2}(\text{sen}4\theta-\text{sen}2\theta)$
- $2\cos3\theta\ \cos\theta=\cos2\theta+\cos4\theta$
- $\text{sen}3x\ \text{sen}7x=\frac{1}{2}(\cos4x-\cos10x)$
- $\cos2x\ \cos8x=\frac{1}{2}(\cos6x+\cos10x)$
- $\text{sen}3\theta\ \text{sen}5\theta=\frac{1}{2}(\cos2\theta-\cos8\theta)$

2.

- $\cos5\theta+\cos\theta=2\cos\left(\frac{5\theta+\theta}{2}\right)\cos\left(\frac{5\theta-\theta}{2}\right)$

$=2\cos3\theta\ \cos2\theta$

- $\text{sen}4x+\text{sen}2x=$

$2\text{sen}\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)$

$=2\text{sen}3x\ \cos x$

- $\cos19^\circ-\cos9^\circ$

$=-2\text{sen}\left(\frac{19^\circ+9^\circ}{2}\right)\text{sen}\left(\frac{19^\circ-9^\circ}{2}\right)$

$=-2\text{sen}14^\circ\ \text{sen}5^\circ$

- $\cos3x+\cos4x=$

$2\cos\left(\frac{3x+4x}{2}\right)\cos\left(\frac{3x-4x}{2}\right)$

$=2\cos\frac{7x}{2}\cos\left(\frac{-x}{2}\right)$

$=2\cos\frac{7x}{2}\cos\frac{x}{2}$

- $\text{sen}\frac{\pi}{9}+\text{sen}\frac{\pi}{10}=2\text{sen}\frac{\pi}{180}\cos\frac{\pi}{180}$

- $\text{sen}4x+\cos8x=\cos(90^\circ-4x)+\cos8x$

$=2\cos\left(\frac{90^\circ-4x+8x}{2}\right)\cos\left(\frac{90^\circ-4x-8x}{2}\right)$

$=2\cos(45^\circ+2x)\cos(45^\circ-6x)$

- $\text{sen}6x+\cos4x=\text{sen}6x+\text{sen}(90^\circ-4x)$

$=2\text{sen}\left(\frac{6x+90-4x}{2}\right)\cos\left(\frac{6x-(90-4x)}{2}\right)$

$=2\text{sen}(45^\circ+x)\cos(5x-45^\circ)$

Razonamiento y demostración

3.

$H=\frac{1-\text{sen}^2x-\text{sen}^2y}{\cos(x-y)}$

Sabemos: $1-\text{sen}^2x=\cos^2x$

$H=\frac{\cos^2x-\text{sen}^2y}{\cos(x-y)}=\frac{2\cos^2x-2\text{sen}^2y}{2\cos(x-y)}$

$H=\frac{(1+\cos2x)-(1-\cos2y)}{2\cos(x-y)}$

$H=\frac{\cos2x+\cos2y}{2\cos(x-y)}=\frac{2\cos(x+y)\cos(x-y)}{2\cos(x-y)}$

$\therefore H=\cos(x+y)$

Clave A

4.

$L=\frac{\text{sen}80^\circ+\text{sen}40^\circ}{\cos80^\circ+\cos40^\circ}$

$L=\frac{2\text{sen}60^\circ\cos20^\circ}{2\cos60^\circ\cos20^\circ}$

$\Rightarrow L=\frac{\text{sen}60^\circ}{\cos60^\circ}=\tan60^\circ=\sqrt{3}$

$\therefore L=\sqrt{3}$

Clave C

5.

$H=\underbrace{1+\cos2x}_{2}+\cos4x+\cos6x$

$H=2\cos^2x+(2\cos5x\cos x)$

$H=2\cos x(\cos x+\cos5x)$

$H=2\cos x(2\cos3x\cos2x)$

$\therefore H=4\cos x\cos2x\cos3x$

Clave B

6.

$M=\underbrace{\text{sen}3x+\text{sen}5x}+\text{sen}8x$

$M=(2\text{sen}4x\cos x)+2\text{sen}4x\cos4x$

$M=2\text{sen}4x(\cos x+\cos4x)$

$M=2\text{sen}4x\left(2\cos\frac{5x}{2}\cos\frac{3x}{2}\right)$

$\therefore M=4\text{sen}4x\cos\frac{5x}{2}\cos\frac{3x}{2}$

Clave B

7.

$K=\frac{\text{sen}50^\circ+\cos50^\circ}{\cos5^\circ}$

$K=\frac{\text{sen}50^\circ+\cos(90^\circ-40^\circ)}{\cos5^\circ}$

$K=\frac{\text{sen}50^\circ+\text{sen}40^\circ}{\cos5^\circ}$

$K=\frac{2\text{sen}45^\circ\cos5^\circ}{\cos5^\circ}=2\text{sen}45^\circ$

$\Rightarrow K=2\left(\frac{\sqrt{2}}{2}\right)=\sqrt{2}$

$\therefore K=\sqrt{2}$

Clave B

8.

$$M = \frac{\text{sen}40° + \text{sen}20°}{\cos 10°}$$

$$M = \frac{2\text{sen}30° \cos 10°}{\cos 10°}$$

$\Rightarrow M = 2\text{sen}30° = 2\left(\frac{1}{2}\right) = 1$

$\therefore M = 1$

Clave A

9.

$S = \cos 20° + \cos 100° + \cos 140°$

$S = (2\cos 60° \cos 40°) + \cos 140°$

$S = 2\left(\frac{1}{2}\right)\cos 40° + \cos 140°$

$S = \cos 40° + \cos 140°$

$S = 2\cos 90° \cos 50°$

Pero: $\cos 90° = 0$

$\Rightarrow S = 2(0)\cos 50° = 0$

$\therefore S = 0$

Clave A

10. Sea:

$M = (\text{sen}38° + \cos 68°)\sec 8°$

$M = (\text{sen}38° + \text{sen}22°)\sec 8°$

$M = (2\text{sen}30° \cos 8°)\sec 8°$

$M = 2\text{sen}30° \underbrace{\cos 8° \sec 8°}_{(1)}$

$\Rightarrow M = 2\left(\frac{1}{2}\right) = 1$

$\therefore M = 1$

Clave A

Resolución de problemas

11.

$p = \underbrace{\cos^2\theta + \cos^2 2\theta + \cos^2 3\theta + ... + \cos^2 k\theta}_{k \text{ términos}}$

$2p = 2\cos^2\theta + 2\cos^2 2\theta + 2\cos^2 3\theta + ... + 2\cos^2 k\theta$

$2p = \cos 2\theta + 1 + \cos 4\theta + 1 + ... + \cos 2k\theta + 1$

$2p = k + \cos 2\theta + \cos 4\theta + ... + \cos 2k\theta$

\# términos : k

primer ángulo: 2θ

último ángulo: $2k\theta$

razón: 2θ

$$2p = k + \frac{\text{sen}\left(\frac{k \cdot 2\theta}{2}\right)}{\text{sen}\left(\frac{2\theta}{2}\right)} \cdot \cos\left(\frac{2\theta° + 2k\theta}{2}\right)$$

$$2p = k + \frac{\text{sen}k\theta}{\text{sen}\theta} \cdot \cos(\theta + k\theta)$$

$$p = \frac{1}{2}\left[\left(k + \frac{\text{sen}k\theta}{\text{sen}\theta}\right)\cos(\theta + k\theta)\right]$$

Clave B

12.

$M = \text{sen}74° \,\text{sen}34° - \text{sen}52° \,\text{sen}88°$

$M = \frac{1}{2}(\cos 40° - \cos 108°) - \frac{1}{2}(\cos 36° - \cos 140°)$

$M = \frac{1}{2}(\cos 40° - (-\cos 72°)) - \frac{1}{2}(\cos 36° - (-\cos 40°))$

$M = \frac{1}{2}\cos 40° + \frac{1}{2}\cos 72° - \frac{1}{2}\cos 36° - \frac{1}{2}\cos 40°$

$$M = \frac{\cos 72° - \cos 36°}{2}$$

$$M = \frac{\frac{\sqrt{5}-1}{4} - \frac{\sqrt{5}+1}{4}}{2} \Rightarrow M = -\frac{1}{4}$$

Clave E

Nivel 2 (página 62) Unidad 3

Comunicación matemática

13. Según teoría tenemos:

Ic - IIa - IIIb

Clave D

14.

- $\text{sen}A - \text{sen}B = 2\text{sen}\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$ (F)
- $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ (V)
- $\cos A - \cos B = -2\text{sen}\left(\frac{A+B}{2}\right)\text{sen}\left(\frac{A-B}{2}\right)$ (V)
- $\text{sen}A + \text{sen}B = 2\text{sen}\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ (F)

$\therefore$ Dos son verdaderas.

Clave B

Razonamiento y demostración

15. Del enunciado: $A + B + C = \pi$ rad

$K = \text{sen}A + \text{sen}B + \text{sen}C$

$K = 2\text{sen}\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + \text{sen}C$

$K = 2\text{sen}\left(\frac{\pi}{2} - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) + \text{sen}C$

$K = 2\cos\frac{C}{2}\cos\left(\frac{A-B}{2}\right) + 2\text{sen}\frac{C}{2}\cos\frac{C}{2}$

$K = 2\cos\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) + \text{sen}\frac{C}{2}\right]$

Como: $A + B + C = \pi$ rad

$\Rightarrow \frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$ rad

$\Rightarrow \text{sen}\frac{C}{2} = \cos\left(\frac{A+B}{2}\right)$

Luego:

$K = 2\cos\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)\right]$

$K = 2\cos\frac{C}{2}\left(2\cos\frac{A}{2}\cos\frac{B}{2}\right)$

$\therefore K = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

Clave C

16. Por dato, A; B y C son los ángulos internos de un triángulo.

$\Rightarrow A + B + C = \pi$ rad

Piden transformar a producto:

$F = \text{sen}2A + \text{sen}2B - \text{sen}2C$

$F = 2\text{sen}\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right) - \text{sen}2C$

$F = 2\text{sen}(A + B)\cos(A - B) - \text{sen}2C$

$F = 2\text{sen}(\pi - C)\cos(A - B) - \text{sen}2C$

$F = 2\text{sen}C\cos(A - B) - 2\text{sen}C\cos C$

$F = 2\text{sen}C[\cos(A - B) - \cos C]$

Pero: $\cos C = -\cos(A + B)$

$\Rightarrow F = 2\text{sen}C[\cos(A - B) + \cos(A + B)]$

$F = 2\text{sen}C(2\cos A\cos B)$

$\therefore F = 4\cos A\cos B\,\text{sen}C$

Clave B

17. Por dato: $x + y = 30°$

$$H = \frac{\text{sen}(x + 3y) + \text{sen}(3x + y)}{\text{sen}2x + \text{sen}2y}$$

$$H = \frac{2\text{sen}(2x + 2y)\cos(y - x)}{2\text{sen}(x + y)\cos(x - y)}$$

$$H = \frac{\text{sen}2(x + y)\cos(x - y)}{\text{sen}(x + y)\cos(x - y)}$$

$$H = \frac{\text{sen}2(30°)}{\text{sen}30°} = \frac{\text{sen}60°}{\text{sen}30°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$\therefore H = \sqrt{3}$

Clave C

18.

$H = \cos 20° + \cos 100° + \cos 220°$

$H = \cos 20° + \cos 100° + \cos(360° - 140°)$

$H = \cos 20° + \cos 100° + \cos 140°$

Por propiedad:

$\cos(x - 120°) + \cos x + \cos(x + 120°) = 0$

Para: $x = 20°$

$\cos(-100°) + \cos 20° + \cos 140° = 0$

$\Rightarrow \cos 20° + \cos 100° + \cos 140° = 0$

$\therefore H = 0$

Clave E

19. Por dato: $x = y + 30°$

$\Rightarrow x - y = 30°$

Piden:

$$P = \frac{sen(x+y)}{sen^2x - sen^2y} = \frac{2sen(x+y)}{2sen^2x - 2sen^2y}$$

$$P = \frac{2sen(x+y)}{(1-\cos 2x)-(1-\cos 2y)}$$

$$P = \frac{2sen(x+y)}{\cos 2y - \cos 2x} = \frac{2sen(x+y)}{2sen(y+x)sen(y-x)}$$

$$P = \frac{2sen(x+y)}{-2sen(x+y)(-sen(x-y))} = \frac{1}{sen(x-y)}$$

$$\Rightarrow P = \frac{1}{sen30°} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$\therefore P = 2$

Clave C

20.

$$A = \frac{\cos(a-3b)-\cos(3a-b)}{sen2a + sen2b}$$

$$A = \frac{-2sen(2a-2b)sen(-a-b)}{2sen(a+b)\cos(a-b)}$$

$$A = \frac{sen(2a-2b)sen(a+b)}{sen(a+b)\cos(a-b)}$$

$$A = \frac{2sen(a-b)\cos(a-b)}{\cos(a-b)} = 2sen(a-b)$$

$\therefore A = 2sen(a - b)$

Clave D

Resolución de problemas

21.

$2sen^2\alpha + 2\cos^2(x-\alpha) + 2sen^2(x+\alpha) = 4$

$1 - \cos 2\alpha + 1 + \cos(2x - 2\alpha) + 1 - \cos(2x + 2\alpha) = 4$

$\cos(2x - 2\alpha) - \cos(2x + 2\alpha) = 1 + \cos 2\alpha$

$2sen2x\, sen2\alpha = 1 + 2\cos 2\alpha$

$$sen2x = \frac{1+2\cos 2\alpha}{2sen2\alpha}$$

Clave A

22.

$$C = \cos\frac{2\pi}{7}\cos\frac{4\pi}{7} + \cos\frac{4\pi}{7}\cos\frac{6\pi}{7} + \cos\frac{6\pi}{7}\cos\frac{2\pi}{7}$$

$$C = \frac{\cos\frac{2\pi}{7}+\cos\frac{6\pi}{7}}{2} + \frac{\cos\frac{2\pi}{7}+\cos\frac{10\pi}{7}}{2} + \frac{\cos\frac{4\pi}{7}+\cos\frac{8\pi}{7}}{2}$$

Pero:

$$\cos\frac{10\pi}{7} = \cos\left(2\pi - \frac{4\pi}{7}\right) = \cos\frac{4\pi}{7}$$

$$\cos\frac{8\pi}{7} = \cos\frac{6\pi}{7}$$

Entonces:

$$C = 2\frac{\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right)}{2}$$

$$C = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$$

n.° de términos = 3

Razón $= \frac{2\pi}{7}$

$$\Rightarrow C = \frac{sen\left(\frac{3}{2}\cdot\frac{2\pi}{7}\right)}{sen\left(\frac{2\pi}{2.7}\right)}\cos\left(\frac{8\pi}{7}\cdot\frac{1}{2}\right)$$

$$C = \frac{sen\left(\frac{3\pi}{7}\right)}{sen\left(\frac{\pi}{7}\right)}\cos\left(\frac{4\pi}{7}\right)$$

Pero: $\cos\left(\frac{4\pi}{7}\right) = \cos\left(\pi - \frac{3\pi}{7}\right) = -\cos\left(\frac{3\pi}{7}\right)$

$$\Rightarrow C = \frac{sen\left(\frac{3\pi}{7}\right)\left[-\cos\left(\frac{3\pi}{7}\right)\right]}{sen\left(\frac{\pi}{7}\right)}$$

$$C = \frac{-2sen\left(\frac{3\pi}{7}\right)\cos\left(\frac{3\pi}{7}\right)}{2sen\left(\frac{\pi}{7}\right)}$$

$$C = \frac{-sen\left(\frac{6\pi}{7}\right)}{2sen\left(\frac{\pi}{7}\right)} = \frac{-sen\left(\pi-\frac{\pi}{7}\right)}{2sen\left(\frac{\pi}{7}\right)}$$

$$C = \frac{-sen\left(\frac{\pi}{7}\right)}{2sen\left(\frac{\pi}{7}\right)} = \frac{-1}{2}$$

Clave D

Nivel 3 (página 63) Unidad 3

Comunicación matemática

23.

En M tenemos:

$P = sen(x + 53°)\cos x$

$2P = 2sen(x + 53°)\cos x$

$2P = sen(x + 53° + x) + sen(x + 53° - x)$

$2P = sen(2x + 53°) + sen53°$

$2P = sen(2x + 53°) + \frac{4}{5}$

$P = \frac{1}{2}(sen(2x + 53°) + \frac{4}{5})$

$-1 \le sen(2x + 53°) \le 1$

$-1 \le sen(2x + 53°) + \frac{4}{5} \le \frac{9}{5}$

$-\frac{1}{10} \le \frac{1}{2}[sen(2x + 53°) + \frac{4}{6}] \le \frac{9}{5}$

$P_{máx.} = 9/10$

$\Rightarrow M = 9/10$

En N tenemos:

$T = sen(x + 37°)sen x$

$2T = 2sen(x + 37°)sen x$

$2T = \cos(x + 37° - x) - \cos(x + 37° + x)$

$2T = \cos 37° - \cos(2x + 37°)$

$T = \frac{1}{2}\left[\frac{4}{5} - \cos(2x + 37°)\right]$

$-1 \le \cos(2x + 37°) \le 1$

$-1 \le -\cos(2x + 37°) \le 1$

$-\frac{1}{5} \le \frac{4}{5} - \cos(2x + 37°) \le \frac{9}{5}$

$-\frac{1}{10} \le \frac{1}{2}\left[\frac{4}{5} - \cos(2x + 37°)\right] \le \frac{9}{10}$

$T_{máx.} = \frac{9}{10} \Rightarrow N = 9/10$

$\therefore M = N$

Clave C

24. Si: $A + B + C = 180°$

$\Rightarrow \cos^2A + \cos^2B + \cos^2C = 1 - 2\cos A\cos B\cos C$

Luego:

$\cos^2A + \cos^2B + \cos^2C = 1$

$\Rightarrow 1 = 1 - 2\cos A\cos B\cos C$

$0 = -2\cos A\cos B\cos C$

Deducimos que:

$\cos A = 0 \lor \cos B = 0 \lor \cos C = 0$

Esto ocurre cuando un ángulo al menos mide 90°.

$\therefore$ Se cumple en un triángulo rectángulo.

Clave D

Razonamiento y demostración

25.

$$R = \frac{\cos 7x + \cos 3x}{sen7x - sen3x}$$

$$R = \frac{2\cos 5x\cos 2x}{2sen2x\cos 5x}$$

$$\Rightarrow R = \frac{\cos 2x}{sen2x} = \cot 2x$$

$\therefore R = \cot 2x$

Clave D

26.

$$T = \frac{\cos x + \cos 7x}{sen x + sen7x} + \frac{2\cos x}{sen5x + sen3x}$$

$$T = \frac{2\cos 4x\cos 3x}{2sen4x\cos 3x} + \frac{2\cos x}{2sen4x\cos x}$$

$$T = \frac{\cos 4x}{sen4x} + \frac{1}{sen4x}$$

$$T = \frac{1+\cos 4x}{sen4x} = \frac{2\cos^2 2x}{2sen2x\cos 2x}$$

$$\Rightarrow T = \frac{\cos 2x}{sen2x} = \cot 2x$$

$\therefore T = \cot 2x$

Clave D

27.

$$P=\frac{\text{sen}7x+\text{sen}3x}{\text{sen}x+\text{sen}9x}\ ;\ 6x=\pi$$

$$P=\frac{2\text{sen}5x\cos 2x}{2\text{sen}5x\cos 4x}$$

$$P=\frac{\cos 2x}{\cos 4x}=\frac{\cos(6x-4x)}{\cos 4x}$$

$$\Rightarrow P=\frac{\cos(\pi-4x)}{\cos 4x}=\frac{-\cos 4x}{\cos 4x}$$

$\therefore P=-1$

Clave B

28.

$$M=\frac{\text{sen}5\theta+\text{sen}3\theta+\text{sen}\theta}{\cos 5\theta+\cos 3\theta+\cos\theta}$$

$$M=\frac{\text{sen}3\theta+\text{sen}5\theta+\text{sen}\theta}{\cos 3\theta+\cos 5\theta+\cos\theta}$$

$$M=\frac{\text{sen}3\theta+2\text{sen}3\theta\cos 2\theta}{\cos 3\theta+2\cos 3\theta\cos 2\theta}$$

$$M=\frac{\text{sen}3\theta(1+2\cos 2\theta)}{\cos 3\theta(1+2\cos 2\theta)}$$

$$\Rightarrow M=\frac{\text{sen}3\theta}{\cos 3\theta}=\tan 3\theta$$

$\therefore M=\tan 3\theta$

Clave A

29.

$R=\text{sen}\alpha+\text{sen}3\alpha+\text{sen}5\alpha+\text{sen}7\alpha$

Primer ángulo: $P=\alpha$

Último ángulo: $U=7\alpha$

n.° de términos: $n=4$

Razón: $r=2\alpha$

$$R=\frac{\text{sen}\frac{nr}{2}}{\text{sen}\frac{r}{2}}\cdot\text{sen}\left(\frac{P+U}{2}\right)$$

Reemplazando los valores tenemos:

$$R=\frac{\text{sen}\frac{4(2\alpha)}{2}}{\text{sen}\frac{2\alpha}{2}}\cdot\text{sen}\left(\frac{\alpha+7\alpha}{2}\right)$$

$$R=\frac{\text{sen}4\alpha}{\text{sen}\alpha}\cdot\text{sen}4\alpha$$

$$R=\frac{2\text{sen}2\alpha\cos 2\alpha}{\text{sen}\alpha}\cdot\text{sen}4\alpha$$

$$R=\frac{2(2\text{sen}\alpha\cos\alpha)\cos 2\alpha}{\text{sen}\alpha}\cdot\text{sen}4\alpha$$

$\therefore R=4\text{sen}4\alpha\cos 2\alpha\cos\alpha$

Clave C

30.

$$A=\frac{\text{sen}2x+\text{sen}4x+\text{sen}6x}{\cos 2x+\cos 4x+\cos 6x}$$

$$A=\frac{\text{sen}4x+(\text{sen}6x+\text{sen}2x)}{\cos 4x+(\cos 6x+\cos 2x)}$$

$$A=\frac{\text{sen}4x+(2\text{sen}4x\cos 2x)}{\cos 4x+(2\cos 4x\cos 2x)}$$

$$A=\frac{\text{sen}4x(1+2\cos 2x)}{\cos 4x(1+2\cos 2x)}=\frac{\text{sen}4x}{\cos 4x}$$

$\therefore A=\tan 4x$

Clave D

31. Sea:

$M=(\cot 2\theta+\tan\theta)(\cos 3\theta+\cos\theta)\text{sen}\theta$

Luego:

$$\cot 2\theta+\tan\theta=\frac{\cos 2\theta}{\text{sen}2\theta}+\frac{\text{sen}\theta}{\cos\theta}$$

$$\cot 2\theta+\tan\theta=\frac{\cos 2\theta\cos\theta+\text{sen}2\theta\,\text{sen}\theta}{\text{sen}2\theta\cos\theta}$$

$$\cot 2\theta+\tan\theta=\frac{\cos(2\theta-\theta)}{\text{sen}2\theta\cos\theta}$$

$$\Rightarrow\cot 2\theta+\tan\theta=\frac{\cos\theta}{\text{sen}2\theta\cos\theta}=\frac{1}{\text{sen}2\theta}$$

Además:

$\cos 3\theta+\cos\theta=2\cos 2\theta\cos\theta$

Reemplazando en M:

$$M=\left(\frac{1}{\text{sen}2\theta}\right)(2\cos 2\theta\cos\theta)\text{sen}\theta$$

$$\Rightarrow M=\frac{\cos 2\theta(\text{sen}2\theta)}{\text{sen}2\theta}=\cos 2\theta$$

$\therefore M=\cos 2\theta$

Clave D

Resolución de problemas

32. La progresión es:

$$\alpha\ \underset{+120^\circ}{;}\ \beta\ \underset{+120^\circ}{;}\ \theta$$

Nos piden:

$S=\cos\alpha+\cos\beta+\cos\theta$

$S=\cos\alpha+\cos(\alpha+120^\circ)+\cos(\alpha+240^\circ)$

$S=\cos\alpha+\cos(\alpha+120^\circ)+\cos(120^\circ-\alpha)$

$$S=\cos\alpha+2\cos\left(\frac{\alpha+120^\circ+(120^\circ-\alpha)}{2}\right)\cos\left(\frac{\alpha+120^\circ-(120^\circ-\alpha)}{2}\right)$$

$S=\cos\alpha+2\cos(120^\circ)\times\cos(\alpha)$

$$S=\cos\alpha+2\left(\frac{-1}{2}\right)\cdot\cos\alpha$$

$S=\cos\alpha-\cos\alpha$

$S=0$

Clave B

33. $P=\tan x\tan 2x+\tan 2x\tan 3x+\tan 3x\tan 4x+\ldots$

Sabemos:

$\tan(a-b)=\tan a-\tan b-\tan(a-b)\tan a\tan b$

$\tan(a-b)\tan a\tan b=\tan a-\tan b-\tan(a-b)$

Aplicamos esta identidad:

$$+\left|\begin{array}{l}\tan x\tan 2x\tan x=\tan 2x-\tan x-\tan x\\ \tan x\tan 3x\tan 2x=\tan 3x-\tan 2x-\tan x\\ \tan x\tan 4x\tan 3x=\tan 4x-\tan 3x-\tan x\\ \quad\vdots\qquad\vdots\qquad\vdots\qquad\vdots\\ \underline{\tan x\tan(n+1)x\tan nx=\tan(n+1)x-\tan nx-\tan x}\end{array}\right.$$

$$\tan x\underbrace{(\tan x\tan 2x+\tan 2x\tan 3x+\ldots)}_{P}=\tan(n+1)x-\tan x-n\tan x$$

$P\tan x=\tan(n+1)x-(n+1)\tan x$

$$P=\frac{\tan(n+1)x-(n+1)\tan x}{\tan x}$$

$\therefore P=\cot x\tan(n+1)x-(n+1)$

Clave E

APLICAMOS LO APRENDIDO (página 64) Unidad 3

1. Piden el dominio de la función:

$g(x) = \dfrac{\text{senx} + 1}{\text{cosx} - \text{senx}}$

El dominio de senx y cosx no presenta restricciones, pero g(x) por ser una fracción, su denominador no puede ser cero, entonces:

$\text{cosx} - \text{senx} \neq 0 \Rightarrow \text{cosx} \neq \text{senx} \Rightarrow 1 \neq \dfrac{\text{senx}}{\text{cosx}}$

$\Rightarrow \text{tanx} \neq 1$

$\therefore x \neq \left\{\dfrac{\pi}{4}; \dfrac{5\pi}{4}; \dfrac{9\pi}{4}; ...\right\}$

$\Rightarrow x \neq (4n+1)\dfrac{\pi}{4}; n \in \mathbb{Z}$

$\therefore \text{Dom}(g) = \mathbb{R} - \{(4n+1)\dfrac{\pi}{4} / n \in \mathbb{Z}\}$

Clave B

2. Piden el rango de la función:

$F(x) = 3 + (\text{senx})(\text{cosx})$

Como observamos, no hay ningún tipo de restricción, por lo que afirmamos que F(x) se halla definido $\forall x \in \mathbb{R}$, entonces $\text{Dom}(F) = \mathbb{R}$.

Luego:

$F(x) = 3 + \dfrac{2\text{senx}\,\text{cosx}}{2}$

$F(x) = 3 + \dfrac{\text{sen2x}}{2}$

Como $x \in \mathbb{R} \Rightarrow (2x) \in \mathbb{R}$

$\Rightarrow -1 \leq \text{sen2x} \leq 1$

$-\dfrac{1}{2} \leq \dfrac{\text{sen2x}}{2} \leq \dfrac{1}{2}$

$3 - \dfrac{1}{2} \leq \underbrace{3 + \dfrac{\text{sen2x}}{2}}_{} \leq 3 + \dfrac{1}{2}$

$\dfrac{5}{2} \leq F(x) \leq \dfrac{7}{2}$

$\therefore \text{Ran}(F) = \left[\dfrac{5}{2}; \dfrac{7}{2}\right]$

Clave D

3. Por dato:

$f(x) = \text{cosx}(\text{cosx} - 4)$ y $\text{Ran}(f) = [a; b]$

Entonces:

$f(x) = \cos^2x - 4\text{cosx}$

$f(x) = \cos^2x - 2(2)\text{cosx} + 2^2 - 2^2$

$\Rightarrow f(x) = (\text{cosx} - 2)^2 - 4$

Luego, la función f(x) está definida $\forall x \in \mathbb{R}$, no presenta restricciones en su dominio.

$\Rightarrow -1 \leq \text{cosx} \leq 1$

$-3 \leq \text{cosx} - 2 \leq -1$

$1 \leq (\text{cosx} - 2)^2 \leq 9$

$-3 \leq \underbrace{(\text{cosx} - 2)^2 - 4}_{} \leq 5$

$-3 \leq f(x) \leq 5$

$\Rightarrow \text{Ran}(f) = [-3; 5]$

Comparando: $a = -3 \wedge b = 5$

Piden:

$H = a^2 + b^2 - ab$

$\Rightarrow H = (-3)^2 + (5)^2 - (-3)(5)$

$\therefore H = 49$

Clave B

4. Piden el dominio de la función:

$F(x) = \text{tan2x} + \text{sec2x} + 2x$

La función F(x) presenta restricciones en su dominio dado que tan2x y sec2x presentan restricciones.

Función de referencia: y = tanx

Dominio: $\mathbb{R} - \{(2n+1)\dfrac{\pi}{2} / n \in \mathbb{Z}\}$

$\Rightarrow 2x \neq (2n+1)\dfrac{\pi}{2} \Rightarrow x \neq (2n+1)\dfrac{\pi}{4}$ ...(a)

Función de referencia: y = secx

Dominio: $\mathbb{R} - \{(2n+1)\dfrac{\pi}{2} / n \in \mathbb{Z}\}$

$\Rightarrow 2x \neq (2n+1)\dfrac{\pi}{2} \Rightarrow x \neq (2n+1)\dfrac{\pi}{4}$ ...(b)

De (a) y (b): $x \neq (2n+1)\dfrac{\pi}{4}$

$\therefore \text{Dom}(F) = \mathbb{R} - \{(2n+1)\dfrac{\pi}{4} / n \in \mathbb{Z}\}$

Clave D

5. Piden el rango de la función:

$H(x) = \text{tanx} + \text{cotx}$

De la función H se observa que aparecen funciones tangente y cotangente, sabemos que no están definidas en $(2n+1)\dfrac{\pi}{2}$ y $n\pi$, respectivamente, uniendo estas dos restricciones tenemos:

$\text{Dom}(H) = \mathbb{R} - \{\dfrac{n\pi}{2} / n \in \mathbb{Z}\}$

Por identidades del ángulo doble:

$H(x) = \text{tanx} + \text{cotx} = 2\text{csc2x}$

$\Rightarrow H(x) = 2\text{csc2x}$

Luego a partir del dominio de H, obtenemos:

$-\infty < \text{csc2x} \leq -1 \vee 1 \leq \text{csc2x} < +\infty$

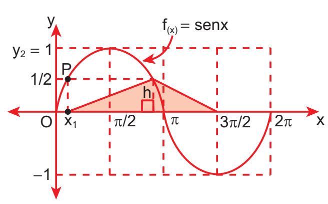
$-\infty < 2\text{csc2x} \leq -2 \vee 2 \leq 2\text{csc2x} < +\infty$

$\Rightarrow \text{Ran}(H) = \langle -\infty; -2], [2; +\infty\rangle$

$\therefore \text{Ran}(H) = \mathbb{R} - \langle -2; 2\rangle$

Clave C

6.



Del gráfico: $P\left(x_1; \dfrac{1}{2}\right) \wedge h = \dfrac{1}{2}$

Además:

P pertenece a la gráfica f(x) = senx.

$\Rightarrow P\left(x_1; \dfrac{1}{2}\right) = P(x_1; \text{senx}_1)$

$\Rightarrow \text{senx}_1 = \dfrac{1}{2}$

Como $x_1 \in \left\langle 0; \dfrac{\pi}{2}\right\rangle \Rightarrow x_1 = \dfrac{\pi}{6}$

Piden:

$A_{\text{somb.}} = \dfrac{(\text{base})(\text{altura})}{2} = \dfrac{\left(\dfrac{3\pi}{2} - x_1\right)(h)}{2}$

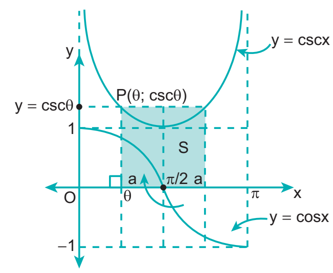
$A_{\text{somb.}} = \dfrac{\left(\dfrac{3\pi}{2} - \dfrac{\pi}{6}\right)(h)}{2} = \dfrac{\left(\dfrac{4\pi}{3}\right)\left(\dfrac{1}{2}\right)}{2}$

$\Rightarrow A_{\text{somb.}} = \dfrac{4\pi}{12}$

$\therefore A_{\text{somb.}} = \dfrac{\pi}{3}u^2$

Clave B

7.



El gráfico presenta simetría con respecto a: $x = \dfrac{\pi}{2}$

Entonces: $\theta + a = \dfrac{\pi}{2} \Rightarrow a = \dfrac{\pi}{2} - \theta$

Luego, trasladamos la porción del área inferior, con la cual el área sombreada será equivalente al área de un rectángulo:

$S = (2a)(y) = 2\left(\dfrac{\pi}{2} - \theta\right)(\csc\theta)$

$\Rightarrow S = (\pi - 2\theta)\csc\theta$

Piden:

$A = \tan2\theta \cot\left(\dfrac{S}{\csc\theta}\right)$

$A = \tan2\theta \cot\left(\dfrac{(\pi - 2\theta)\csc\theta}{\csc\theta}\right)$

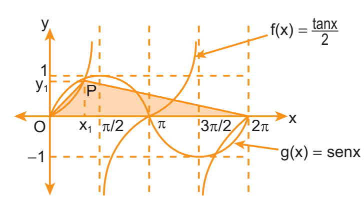
$\Rightarrow A = \tan2\theta \cot(\pi - 2\theta)$

$A = \tan2\theta(-\cot2\theta) = -\underbrace{\tan2\theta\cot2\theta}_{(1)}$

$\therefore A = -1$

Clave E

8.



Del gráfico: $P(x_1; y_1)$

Además: $P\left(x_1; \dfrac{1}{2}\text{tanx}_1\right) = P(x_1; \text{senx}_1)$

$\Rightarrow \dfrac{1}{2}\text{tanx}_1 = \text{senx}_1; \left(0 < x_1 < \dfrac{\pi}{2}\right)$

$\dfrac{\text{senx}_1}{2\text{cosx}_1} = \text{senx}_1$

$\text{cosx}_1 = \dfrac{1}{2} \Rightarrow x_1 = \dfrac{\pi}{3}$

Luego:

$P(x_1; y_1) = P\left(\frac{\pi}{3}; \text{sen}\frac{\pi}{3}\right) = P\left(\frac{\pi}{3}; \frac{\sqrt{3}}{2}\right)$

$\Rightarrow y_1 = \frac{\sqrt{3}}{2}$

Piden:

$A_{somb.} = \frac{(\text{base})(\text{altura})}{2} = \frac{(2\pi)(y_1)}{2}$

$\Rightarrow A_{somb.} = \frac{(2\pi)\left(\frac{\sqrt{3}}{2}\right)}{2}$

$\therefore A_{somb.} = \frac{\sqrt{3}\,\pi}{2}$

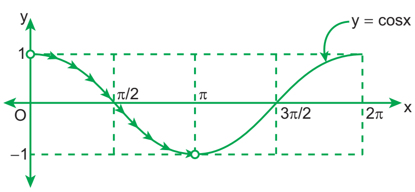
Clave C

9.

I. La función $y = F(x) = \text{senx} + 1$, tiene como dominio: $\mathbb{R} - \{n\pi / n \in \mathbb{Z}\}$ (F)

La función $y = F(x) = \text{senx} + 1$ no presenta restricciones dado que $\text{Dom(senx)} = \mathbb{R}$
$\Rightarrow \text{Dom}(F) = \mathbb{R}$

II. La función $y = F(x) = \cos x$, es creciente en el intervalo $\langle 0; \pi\rangle$ (F)



$y = F(x) = \cos x$ es decreciente en $\langle 0; \pi\rangle$.

III. La función $y = F(x) = \cos x + 47$, es par (V)
$F(x) = \cos x + 47$
$F(-x) = \cos(-x) + 47 = \cos x + 47$
$\Rightarrow F(x) = F(-x)$

$\therefore$ Por lo tanto, $F(x) = \cos x + 7$ es una función par.

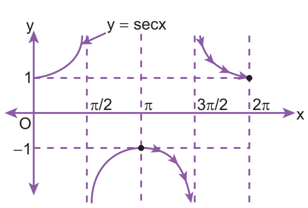
Clave C

10.

I. La función $y = F(x) = \text{cotx}$, tiene como dominio: $\mathbb{R} - \{(2n+1)\frac{\pi}{2} / n \in \mathbb{Z}\}$ (F)

La función $y = F(x) = \text{cotx}$ tiene como dominio: $\mathbb{R} - \{n\pi / n \in \mathbb{Z}\}$

II. La función $y = F(x) = \text{secx}$ es decreciente en los intervalos $\langle \pi; \frac{3\pi}{2}\rangle \cup \langle \frac{3\pi}{2}; 2\pi\rangle$ (V)



III. La función $y = F(x) = \csc x - 5x$, es impar (V)
$F(x) = \csc x - 5x$
$F(-x) = \csc(-x) - 5(-x)$
$F(-x) = -\csc x + 5x = -\underbrace{(\csc x - 5x)}_{F(x)}$
$\Rightarrow F(-x) = -F(x)$
$\therefore F(x) = \csc x - 5x$ es una función impar.

Clave E

11. $f(x) = 2[\text{senx}(2\cos 2x + 1) - 2\text{senx}][\cos x(2\cos 2x - 1) + 2\cos x]$
$= 2\text{senx}[2\cos 2x - 1]\cos x[2\cos 2x + 1]$
$= 2\cos x(2\cos 2x - 1)\text{senx}(2\cos 2x + 1)$
$= 2\cos 3x\,\text{sen}3x$
$= \text{sen}6x$

Luego: $T = \frac{2\pi}{6} \Rightarrow T = \frac{\pi}{3}$

Clave B

12. $\cos x + \cos 2x + \cos 3x \neq 0$
$2\cos 2x \,.\, \cos x + \cos 2x \neq 0$
$\cos 2x(2\cos x + 1) \neq 0$

$\cos 2x \neq 0 \Rightarrow 2x \neq (2n+1)\frac{\pi}{2}$

$x \neq (2n+1)\frac{\pi}{4}$

$2\cos x + 1 \neq 0 \Rightarrow \cos x \neq -\frac{1}{2}$

$\Rightarrow x \neq \frac{2\pi}{3} + 2n\pi \wedge x \neq \frac{4\pi}{3} + 2n\pi$

Luego: $g(x) = \frac{2\text{sen}2x\cos x + \text{sen}2x}{2\cos 2x\cos x + \cos 2x}$

$= \frac{\text{sen}2x(2\cos x + 1)}{\cos 2x(2\cos x + 1)} = \tan 2x$

$\therefore T = \frac{\pi}{2}$

Clave C

13. $f(x) = A\cos Bx$
Valor máximo = 3
Valor mínimo = –3
$\text{Amplitud} = A = \frac{3-(-3)}{2} \Rightarrow A = 3$
Luego: $T = \pi$
Además: $T = \frac{2\pi}{B} \Rightarrow \frac{2\pi}{B} = \pi \Rightarrow B = 2$

Clave D

14. $y = f(x) = a\text{senbx}; x \in [0; +\infty\rangle$
$x = 5\pi; y = a \Rightarrow a = a\text{senb}(5\pi)$
$\Rightarrow \text{sen}(5b\pi) = 1$
$\Rightarrow 5b\pi = 2k\pi + \frac{\pi}{2}; k \in \mathbb{Z}$
$5b\pi = 2\pi + \frac{\pi}{2} \Rightarrow b = \frac{1}{2}$

$x = \frac{\pi}{3}; y = 0{,}8: 0{,}8 = a\text{senb}\left(\frac{\pi}{3}\right)$

$0{,}8 = a\text{sen}\frac{1}{2}\left(\frac{\pi}{3}\right)$

$0{,}8 = a\text{sen}\frac{\pi}{6}$

$0{,}8 = a\left(\frac{1}{2}\right) \Rightarrow a = 1{,}6$

$\therefore M = 5a + 4b = 5(1{,}6) + 4\left(\frac{1}{2}\right) = 10$

Clave A

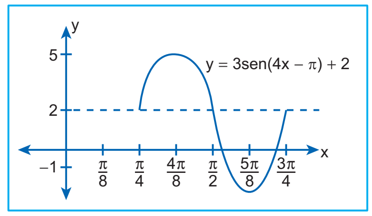
PRACTIQUEMOS

Nivel 1 (página 66) Unidad 3

Comunicación matemática

1.

2.



Razonamiento y demostración

3. $F(x) = 2\text{senx} + 3$
$\text{Dom(senx)} = \mathbb{R}$
$\Rightarrow \text{Dom}(F) = \mathbb{R}$
Como en el dominio de F no hay restricciones:
$\Rightarrow -1 \leq \text{senx} \leq 1$
$-2 \leq 2\text{senx} \leq 2$
$1 \leq 2\text{senx} + 3 \leq 5$
$\Rightarrow \text{Ran}(F) = [1; 5]$
Piden:
$\text{Dom}(F) \cap \text{Ran}(F) = \mathbb{R} \cap [1; 5]$
$\therefore \text{Dom}(F) \cap \text{Ran}(F) = [1; 5]$

Clave B

4.

$G(x) = \cos^4\frac{x}{2} - \text{sen}^4\frac{x}{2}; x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

Reduciendo la regla de correspondencia:

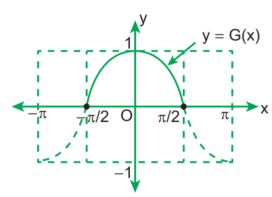
$G(x) = \underbrace{\left(\cos^2\frac{x}{2} + \text{sen}^2\frac{x}{2}\right)}_{1}\left(\cos^2\frac{x}{2} - \text{sen}^2\frac{x}{2}\right)$

$G(x) = \cos^2\frac{x}{2} - \text{sen}^2\frac{x}{2} = \cos 2\left(\frac{x}{2}\right)$

$\Rightarrow G(x) = \cos x$

Además, la función original no presenta ninguna restricción en su dominio.

Luego:



Clave A

5. Piden el rango de la función f.

$f(x) = \frac{3}{2 + \cos x}$

En la función f se observa que aparece la función coseno y sabemos que está definida en $\mathbb{R}$, además el denominador no afecta al dominio ya que $(2 + \cos x)$ es siempre diferente de cero para todo $x \in \mathbb{R}$.

$\Rightarrow \text{Domf} = \mathbb{R}$
$\Rightarrow -1 \leq \cos x \leq 1$
$1 \leq 2 + \cos x \leq 3$
$\frac{1}{3} \leq \frac{1}{2 + \cos x} \leq 1$
$1 \leq \frac{3}{2 + \cos x} \leq 3 \Rightarrow 1 \leq f(x) \leq 3$
$\therefore \text{Ranf} = [1; 3]$

Clave A

6. Por dato:

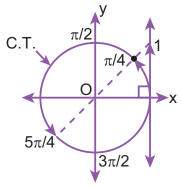
$f(x) = \text{sen}x \ \wedge \ g(x) = \cos x$

Además: $f(x) = g(x)$

Entonces: $\text{sen}x = \cos x$

$\Rightarrow \dfrac{\text{sen}x}{\cos x} = 1 \Rightarrow \tan x = 1$

Analizando en la C.T.:



Entonces:

$x = \dfrac{\pi}{4}; \dfrac{5\pi}{4}; \dfrac{9\pi}{4}; \dfrac{13\pi}{4}; ...$

Piden los valores de $x \in \langle 0; 2\pi \rangle$.

$\therefore x = \left\{\dfrac{\pi}{4}; \dfrac{5\pi}{4}\right\}$

Clave B

7. Piden: el rango de la función f.

$f(x) = \cos^2 x - 2\cos x$
$f(x) = \cos^2 x - 2\cos x + 1 - 1$
$f(x) = (\cos x - 1)^2 - 1$

Como la función cosx no presenta restricciones en su dominio, entonces:

$-1 \leq \cos x \leq 1$
$-2 \leq \cos x - 1 \leq 0$
$0 \leq (\cos x - 1)^2 \leq 4$
$-1 \leq \underbrace{(\cos x - 1)^2 - 1}_{} \leq 3$
$-1 \leq f(x) \leq 3$

$\therefore \text{Ranf} = [-1; 3]$

Clave E

8. Del gráfico se tiene que la función $y = \text{sen}x$ pasa por los puntos $Q\left(\dfrac{3\pi}{4}; y_1\right)$ y $P\left(\dfrac{7\pi}{4}; y_2\right)$.

Entonces se cumple:
Para el punto Q:

$y = y_1 = \text{sen}\left(\dfrac{3\pi}{4}\right)$

$\Rightarrow y_1 = \text{sen}135° = \dfrac{\sqrt{2}}{2} \Rightarrow y_1 = \dfrac{\sqrt{2}}{2}$

Para el punto P:

$y = y_2 = \text{sen}\left(\dfrac{7\pi}{4}\right)$

$\Rightarrow y_1 = \text{sen}315° = -\dfrac{\sqrt{2}}{2} \Rightarrow y_2 = -\dfrac{\sqrt{2}}{2}$

Piden:

$y_1 + y_2 = \left(\dfrac{\sqrt{2}}{2}\right) + \left(-\dfrac{\sqrt{2}}{2}\right) = \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}$

$\therefore y_1 + y_2 = 0$

Clave C

Resolución de problemas

9. Para que una función sea par se debe cumplir: $F(x) = F(-x)$

A) $F(x) = |\text{sen}x|$
$F(-x) = |\text{sen}(-x)| = |-\text{sen}x|$
$F(-x) = |\text{sen}x| \Rightarrow F(x) = F(-x)$

B) $G(x) = \cos|x|$
$G(-x) = \cos|-x| = \cos|x|$
$G(-x) = \cos|x| \Rightarrow G(x) = G(-x)$

C) $H(x) = \text{sen}|x|$
$H(-x) = \text{sen}|-x| = \text{sen}|x|$
$H(-x) = \text{sen}|x| \Rightarrow H(x) = H(-x)$

D) $G(x) = \cos x - \text{sen}x$
$G(-x) = \cos(-x) - \text{sen}(-x)$
$G(-x) = (\cos x) - (-\text{sen}x)$
$G(-x) = \cos x + \text{sen}x \Rightarrow G(x) \neq G(-x)$

E) $F(x) = |\cos x| - |\text{sen}x|$
$F(-x) = |\cos(-x)| - |\text{sen}(-x)|$
$F(-x) = |(\cos x)| - |(-\text{sen}x)|$
$F(-x) = |\cos x| - |\text{sen}x| \Rightarrow F(x) = F(-x)$

Vemos que: $G(x) = \cos x - \text{sen}x$ no es una función par.

Clave D

10. Por dato: el punto $\left(\dfrac{\pi}{3}; \dfrac{2n-1}{2n+1}\right)$ pertenece a la gráfica de la función $y = \cos x$.

Sabemos que cualquier punto de la gráfica $y = \cos x$ tiene la forma: $(x; y) = (x; \cos x)$

Luego:

$\left(\dfrac{\pi}{3}; \dfrac{2n-1}{2n+1}\right) = (x; y) = (x; \cos x)$

$\Rightarrow x = \dfrac{\pi}{3} \ \wedge \ \cos x = \dfrac{2n-1}{2n+1}$

Entonces:

$\cos\dfrac{\pi}{3} = \dfrac{2n-1}{2n+1}$

$\cos 60° = \dfrac{2n-1}{2n+1} \Rightarrow \dfrac{1}{2} = \dfrac{2n-1}{2n+1}$

$2n + 1 = 4n - 2$
$3 = 2n$

$\therefore n = \dfrac{3}{2}$

Clave B

Nivel 2 (página 66) Unidad 3

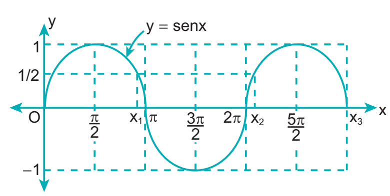
Comunicación matemática

11.

12.

Razonamiento y demostración

13.



Del gráfico:

$\dfrac{5\pi}{2} + \dfrac{\pi}{2} = x_3 \Rightarrow x_3 = 3\pi$

Además:

$\dfrac{1}{2} = \text{sen}x_1; \ \dfrac{\pi}{2} < x_1 < \pi$

Sabemos: $\text{sen}\dfrac{\pi}{6} = \text{sen}\left(\pi - \dfrac{\pi}{6}\right) = \dfrac{1}{2}$

$\Rightarrow x_1 = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$

$\dfrac{1}{2} = \text{sen}x_2; \ 2\pi < x_2 < \dfrac{5\pi}{2}$

Sabemos: $\text{sen}\dfrac{\pi}{6} = \text{sen}\left(2\pi + \dfrac{\pi}{6}\right) = \dfrac{1}{2}$

$\Rightarrow x_2 = 2\pi + \dfrac{\pi}{6} = \dfrac{13\pi}{6}$

Piden:

$x_1 + x_2 + x_3 = \dfrac{5\pi}{6} + \dfrac{13\pi}{6} + 3\pi$

$\therefore x_1 + x_2 + x_3 = 6\pi$

Clave A

14. La tangentoide en x, está representada por la regla de correspondencia: $y = \tan x$

Por dato: $\left(\dfrac{\pi}{4}; y_1\right); \left(\dfrac{3\pi}{4}; y_2\right); \left(\dfrac{4\pi}{3}; y_3\right)$

pertenecen a la tangentoide.
Entonces:

$y_1 = \tan\dfrac{\pi}{4} = 1$

$y_2 = \tan\dfrac{3\pi}{4} = -\tan\dfrac{\pi}{4} = -1$

$y_3 = \tan\dfrac{4\pi}{3} = \tan\dfrac{\pi}{3} = \sqrt{3}$

Piden:

$\dfrac{y_3 - y_1}{y_3 + y_2} = \dfrac{\sqrt{3} - 1}{\sqrt{3} + (-1)} = \dfrac{\sqrt{3} - 1}{\sqrt{3} - 1} = 1$

$\therefore \dfrac{y_3 - y_1}{y_3 + y_2} = 1$

Clave A

15. Del gráfico se tiene que la función $y = \cos x$ pasa por el punto $P\left(-\dfrac{\pi}{4}; y\right)$

Entonces para el punto P se cumple:

$y = \cos\left(-\dfrac{\pi}{4}\right) = \cos\dfrac{\pi}{4}$

$\Rightarrow y = \cos 45° = \dfrac{\sqrt{2}}{2}$

Por lo tanto, las coordenadas del punto P serán:

$P(x; y) = P\left(-\dfrac{\pi}{4}; \dfrac{\sqrt{2}}{2}\right)$

Clave A

16. Piden el dominio de la función f.

$f(x) = \cos x + \sqrt{\text{sen}x - 1}; \ (k \in \mathbb{Z})$

Luego:
Por la función cosx y senx; x no presenta restricciones.
Por las funciones raíz cuadrada:
$\text{sen}x - 1 \geq 0 \Rightarrow \text{sen}x \geq 1$ ...(I)
Pero: $-1 \leq \text{sen}x \leq 1$ ...(II)

De (I) y (II) se deduce:
$\text{sen}x = 1$

$\Rightarrow x = \dfrac{\pi}{2}; \dfrac{5\pi}{2}; \dfrac{9\pi}{2}; \dfrac{13\pi}{2}; ...$

$\Rightarrow x = (4k + 1)\dfrac{\pi}{2}; \ k \in \mathbb{Z}$

$\therefore \text{Domf} = \{(4k + 1)\dfrac{\pi}{2}; k \in \mathbb{Z}\}$

Clave E

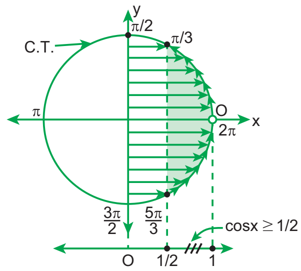
17. Piden el dominio de la función F.

$$F = \left\{(x;y)/y = \sqrt{\cos x - \frac{1}{2}};\ 0 < x < 2\pi\right\}$$

Luego:
Por la función cosx; x no presenta restricciones.
Por la función raíz cuadrada:

$\cos x - \frac{1}{2} \geq 0 \Rightarrow \cos x \geq \frac{1}{2}$

Analizando en la C.T. y teniendo en cuenta el intervalo dado de $\langle 0; 2\pi\rangle$ se tiene:

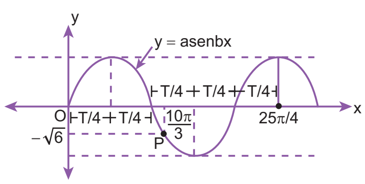


Entonces: $x \in \left\langle 0; \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}; 2\pi\right\rangle$

$\therefore \text{DomF} = \left\langle 0; \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}; 2\pi\right\rangle$

Clave D

18.



Del gráfico:

$5\left(\frac{T}{4}\right) = \frac{25\pi}{4}$; donde T es el período de la función.

$\Rightarrow T = 5\pi$

Además se cumple: $f(x + T) = f(x) = y$

$\Rightarrow$ $a\text{sen}b(x + T) = a\text{sen}bx$

$\text{sen}(bx + bT) = \text{sen}bx$

$\text{sen}(bT + bx) = \text{sen}(2\pi + bx)$

Comparando: $bT = 2\pi$

$\Rightarrow b(5\pi) = 2\pi \Rightarrow b = \frac{2}{5}$

La gráfica pasa por el punto P: $\left(\frac{10\pi}{3}; -\sqrt{6}\right)$.

$\Rightarrow y = -\sqrt{6} = a\text{sen}b\left(\frac{10\pi}{3}\right)$

$\Rightarrow -\sqrt{6} = a\text{sen}\frac{2}{5}\left(\frac{10\pi}{3}\right)$

$-\sqrt{6} = a\text{sen}\frac{4\pi}{3} = a\text{sen}240°$

$-\sqrt{6} = a\left(-\frac{\sqrt{3}}{2}\right)$

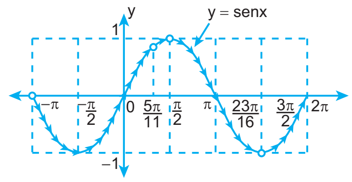
$\frac{2\sqrt{6}}{\sqrt{3}} = a \Rightarrow a = 2\sqrt{2}$

$\therefore a = 2\sqrt{2} \wedge b = \frac{2}{5}$

Clave B

Resolución de problemas

19.



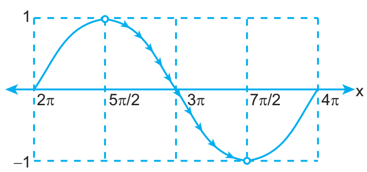
En el intervalo:

$\left\langle -\frac{\pi}{2}; \frac{\pi}{2}\right\rangle$ la función es creciente.

$\left\langle \frac{5\pi}{11}; \frac{23\pi}{16}\right\rangle$ la función es creciente y decreciente.

$\left\langle \frac{3\pi}{2}; 2\pi\right\rangle$ la función es creciente.

$\langle -\pi; 0\rangle$ la función es decreciente y creciente.



En el intervalo $\left\langle \frac{5\pi}{2}; \frac{7\pi}{2}\right\rangle$ la función es decreciente.

Clave E

20. Por dato: el punto $\left(\frac{\pi}{6}; \frac{2n-1}{2n+1}\right)$ pertenece a la gráfica de la función $y = \text{sen}x$.

Sabemos que cualquier punto de la gráfica $y = \text{sen}x$ tiene la forma: $(x; y) = (x; \text{sen}x)$

Luego:

$\left(\frac{\pi}{6}; \frac{2n-1}{2n+1}\right) = (x; y) = (x; \text{sen}x)$

$\Rightarrow x = \frac{\pi}{6} \wedge \text{sen}x = \frac{2n-1}{2n+1}$

Entonces:

$\text{sen}\frac{\pi}{6} = \frac{2n-1}{2n+1}$

$\text{sen}30° = \frac{2n-1}{2n+1} \Rightarrow \frac{1}{2} = \frac{2n-1}{2n+1}$

$2n + 1 = 4n - 2$

$3 = 2n$

$\therefore n = \frac{3}{2}$

Clave B

Nivel 3 (página 67) Unidad 3

Comunicación matemática

21.

22.

Razonamiento y demostración

23. Del gráfico que se da en la pregunta, se tiene que la función $y = 2\text{sen}2x$ pasa por los puntos $P\left(\frac{\pi}{6}; a\right)$ y $Q\left(\frac{7\pi}{8}; b\right)$.

Entonces se cumple:
Para el punto P:

$y = a = 2\text{sen}2\left(\frac{\pi}{6}\right)$

$\Rightarrow a = 2\text{sen}\frac{\pi}{3} = 2\text{sen}60°$

$\Rightarrow a = 2\left(\frac{\sqrt{3}}{2}\right) \Rightarrow a = \sqrt{3}$

Para el punto Q:

$y = b = 2\text{sen}2\left(\frac{7\pi}{8}\right)$

$\Rightarrow b = 2\text{sen}\frac{7\pi}{4} = 2\text{sen}315°$

$\Rightarrow b = 2\left(-\frac{\sqrt{2}}{2}\right) \Rightarrow b = -\sqrt{2}$

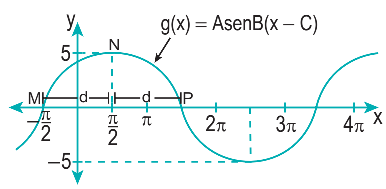
Piden:

$a - b = (\sqrt{3}) - (-\sqrt{2}) = \sqrt{3} + \sqrt{2}$

$\therefore a - b = \sqrt{3} + \sqrt{2}$

Clave A

24.



De la gráfica se deduce: $A; B \in \mathbb{R}^+$

$\text{Domg} = \mathbb{R} \wedge \text{Rang} = [-5; 5]$

Además: $d = \left|-\frac{\pi}{2}\right| + \frac{\pi}{2} = -\left(-\frac{\pi}{2}\right) + \frac{\pi}{2}$

$\Rightarrow d = \frac{\pi}{2} + \frac{\pi}{2} = \pi \Rightarrow d = \pi$

Entonces se tiene los puntos:

$M(x_1; y_1) = M\left(-\frac{\pi}{2}; 0\right) \Rightarrow y_1 = g(x_1) = g\left(-\frac{\pi}{2}\right)$

$0 = A\text{sen}B\left(-\frac{\pi}{2} - C\right) \Rightarrow \text{sen}B\left(-\frac{\pi}{2} - C\right) = \text{sen}0$

$\Rightarrow -\frac{\pi}{2} - C = 0 \Rightarrow C = -\frac{\pi}{2}$

$N(x_2; y_2) = N\left(\frac{\pi}{2}; 5\right) \Rightarrow y_2 = g(x_2) = g\left(\frac{\pi}{2}\right)$

$5 = A\text{sen}B\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) \Rightarrow A\text{sen}B\pi = 5 \quad \text{...(I)}$

$P(x_3; y_3) = P\left(\frac{\pi}{2} + d; 0\right) = P\left(\frac{\pi}{2} + \pi; 0\right)$

$P(x_3; y_3) = P\left(\frac{3\pi}{2}; 0\right) \Rightarrow y_3 = g(x_3) = g\left(\frac{3\pi}{2}\right)$

$0 = A\text{sen}B\left(\frac{3\pi}{2} - \left(-\frac{\pi}{2}\right)\right) \Rightarrow \text{sen}2B\pi = \text{sen}\pi$

$\Rightarrow 2B\pi = \pi \Rightarrow B = \frac{1}{2}$

Reemplazando en (I):

$\Rightarrow A\text{sen}\frac{\pi}{2} = 5$

$A(1) = 5 \Rightarrow A = 5$

$\therefore A = 5; B = \frac{1}{2}; C = -\frac{\pi}{2}$

Clave A

25.

$F(x) = \dfrac{\text{sen}2x}{2\cos x}$; $x \in [0; 2\pi]$

Simplificando la expresión:

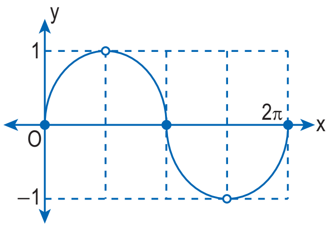
$F(x) = \dfrac{2\,\text{sen}x\cos x}{2\cos x} = \text{sen}x$

$\Rightarrow F(x) = \text{sen}x$

Pero: $\cos x \neq 0$

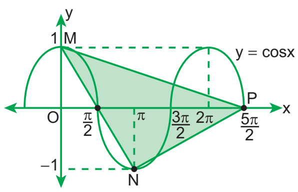
$\Rightarrow x \neq \dfrac{\pi}{2}; \dfrac{3\pi}{2}$

Graficando:



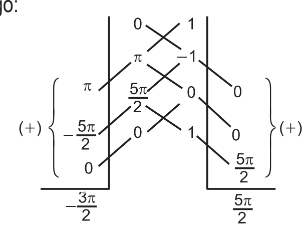
Clave E

26.



Del gráfico, las coordenadas de los puntos M, N y P serán (0; 1), (π; −1) y $\left(\dfrac{5\pi}{2}; 0\right)$, respectivamente.

Luego:

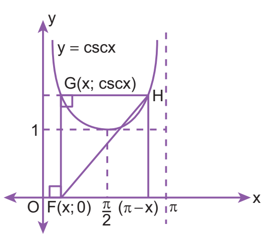


$\Rightarrow A_{\Delta MNP} = \dfrac{\left|\dfrac{5\pi}{2} - \left(-\dfrac{3\pi}{2}\right)\right|}{2} = \dfrac{|4\pi|}{2} = \dfrac{(4\pi)}{2}$

$\therefore A_{\Delta MNP} = 2\pi\ u^2$

Clave C

27.



La gráfica y = cscx, presenta simetría respecto a la recta $x = \dfrac{\pi}{2}$.

Luego las coordenadas del punto H serán: $(\pi - x; \csc x)$

Sea el baricentro del triángulo FGH: $(x_1; y_1)$

Entonces:

$x_1 = \dfrac{x + x + (\pi - x)}{3} \Rightarrow x_1 = \dfrac{\pi + x}{3}$

$y_1 = \dfrac{0 + \csc x + \csc x}{3} \Rightarrow y_1 = \dfrac{2\csc x}{3}$

Por dato: $y_1 = \dfrac{2\sqrt{2}}{3}$

$\Rightarrow \dfrac{2\csc x}{3} = \dfrac{2\sqrt{2}}{3} \Rightarrow \csc x = \sqrt{2}$

Sabemos: $\csc 45° = \csc\dfrac{\pi}{4} = \sqrt{2}$

Además de la gráfica: $0 < x < \dfrac{\pi}{2}$

$\Rightarrow x = \dfrac{\pi}{4}$

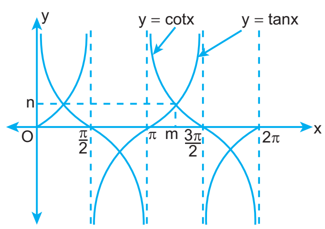
Piden: la abscisa del baricentro.

$x_1 = \dfrac{\pi + x}{3} = \dfrac{\pi + \frac{\pi}{4}}{3} = \dfrac{5\pi}{12}$

$\therefore x_1 = \dfrac{5\pi}{12}$

Clave A

28.



$\Rightarrow y = \tan(m) = \cot(m) \ \wedge \ y = n$

$\Rightarrow \tan(m) = \cot(m)$

$\tan^2(m) = 1 \Rightarrow |\tan(m)| = 1$

$\Rightarrow \tan(m) = 1$ o $\tan(m) = -1$

Como: $\pi < m < \dfrac{3\pi}{2} \Rightarrow \tan(m) > 0$

$\Rightarrow \tan(m) = 1$

Sabemos:

$\tan\dfrac{\pi}{4} = \tan\left(\pi + \dfrac{\pi}{4}\right) = 1$

$\Rightarrow m = \pi + \dfrac{\pi}{4} = \dfrac{5\pi}{4} \Rightarrow m = \dfrac{5\pi}{4}$

Luego: $n = \tan\dfrac{5\pi}{4} \Rightarrow n = 1$

Piden:

$E = \sec\dfrac{m}{3} - n\csc\dfrac{m}{5}$

$E = \sec\dfrac{1}{3}\left(\dfrac{5\pi}{4}\right) - (1)\csc\dfrac{1}{5}\left(\dfrac{5\pi}{4}\right)$

$E = \sec\dfrac{5\pi}{12} - \csc\dfrac{\pi}{4} = \sec 75° - \csc 45°$

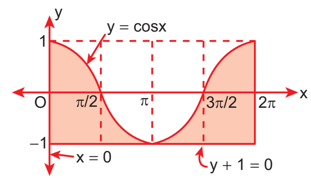
$\Rightarrow E = (\sqrt{6} + \sqrt{2}) - (\sqrt{2}) = \sqrt{6}$

$\therefore E = \sqrt{6}$

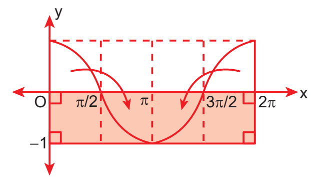
Clave A

Resolución de problemas

29.



El área de la región sombreada será equivalente por simetría:



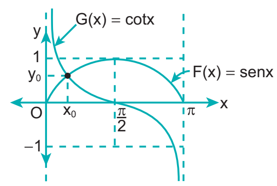
$\Rightarrow A_{\text{somb.}} = (\text{base})(\text{altura})$

$A_{\text{somb.}} = (2\pi)(1) = 2\pi$

$\therefore A_{\text{somb.}} = 2\pi\ u^2$

Clave A

30. Del enunciado:



Entonces:

$y_0 = F(x_0) = \text{sen}x_0$ ...(1)

$y_0 = G(x_0) = \cot x_0$ ...(2)

De (1) y (2): $\text{sen}x_0 = \cot x_0$

$\text{sen}x_0 = \dfrac{\cos x_0}{\text{sen}x_0}$

$\text{sen}^2x_0 = \cos x_0$

$\Rightarrow 1 - \cos^2x_0 = \cos x_0$

Como: $x_0 \in IC \Rightarrow \cos x_0 > 0$

$\Rightarrow \dfrac{1}{\cos x_0} - \dfrac{\cos^2x_0}{\cos x_0} = \dfrac{\cos x_0}{\cos x_0}$

$\dfrac{1}{\cos x_0} - \cos x_0 = 1$

$\therefore \sec x_0 - \cos x_0 = 1$

Clave A

31. Reducimos:

$f(x) = \text{sen}x - [\underbrace{1 - \text{sen}^2x}_{} - \cos^2x]$

$f(x) = \text{sen}x - [\cos^2x - \cos^2x]$

$\Rightarrow f(x) = \text{sen}x$

Observando que la gráfica representa a la función f(x) = senx.

Calculamos el área de la región sombreada por simetría:

$A_{\text{somb.}} = \pi|1| = \pi\ u^2$

Clave C

MARATÓN MATEMÁTICA (página 69)

1. En k:

$$k = 2\text{sen}\frac{3\alpha}{2} \times \text{sen}\frac{\alpha}{2} = -(\cos 2\alpha - \cos\alpha)$$

$k = \cos\alpha - \cos 2\alpha$
$k = \cos\alpha - (2\cos^2\alpha - 1)$
$k = \cos\alpha - 2\cos^2\alpha + 1$

$$k = \frac{1}{\sec\alpha} - 2\left(\frac{1}{\sec\alpha}\right)^2 + 1$$

$$k = \frac{1}{4} - 2\left(\frac{1}{4}\right)^2 + 1$$

$$k = \frac{9}{8}$$

Clave B

2. Sabemos:

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} = \frac{3(3) - (3)^3}{1 - 3(3)^2}$$

$$\tan 3x = \frac{9 - 27}{1 - 27} = \frac{9}{13}$$

$13\text{sen}3x = 9\cos 3x$
$0 = 13\text{sen}3x - 9\cos 3x$
$\therefore\ D = 0$

Clave D

3. Nos piden:

$$M = \frac{\text{sen}6x}{4\text{sen}^3x - 3\text{sen}x} = \frac{\text{sen}6x}{(3\text{sen}x - \text{sen}3x) - 3\text{sen}x}$$

$$M = \frac{\text{sen}6x}{-\text{sen}3x} = \frac{2\text{sen}3x\ .\ \cos 3x}{-\text{sen}3x} = -2\cos 3x$$

$\therefore\ M = -2\cos 3x$

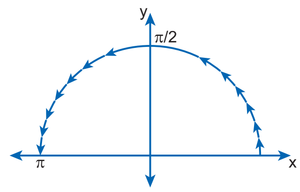
Clave A

4. Analizamos:

$1 + \text{sen}x - \cos^2 x > 0$
$1 + \text{sen}x - (1 - \text{sen}^2 x) > 0 \Rightarrow \text{sen}x + \text{sen}^2 x > 0$
$\text{sen}x(1 + \text{sen}x) > 0$

$\text{sen}x \neq 0$; $\text{sen}x \neq -1$; $\text{sen}x > 0$

Analizamos en la CT:



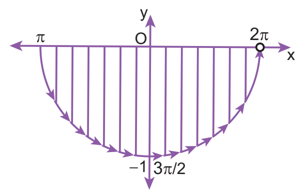
$\therefore\ \text{Dom}(f) = \langle 0; \pi\rangle$

Clave D

5. Analizamos: $-\cos^2 x - 2\text{sen}x + 1 \geq 0$
$-(1 - \text{sen}^2 x) - 2\text{sen}x + 1 \geq 0$
$\text{sen}x(\text{sen}x - 2) \geq 0$

Por teoría de funciones, obtenemos:
$-1 \leq \text{sen}x \leq 0$

Analizamos en la CT:



$\therefore\ \text{Dom}(f) = [\pi; 2\pi\rangle$

Clave E

6. $-1 \leq \text{sen}(3x) \leq 1$
$0 \leq \text{sen}^2(3x) \leq 1$
$0 \leq 2\text{sen}^2(3x) \leq 2$
$1 \leq 2\text{sen}^2(3x) + 1 \leq 3$
$1 \leq f(x) \leq 3$
$\therefore\ \text{Ran}(f) = [1; 3]$

Clave A

7. Damos forma a la expresión:

$$A = 2\left(\frac{\sqrt{3}}{2}\right)\cos 20° - \cos 50°$$

$A = 2\cos 30°\cos 20° - \cos 50°$
$A = \cos(30° + 20°) + \cos(30° - 20°) - \cos 50°$
$A = \cos 50° + \cos 10° - \cos 50°$
$\therefore\ A = \cos 10°$

Clave C

8. De la expresión tenemos:

$$P = 2\left(\frac{\sqrt{2}}{2}\right) \times \cos 15° - \text{sen}60°$$

$P = 2\text{sen}45°\ .\ \cos 15° - \text{sen}60°$
$P = \text{sen}(45° + 15°) + \text{sen}(45° - 15°) - \text{sen}60°$
$P = \text{sen}60° + \text{sen}30° - \text{sen}60° = \text{sen}30°$

$$\therefore\ P = \frac{1}{2}$$

Clave B

9. $\tan 2\theta = \dfrac{a + b}{b} \times \tan\theta$

$$\frac{2\tan\theta}{1 - \tan^2\theta} = \left(\frac{a + b}{b}\right) \times \tan\theta$$

$$\tan 2\theta = \left(\frac{a - b}{a + b}\right) \quad \ldots (1)$$

$$\tan 3\theta = \frac{x + a + b}{b\cot\theta}$$

$$\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \left(\frac{x + a + b}{b}\right) \times \tan\theta$$

$$\frac{3 - \tan^2\theta}{1 - 3\tan^2\theta} = \frac{x + a + b}{b} \quad \ldots (2)$$

(1) en (2):

$$\frac{3 - \left(\frac{a - b}{a + b}\right)}{1 - 3\left(\frac{a - b}{a + b}\right)} = \frac{x + a + b}{b}$$

$$\Rightarrow \frac{x + a + b}{b} = \frac{a + 2b}{2b - a}$$

$$\therefore\ x = \frac{a^2}{2b - a}$$

Clave A

APLICAMOS LO APRENDIDO (página 71) Unidad 4

1. $F(x) = 4\text{arcsen}\left(\frac{x+1}{2}\right)$

Para el dominio:

$-1 \leq \frac{x+1}{2} \leq 1$

$-2 \leq x+1 \leq 2$

$-3 \leq x \leq 1$

$\therefore \text{Dom}(F) = [-3; 1]$

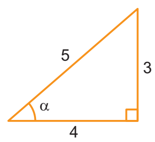
Clave B

2. $N = \text{sen}(\text{arcsen}\frac{3}{5} + \text{arccos}\frac{5}{13})$

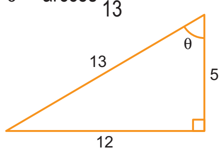
Sea:

$\alpha = \text{arcsen}\frac{3}{5}$

$\Rightarrow \text{sen}\alpha = \frac{3}{5}$



$\theta = \text{arccos}\frac{5}{13}$



$\Rightarrow \cos\theta = \frac{5}{13}$

Luego:

$N = \text{sen}(\alpha + \theta)$

$N = \text{sen}\alpha\cos\theta + \cos\alpha\,\text{sen}\theta$

$N = \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$

$\Rightarrow N = \frac{15}{65} + \frac{48}{65} = \frac{63}{65}$

$\therefore N = \frac{63}{65}$

Clave D

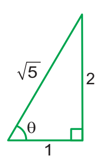
3. Haciendo: $\arctan 2 = \theta$

$\Rightarrow \tan\theta = 2$

Luego, nos piden:

$E = \text{sen}\theta$

$\therefore E = \frac{2}{\sqrt{5}}$

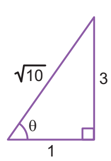


Clave E

4. $E = \cos(2\arctan 3)$

Sea: $\arctan 3 = \theta$

$\Rightarrow \tan\theta = 3$



Luego, nos piden:

$E = \cos(2\theta)$

$E = \cos 2\theta = 2\cos^2\theta - 1$

$\Rightarrow E = 2\left(\frac{1}{\sqrt{10}}\right)^2 - 1 = \frac{1}{5} - 1$

$\therefore E = -\frac{4}{5}$

Clave E

5. Haciendo: $\arctan\sqrt{3} = \theta$

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = 60°$

Luego, del dato:

$\sec(2x - \theta) = \sqrt{2} = \sec 45°$

$\Rightarrow 2x - \theta = 45°$

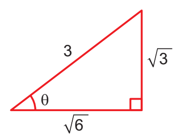
$x - 60° = 45°$

$x = \frac{105°}{2} = 52°30'$

Clave B

6. Haciendo: $\text{arcsen}\frac{\sqrt{3}}{3} = \theta$

$\Rightarrow \text{sen}\theta = \frac{\sqrt{3}}{3}$



Luego, nos piden:

$E = \text{sen}2\theta = 2\text{sen}\theta\cos\theta$

$\text{sen}2\theta = 2\left(\frac{\sqrt{3}}{3}\right)\left(\frac{\sqrt{6}}{3}\right)$

$\therefore E = \frac{2\sqrt{2}}{3}$

Clave C

7. Piden: Dom(f)

$f(x) = 4\arccos\left(\frac{7x+1}{8}\right) - \frac{\pi}{5}$

Entonces:

$-1 \leq \left(\frac{7x+1}{8}\right) \leq 1$

$-8 \leq 7x + 1 \leq 8$

$-9 \leq 7x \leq 7$

$-\frac{9}{7} \leq x \leq 1$

$\therefore \text{Dom}(f) = \left[-\frac{9}{7}; 1\right]$

Clave D

8. Piden: x

$\underbrace{\arctan\frac{1}{7} + \arctan\frac{1}{8}} + \arctan\frac{1}{18} = \arctan x$

$\arctan\left[\frac{\frac{1}{7} + \frac{1}{8}}{1 - \frac{1}{7}\cdot\frac{1}{8}}\right] + \arctan\frac{1}{18} = \arctan x$

$\underbrace{\arctan\left(\frac{3}{11}\right) + \arctan\frac{1}{18}} = \arctan x$

$\arctan\left[\frac{\frac{3}{11} + \frac{1}{18}}{1 - \frac{3}{11}\cdot\frac{1}{18}}\right] = \arctan x$

$\Rightarrow \arctan\left(\frac{1}{3}\right) = \arctan x$

$\therefore x = \frac{1}{3}$

Clave A

9. Piden: m

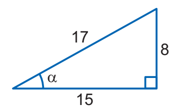
Por dato:

$\arctan\left(\frac{m}{8}\right) = \text{arcsen}\frac{8}{17} = \alpha$

Entonces:

$\text{arcsen}\frac{8}{17} = \alpha$

$\Rightarrow \text{sen}\alpha = \frac{8}{17}$



Además:

$\arctan\left(\frac{m}{8}\right) = \alpha$

$\Rightarrow \underbrace{\tan\alpha} = \frac{m}{8}$

$\frac{8}{15} = \frac{m}{8}$

$\therefore m = \frac{64}{15}$

Clave D

10. Piden: x

Por dato:

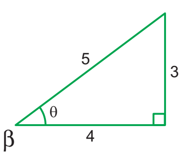
$\text{arcsen}x = \arctan\frac{3}{4} + \frac{1}{2}\arctan\left(-\frac{5}{12}\right)$

$\text{arcsen}x = \arctan\frac{3}{4} - \frac{1}{2}\arctan\frac{5}{12}$

Sea:

• $\arctan\frac{3}{4} = \theta$

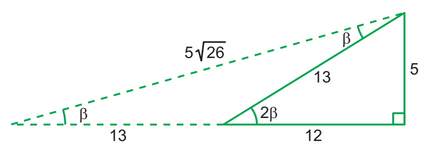
$\Rightarrow \tan\theta = \frac{3}{4}$



• $\frac{1}{2}\arctan\frac{5}{12} = \beta$

$\arctan\frac{5}{12} = 2\beta$

$\Rightarrow \tan 2\beta = \frac{5}{12}$



Luego:

$\text{arcsen}x = \theta - \beta$

$\Rightarrow x = \text{sen}(\theta - \beta)$

$x = \text{sen}\theta\cos\beta - \cos\theta\,\text{sen}\beta$

$x = \left(\frac{3}{5}\right)\left(\frac{25}{5\sqrt{26}}\right) - \left(\frac{4}{5}\right)\left(\frac{5}{5\sqrt{26}}\right)$

$x = \frac{15}{5\sqrt{26}} - \frac{4}{5\sqrt{26}} = \frac{11}{5\sqrt{26}}$

$\therefore x = \frac{11\sqrt{26}}{130}$

Clave A

11. $E = \tan\left[m\left(\frac{\pi}{2} - \text{arcsec}m\right)\right]$

Por dato:

$\arctan\sqrt{3} = m\arctan\left(\frac{\sqrt{3}}{3}\right) = \alpha$

Entonces:

$\arctan\sqrt{3} = \alpha$

$\Rightarrow \tan\alpha = \sqrt{3} \Rightarrow \alpha = \dfrac{\pi}{3}$ ...(1)

$m \,.\, \arctan\left(\dfrac{\sqrt{3}}{3}\right) = \alpha$

$\Rightarrow \tan\left(\dfrac{\alpha}{m}\right) = \dfrac{\sqrt{3}}{3} \Rightarrow \dfrac{\alpha}{m} = \dfrac{\pi}{6}$ ...(2)

Reemplazando (1) en (2):

$\dfrac{\left(\frac{\pi}{3}\right)}{m} = \dfrac{\pi}{6} \Rightarrow \dfrac{\pi}{3m} = \dfrac{\pi}{6}$

$\Rightarrow 3m = 6 \Rightarrow m = 2$

Luego:

$E = \tan\left[m\left(\dfrac{\pi}{2} - \text{arcsec}m\right)\right]$

$E = \tan\left[2\left(\dfrac{\pi}{2} - \text{arc sec}2\right)\right]$

$E = \tan(\pi - 2\text{arcsec}2)$

$E = -\tan(2\text{arcsec}2)$

Pero: $\text{arcsec}2 = \dfrac{\pi}{3}$

$\Rightarrow E = -\tan\left(\dfrac{2\pi}{3}\right) = -\tan120°$

$E = -(-\tan60°) = \tan60° = \sqrt{3}$

$\therefore E = \sqrt{3}$

Clave B

12. $E = \text{arcsen}\{\cos[\arctan(\cot30°)]\}$

Luego:

$\arctan(\cot30°) = \arctan(\sqrt{3}) = \dfrac{\pi}{3}$

Entonces:

$E = \text{arcsen}\left(\cos\dfrac{\pi}{3}\right)$

$E = \text{arcsen}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{6} = 30°$

$\therefore E = 30°$

Clave C

13. Piden: Ran(g)

$g(x) = 8\arccos\left(\dfrac{3x+1}{2}\right) - \dfrac{\pi}{4}$

Analizamos el dominio:

$-1 \leq \dfrac{3x+1}{2} \leq 1$

$-2 \leq 3x + 1 \leq 2$

$-3 \leq 3x \leq 1 \Rightarrow -1 \leq x \leq \dfrac{1}{3}$

Además:

$0 \leq \arccos\left(\dfrac{3x+1}{2}\right) \leq \pi$

$0 \leq 8\arccos\left(\dfrac{3x+1}{2}\right) \leq 8\pi$

$-\dfrac{\pi}{4} \leq \underbrace{8\arccos\left(\dfrac{3x+1}{2}\right) - \dfrac{\pi}{4}} \leq \dfrac{31\pi}{4}$

$-\dfrac{\pi}{4} \leq g(x) \leq \dfrac{31\pi}{4}$

$\therefore \text{Ran}(g) = \left[-\dfrac{\pi}{4}; \dfrac{31\pi}{4}\right]$

Clave D

14. Piden: Ran(f)

$f(x) = \dfrac{\pi}{3} + 3\text{arcsen}x$

Sabemos:

$-\dfrac{\pi}{2} \leq \text{arcsen}x \leq \dfrac{\pi}{2}$

$-\dfrac{3\pi}{2} \leq 3\text{arcsen}x \leq \dfrac{3\pi}{2}$

$-\dfrac{7\pi}{6} \leq 3\text{arcsen}x + \dfrac{\pi}{3} \leq \dfrac{11\pi}{6}$

$-\dfrac{7\pi}{6} \leq f(x) \leq \dfrac{11\pi}{6}$

$\therefore \text{Ran}(f) = \left[-\dfrac{7\pi}{6}; \dfrac{11\pi}{6}\right]$

Clave C

PRACTIQUEMOS
Nivel 1 (página 73) Unidad 4

Comunicación matemática

1.

2.

Razonamiento y demostración

3. Por dato:

$\theta = \text{arcsen}\dfrac{\sqrt{3}}{2} + \arccos1$

Sabemos:

$\text{arcsen}\dfrac{\sqrt{3}}{2} = \dfrac{\pi}{3} \ \wedge\ \arccos1 = 0$

$\Rightarrow \theta = \dfrac{\pi}{3} + 0 = \dfrac{\pi}{3}$

Piden:

$\text{sen}\theta + \cos\theta = \text{sen}\dfrac{\pi}{3} + \cos\dfrac{\pi}{3}$

$\Rightarrow \text{sen}\theta + \cos\theta = \dfrac{\sqrt{3}}{2} + \dfrac{1}{2}$

$\therefore \text{sen}\theta + \cos\theta = \dfrac{\sqrt{3}+1}{2}$

Clave D

4. $M = \text{arcsec}(2) + \text{arccsc}\left(\dfrac{2\sqrt{3}}{3}\right)$

Sea:

$\text{arcsec}(2) = \alpha \Rightarrow \sec\alpha = 2$

$\Rightarrow \alpha = \dfrac{\pi}{3}$

$\text{arccsc}\left(\dfrac{2\sqrt{3}}{3}\right) = \theta \Rightarrow \csc\theta = \dfrac{2\sqrt{3}}{3}$

$\Rightarrow \theta = \dfrac{\pi}{3}$

Luego:

$M = \alpha + \theta = \left(\dfrac{\pi}{3}\right) + \left(\dfrac{\pi}{3}\right)$

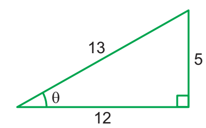
$\therefore M = \dfrac{2\pi}{3}$

Clave E

5. $M = \text{sen}\left(\arctan\dfrac{5}{12}\right)$

Sea: $\arctan\dfrac{5}{12} = \theta$

$\Rightarrow \tan\theta = \dfrac{5}{12}$



Luego: $M = \text{sen}(\theta)$

$\Rightarrow M = \text{sen}\theta = \dfrac{5}{13}$

$\therefore M = \dfrac{5}{13}$

Clave B

6. Por dato: $\text{arcsen}a + \arccos b = \dfrac{\pi}{3}$

Piden:

$K = \arccos a + \text{arcsen}b$

$K = \left(\dfrac{\pi}{2} - \text{arcsen}a\right) + \left(\dfrac{\pi}{2} - \arccos b\right)$

$K = \pi - (\text{arcsen}a + \arccos b)$

$K = \pi - \left(\dfrac{\pi}{3}\right) = \dfrac{2\pi}{3}$

$\therefore K = \dfrac{2\pi}{3}$

Clave E

7. $M = \arctan\dfrac{5}{6} + \arctan\dfrac{1}{11}$

$M = \arctan\left(\dfrac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6}\cdot\frac{1}{11}}\right) + n\pi$

Como: $\dfrac{5}{6}\cdot\dfrac{1}{11} < 1 \Rightarrow n = 0$

$\Rightarrow M = \arctan\left(\dfrac{\frac{61}{66}}{\frac{61}{66}}\right) = \arctan1$

$\therefore M = \arctan1 = \dfrac{\pi}{4}$

Clave B

8. $\theta = \text{arcsen}(x^2 + 1)$

Sabemos: $x^2 \geq 0$ ...(1)

Además por la función arcsen:

$-1 \leq x^2 + 1 \leq 1$

$\Rightarrow -2 \leq x^2 \leq 0$ ...(2)

De (1) y (2): $x^2 = 0 \Rightarrow x = 0$

Luego:

$\theta = \text{arcsen}(0^2 + 1)$

$\Rightarrow \theta = \text{arcsen}(1) \Rightarrow \theta = \dfrac{\pi}{2}$

Piden:

$\cos\theta = \cos\dfrac{\pi}{2} = 0$

$\therefore \cos\theta = 0$

Clave E

9. Piden: Dom(f)

$f(x) = \text{arcsen}x + \text{arcsen}2x$

Sabemos: $y = f^*(x) = \text{arcsen}x$

$\text{Dom}(f^*) = [-1; 1]$

Además:

$\text{arcsen}x \Rightarrow -1 \le x \le 1 \quad ...(\alpha)$

$\text{arcsen}2x \Rightarrow -1 \le 2x \le 1$

$\Rightarrow -\dfrac{1}{2} \le x \le \dfrac{1}{2} \quad ...(\beta)$

De (α) y (β): $-\dfrac{1}{2} \le x \le \dfrac{1}{2}$

$\therefore \text{Dom}(f) = \left[-\dfrac{1}{2}; \dfrac{1}{2}\right]$

Clave B

Resolución de problemas

10. Por teoría sabemos:

$\text{arcsen}x \Leftrightarrow x \in [-1; 1]$

$\text{arccos}x \Leftrightarrow x \in [-1; 1]$

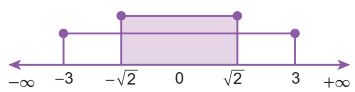
Entonces:

$-1 \le x + 2 \le 1 \ \wedge \ -1 \le x^2 - 1 \le 1$

$-3 \le x \le 3 \ \wedge \ 0 \le x^2 \le 2$

$-3 \le x \le 3 \wedge -\sqrt{2} \le x \le \sqrt{2}$

Intersecamos:



$\therefore x \in [-\sqrt{2}; \sqrt{2}]$

$\Rightarrow \text{Dom}f = [-\sqrt{2}; \sqrt{2}]$

Clave A

11. Para:

- $\text{arcsen}x \Leftrightarrow x \in [-1; 1]$

$-1 \le \dfrac{1-x}{1+x} \le 1$

$-1 \le \dfrac{2-(1+x)}{1+x} \le 1$

$-1 \le \dfrac{2}{1+x} - 1 \le 1$

$0 \le \dfrac{2}{1+x} \le 2$

$0 \le \dfrac{1}{1+x} \le 1$

$1 \le 1 + x < +\infty \Rightarrow 0 \le x < +\infty$

Debemos tomar en cuenta que en una división el divisor debe ser diferente de cero.

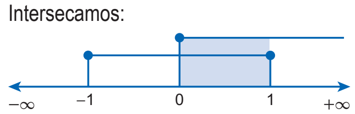
$\Rightarrow \text{arccos}\left(\dfrac{1-x}{1+x}\right) \ne 0$

$\dfrac{1-x}{1+x} \ne 0$

$1 - x \ne 0$

$1 \ne x$

Intersecamos:



$\text{Dom}f = [0; 1] - \{1\}$

$\therefore \ \text{Dom}f = [0; 1\rangle$

Clave E

Nivel 2 (página 74) Unidad 4

Comunicación matemática

12.

13.

Razonamiento y demostración

14. Por dato: $0 < \dfrac{a}{b} < \dfrac{\pi}{2}$

Además:

$a = \text{barcsen}\dfrac{2cx}{d} \Rightarrow -1 \le \dfrac{2cx}{d} \le 1 \ ...(\alpha)$

$\text{arcsen}\dfrac{2cx}{d} = \dfrac{a}{b}$

$\Rightarrow \text{sen}\dfrac{a}{b} = \dfrac{2cx}{d}$

Como: $0 < \dfrac{a}{b} < \dfrac{\pi}{2}$

$\Rightarrow 0 < \text{sen}\dfrac{a}{b} < 1$

$0 < \dfrac{2cx}{d} < 1 \quad ...(\beta)$

De (α) y (β): $0 < \dfrac{2cx}{d} < 1$

$\Rightarrow 0 < 2cx < d$

$0 < 2x < \dfrac{d}{c}$

$0 < x < \dfrac{d}{2c}$

$\therefore x \in \left\langle 0; \dfrac{d}{2c} \right\rangle$

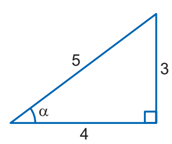
Clave A

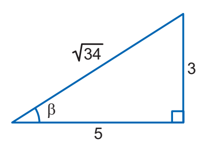
15. $P = \text{arccos}\left(\dfrac{4}{5}\right) + \text{arctan}\left(\dfrac{3}{5}\right)$

Sea:

$\text{arccos}\left(\dfrac{4}{5}\right) = \alpha \quad \wedge \quad \text{arctan}\left(\dfrac{3}{5}\right) = \beta$

$\Rightarrow \quad \cos\alpha = \dfrac{4}{5} \qquad \Rightarrow \tan\beta = \dfrac{3}{5}$





Entonces: $P = \alpha + \beta$

Luego:

$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\tan(\alpha + \beta) = \dfrac{\left(\frac{3}{4}\right) + \left(\frac{3}{5}\right)}{1 - \left(\frac{3}{4}\right)\left(\frac{3}{5}\right)} = \dfrac{\frac{27}{20}}{\frac{11}{20}}$

$\tan(\alpha + \beta) = \dfrac{27}{11}$

$\Rightarrow (\alpha + \beta) = \text{arctan}\left(\dfrac{27}{11}\right)$

$\therefore P = \text{arctan}\left(\dfrac{27}{11}\right)$

Clave B

16. Por dato:

$\theta = \text{arctan}\left(\dfrac{m}{n}\right) - \text{arctan}\left(\dfrac{m-n}{m+n}\right)$

Sea:

$\text{arctan}\left(\dfrac{m}{n}\right) = \alpha \Rightarrow \tan\alpha = \dfrac{m}{n}$

$\text{arctan}\left(\dfrac{m-n}{m+n}\right) = \beta \Rightarrow \tan\beta = \dfrac{m-n}{m+n}$

Entonces: $\theta = \alpha - \beta$

$\Rightarrow \tan\theta = \tan(\alpha - \beta)$

$\tan\theta = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\tan\theta = \dfrac{\left(\frac{m}{n}\right) - \left(\frac{m-n}{m+n}\right)}{1 + \left(\frac{m}{n}\right)\left(\frac{m-n}{m+n}\right)}$

$\tan\theta = \dfrac{\left(\frac{m^2+n^2}{n(m+n)}\right)}{\left(\frac{m^2+n^2}{n(m+n)}\right)}$

$\therefore \tan\theta = 1$

Clave C

17. $K = \text{arccos}\left(-\dfrac{1}{2}\right) + \text{arcsen}\left(\dfrac{1}{2}\right) + \text{arctan}\left(\sqrt{3}\right)$

Por propiedad: $\text{arccos}(-x) = \pi - \text{arccos}x$

$K = \pi - \text{arccos}\left(\dfrac{1}{2}\right) + \text{arcsen}\left(\dfrac{1}{2}\right) + \text{arctan}\left(\sqrt{3}\right)$

$K = \pi - \left(\dfrac{\pi}{3}\right) + \left(\dfrac{\pi}{6}\right) + \left(\dfrac{\pi}{3}\right)$

$\Rightarrow K = \pi + \dfrac{\pi}{6} = \dfrac{7\pi}{6}$

$\therefore K = \dfrac{7\pi}{6}$

Clave B

18. $\text{arctan}x + \text{arctan}(1 - x) = \text{arctan}\dfrac{4}{3}$

Sea:

$\text{arctan}x = \alpha \Rightarrow \tan\alpha = x$

$\text{arctan}(1 - x) = \beta \Rightarrow \tan\beta = (1 - x)$

$\text{arctan}\dfrac{4}{3} = \theta \Rightarrow \tan\theta = \dfrac{4}{3}$

Entonces: $\alpha + \beta = \theta$

$\Rightarrow \tan(\alpha + \beta) = \tan\theta$

$\dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \tan\theta$

$\dfrac{x + (1 - x)}{1 - x(1 - x)} = \dfrac{4}{3}$

$\Rightarrow 3 = 4 - 4x + 4x^2$

$0 = 4x^2 - 4x + 1$

$0 = (2x - 1)^2$

$\Rightarrow 2x - 1 = 0$

$2x = 1$

$\therefore x = \dfrac{1}{2}$

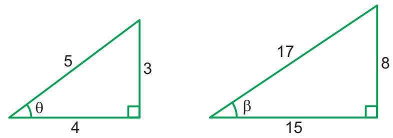
Clave C

19. $Q = \text{arcsen}\frac{3}{5} + \text{arcsen}\frac{8}{17} - \text{arcsen}\frac{77}{85}$

Sea:

$\text{arcsen}\frac{3}{5} = \theta \;\wedge\; \text{arcsen}\frac{8}{17} = \beta$

$\Rightarrow \text{sen}\theta = \frac{3}{5} \qquad \Rightarrow \text{sen}\beta = \frac{8}{17}$



$\text{sen}(\theta + \beta) = \text{sen}\theta\cos\beta + \cos\theta\text{sen}\beta$

$\text{sen}(\theta + \beta) = \left(\frac{3}{5}\right)\left(\frac{15}{17}\right) + \left(\frac{4}{5}\right)\left(\frac{8}{17}\right)$

$\text{sen}(\theta + \beta) = \frac{77}{85}$

$\Rightarrow \theta + \beta = \text{arcsen}\frac{77}{85}$

Entonces:

$Q = \theta + \beta - \text{arcsen}\frac{77}{85}$

$Q = \left(\text{arcsen}\frac{77}{85}\right) - \text{arcsen}\frac{77}{85}$

$\therefore Q = 0$

Clave B

20. $g(x) = 4\arctan x - \frac{\pi}{2}$

Sabemos:

$y = f^*(x) = \arctan x$

$\text{Dom}(f^*) = \langle -\infty; +\infty \rangle$

$\text{Ran}(f^*) = \left\langle -\frac{\pi}{2}; \frac{\pi}{2} \right\rangle$

$\Rightarrow -\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$

$-2\pi < 4\arctan x < 2\pi$

$-2\pi - \frac{\pi}{2} < \underbrace{4\arctan x - \frac{\pi}{2}} < 2\pi - \frac{\pi}{2}$

$-\frac{5\pi}{2} < g(x) < \frac{3\pi}{2}$

$\therefore \text{Ran}(g) = \left\langle -\frac{5\pi}{2}; \frac{3\pi}{2} \right\rangle$

Clave E

21. Por dato: $x > 0$

Además: $\arccos\left(\sqrt{3}\,x\right) + \arccos x = \frac{\pi}{2}$

Sea:

$\arccos\left(\sqrt{3}\,x\right) = \alpha \Rightarrow \cos\alpha = \left(\sqrt{3}\,x\right)$

$\arccos x = \theta \Rightarrow \cos\theta = x$

Luego: $\alpha + \theta = \frac{\pi}{2}$

$\Rightarrow \cos\alpha = \text{sen}\theta$

$\cos^2\alpha = \text{sen}^2\theta$

$\cos^2\alpha = 1 - \cos^2\theta$

$\left(\sqrt{3}\,x\right)^2 = 1 - (x)^2$

$3x^2 = 1 - x^2$

$4x^2 - 1 = 0$

$\Rightarrow (2x + 1)(2x - 1) = 0$

$\Rightarrow x = -\frac{1}{2} \;\vee\; x = \frac{1}{2}$; como $x > 0$

$\therefore x = \frac{1}{2}$

Clave B

22. $M = \text{arcsen}\left(\frac{\sqrt{3}}{2}\right) + \arccos 1 + \arctan\sqrt{3}$

Sabemos:

$\text{arcsen}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

$\arccos 1 = 0$

$\arctan\sqrt{3} = \frac{\pi}{3}$

Reemplazando:

$M = \frac{\pi}{3} + 0 + \frac{\pi}{3} = \frac{2\pi}{3}$

$\therefore M = \frac{2\pi}{3}$

Clave C

Resolución de problemas

23. Sabemos:

$\text{arcsen}(m) \Leftrightarrow m \in [-1; 1]$

Entonces:

$-1 \leq \text{sen}x - 1/2 \leq 1$

$-\frac{1}{2} \leq \text{sen}x \leq \frac{3}{2}$

Además:

$-1 \leq \text{sen}x \leq 1$

$\Rightarrow x \in \left[0; \frac{7\pi}{6}\right] \cup \left[\frac{11\pi}{6}; 2\pi\right]$

Pero $x \in \langle 0; 2\pi \rangle$

$\Rightarrow x \in \left\langle 0; \frac{7\pi}{6}\right] \cup \left[\frac{11\pi}{6}; 2\pi\right\rangle$

Clave B

24. Sabemos:

$-1 \leq \text{sen}^2x - 1 \leq 1$

$0 \leq \text{sen}^2x \leq 1$

Entonces:

Para $\text{sen}^2x = 0$

$f(x) = \text{arcsen}(0 - 1)$

$f(x) = \text{arcsen}(-1)$

$f(x) = -\frac{\pi}{2}$ (mín.)

Para $\text{sen}^2x = 1$

$f(x) = \text{arcsen}(1 - 1)$

$f(x) = \text{arcsen}(0)$

$f(x) = 0$ (máx.)

$f(x)_{\text{máx.}} + f(x)_{\text{mín.}} = 0 - \frac{\pi}{2}$

$= -\frac{\pi}{2}$

Clave D

Nivel 3 (página 75) Unidad 4

Comunicación matemática

25. V

F; $\arctan x + \text{arccot} x = \pi/2$

V

V

$\therefore$ Tres son verdaderas.

Clave B

26. M:

$\text{arcsen}(-x) + 2\text{arcsen}x = \pi/6$

$-\text{arcsen}x + 2\text{arcsen}x = \pi/6$

$\text{arcsen}x = \pi/6$

$\text{sen}(\text{arcsen}x) = \text{sen}(\pi/6)$

$\therefore x = 1/2$

N:

$\arccos(-x) + 2\arccos x = \frac{7\pi}{6}$

$\pi - \arccos x + 2\arccos x = \frac{7\pi}{6}$

$\arccos x = \pi/6$

$\cos(\arccos x) = \cos \pi/6$

$\therefore x = \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{M}{N} = \tan 30°$

Clave B

Razonamiento y demostración

27. Piden: β

Por dato:

$p = q\arctan\left(\frac{m\beta}{n}\right)$

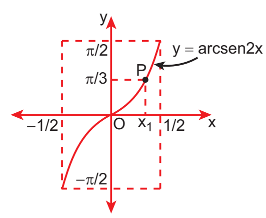
$\Rightarrow \arctan\left(\frac{m\beta}{n}\right) = \frac{p}{q}$

$\left(\frac{m\beta}{n}\right) = \tan\left(\frac{p}{q}\right)$

$\therefore \beta = \frac{n}{m}\tan\left(\frac{p}{q}\right)$

Clave B

28.



Del gráfico:

$y = \frac{\pi}{3} = \text{arcsen}2x_1$

$\Rightarrow \text{arcsen}2x_1 = \frac{\pi}{3}$

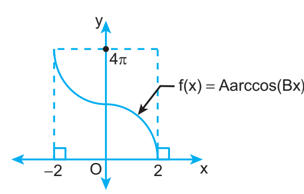
$2x_1 = \text{sen}\frac{\pi}{3}$

$2x_1 = \frac{\sqrt{3}}{2}$

$\therefore x_1 = \frac{\sqrt{3}}{4}$

Clave B

29.



Del gráfico: $B > 0 \;\wedge\; A > 0$

$\text{Dom}(f) = [-2; 2] \wedge \text{Ran}(f) = [0; 4\pi]$

Sabemos: Dom(arccos): $[-1; 1]$

$\Rightarrow -1 \leq Bx \leq 1$

$-\frac{1}{B} \leq x \leq \frac{1}{B} \Rightarrow \text{Dom(f)} = \left[-\frac{1}{B}; \frac{1}{B}\right]$

Comparando el dominio: $B = \frac{1}{2}$
Además:
Ran(arccos) = $[0; \pi]$

$\Rightarrow 0 \leq \arccos\left(\frac{1}{2}x\right) \leq \pi$

$\Rightarrow 0 \leq A\arccos\left(\frac{x}{2}\right) \leq A\pi$

$\Rightarrow \text{Ran(f)} = [0; A\pi]$

Comparando el rango: $A = 4$
Piden:

$A \cdot B = (4)\left(\frac{1}{2}\right) = 2$

$\therefore A \cdot B = 2$

Clave B

30. $f(x) = 2\text{arcsen}\left(\frac{x}{2}\right) + \pi$

Para el dominio:

$-1 \leq \frac{x}{2} \leq 1$

$-2 \leq x \leq 2 \Rightarrow \text{Dom(f)} = [-2; 2]$

Además:

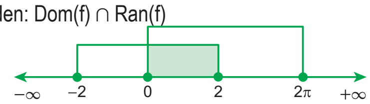
$-\frac{\pi}{2} \leq \text{arcsen}\left(\frac{x}{2}\right) \leq \frac{\pi}{2}$

$-\pi \leq 2\text{arcsen}\left(\frac{x}{2}\right) \leq \pi$

$0 \leq 2\text{arcsen}\left(\frac{x}{2}\right) + \pi \leq 2\pi$

$\Rightarrow \text{Ran(f)} = [0; 2\pi]$

Piden: Dom(f) ∩ Ran(f)



$\therefore \text{Dom(f)} \cap \text{Ran(f)} = [0; 2]$

Clave E

31. Por dato:

arcsen1 + arccosx = arccos0
Sabemos:

$\text{arcsen}1 = \frac{\pi}{2} \wedge \arccos 0 = \frac{\pi}{2}$

Reemplazando tenemos:

$\frac{\pi}{2} + \arccos x = \frac{\pi}{2}$
$\Rightarrow \arccos x = 0$
$\Rightarrow x = \cos 0 = 1$
$\therefore x = 1$

Clave A

32. $L = 2\text{arcsen}(-1) + \frac{1}{2}\arccos\left(-\frac{\sqrt{3}}{2}\right)$

Sabemos:

$\text{arcsen}(-1) = -\text{arcsen}(1) = -\frac{\pi}{2}$

$\Rightarrow \text{arcsen}(-1) = -\frac{\pi}{2}$

$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \pi - \arccos\left(\frac{\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6}$

$\Rightarrow \arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

Reemplazando en L:

$L = 2\left(-\frac{\pi}{2}\right) + \frac{1}{2}\left(\frac{5\pi}{6}\right)$

$\Rightarrow L = -\pi + \frac{5\pi}{12} = -\frac{7\pi}{12}$

$\therefore L = -\frac{7\pi}{12}$

Clave D

33. Piden:

$\theta = \arctan(\tan 100°) - \text{arccot}(\cot 300°)$

Por propiedad:

$\arctan(\tan x) = x$, si: $x \in \left\langle -\frac{\pi}{2}; \frac{\pi}{2} \right\rangle$

$\text{arccot}(\cot x) = x$, si: $x \in \langle 0; \pi \rangle$

Observamos que 100° y 300° no se encuentran en los intervalos para aplicar la propiedad respectiva, para ello buscamos los equivalentes de:

- $\tan 100° = \tan(180° - 80°) = -\tan 80°$
  $\Rightarrow \tan 100° = -\tan 80°$
- $\cot 300° = \cot(360° - 60°) = -\cot 60°$
  $\Rightarrow \cot 300° = -\cot 60°$

Luego:
$\theta = \arctan(-\tan 80°) - \text{arccot}(-\cot 60°)$
$\theta = (-\arctan(\tan 80°)) - (\pi - \text{arccot}(\cot 60°))$
$\theta = -\arctan(\tan 80°) - \pi + \text{arccot}(\cot 60°)$

Ahora 80° y 60° si se encuentran en los intervalos para aplicar la propiedad respectiva, entonces:

$\theta = -(80°) - (180°) + (60°) = -200°$
$\therefore \theta = -200°$

Clave E

Resolución de problemas

34. Para $x = 1$:

$f(1) = A\arccos(1) + B = \pi$
$A(0) + B = \pi$
$\Rightarrow B = \pi$

Para $x = -1$:
$f(-1) = A\arccos(-1) + \beta = 4\pi$
$A(\pi) + \pi = 4\pi$
$A\pi = 3\pi$
$\Rightarrow A = 3$

Nos piden:
$Q = A + \cos B$
$Q = 3 + \cos(\pi)$
$Q = 3 - 1 = 2$

Clave C

35. Tenemos:

$\pi - 3\text{arcsen}x \geq 0$
$\pi \leq 3\text{arcsen}x$
$\frac{\pi}{3} \geq \text{arcsen}x$

$\text{sen}\left(\frac{\pi}{3}\right) \geq x$

$\frac{\sqrt{3}}{2} \geq x$

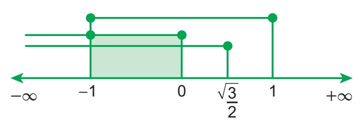
$\arccos(-x) - \arccos x \geq 0$
$\pi - 2\arccos x \geq 0$
$\pi \geq 2\arccos x$

$\frac{\pi}{2} \geq \arccos x$

$\cos\frac{\pi}{2} \geq \cos(\arccos x)$

$0 \geq x$

Intersecamos los dominios obtenidos:



Domf = $[-1; 0]$

Clave A

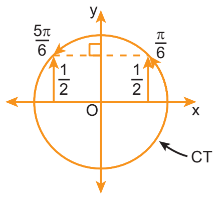
APLICAMOS LO APRENDIDO (página 76) Unidad 4

1. $2\text{sen}2x - 1 = 0$

$2\text{sen}2x = 1$

$\Rightarrow \text{sen}2x = \frac{1}{2}$

Analizando los valores en la CT:



En el intervalo de $\langle 0; 2\pi\rangle$ se tiene:

$2x = \frac{\pi}{6} \quad \vee \quad 2x = \frac{5\pi}{6}$

$\Rightarrow x = \frac{\pi}{12} \quad \vee \quad x = \frac{5\pi}{12}$

Piden: la solución principal, que es el menor valor positivo que satisface la ecuación.

$\therefore x = \frac{\pi}{12} = 15°$

Clave C

2. $\tan x = \sqrt{3}$; $x \in \langle 0°; 180°\rangle$

Entonces: $VP = \arctan(\sqrt{3}) = \frac{\pi}{3}$

Empleando la expresión general para la tangente:

$x_G = k\pi + VP$; $k \in \mathbb{Z}$

$x_G = k\pi + \frac{\pi}{3}$

$\Rightarrow x = \left\{k\pi + \frac{\pi}{3} / k \in \mathbb{Z}\right\}$

Evaluando:

$k = -1 \Rightarrow x = -\frac{2\pi}{3} = -120° \notin \langle 0°; 180°\rangle$

$k = 0 \Rightarrow x = \frac{\pi}{3} = 60° \in \langle 0°; 180°\rangle$

$k = 1 \Rightarrow x = \frac{4\pi}{3} = 240° \notin \langle 0°; 180°\rangle$

$\therefore x = 60°$

Clave C

3. $\text{sen}^2\theta + \text{sen}\theta = \cos^2\theta$

$\text{sen}^2\theta + \text{sen}\theta = 1 - \text{sen}^2\theta$

Entonces:

$2\text{sen}^2\theta + \text{sen}\theta - 1 = 0$

$(2\text{sen}\theta - 1)(\text{sen}\theta + 1) = 0$

$\Rightarrow \text{sen}\theta = \frac{1}{2} \vee \text{sen}\theta = -1$

Por dato: $90° \leq \theta \leq 180°$ ... (I)

Si: $\text{sen}\theta = -1 \Rightarrow \theta = 270°$ (no cumple (I))

Si: $\text{sen}\theta = \frac{1}{2} \Rightarrow \theta = 30° \vee \theta = 150°$

De (I): θ no puede ser agudo.

$\therefore \theta = 150°$

Clave E

4. $\text{sen}x + \cos x = 0$

$\text{sen}x = -\cos x$

$\frac{\text{sen}x}{\cos x} = -1$

$\Rightarrow \tan x = -1$

Entonces: $VP = \arctan(-1) = -\frac{\pi}{4}$

Usando la expresión general para la tangente:

$x_G = k\pi + VP$; $k \in \mathbb{Z}$

$\Rightarrow x_G = k\pi + \left(-\frac{\pi}{4}\right)$

$\Rightarrow x = \left\{k\pi - \frac{\pi}{4} / k \in \mathbb{Z}\right\}$

Evaluando:

Para: $k = 0 \Rightarrow x = -\frac{\pi}{4} = -45°$

Para: $k = 1 \Rightarrow x = \frac{3\pi}{4} = 135°$

Para: $k = 2 \Rightarrow x = \frac{7\pi}{4} = 315°$

Por lo tanto, la menor solución positiva es 135°.

Clave A

5. $\text{sen}^2x - 2\text{sen}x - 3 = 0$

$(\text{sen}x - 3)(\text{sen}x + 1) = 0$

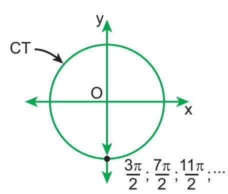
$\Rightarrow \text{sen}x = 3 \vee \text{sen}x = -1$

Como: $-1 \leq \text{sen}x \leq 1$

Entonces, en $\text{sen}x = 3$ no existe solución en los $\mathbb{R}$.

Luego: $\text{sen}x = -1$

Analizando los valores en la CT:



$\Rightarrow x = \left\{\frac{3\pi}{2}; \frac{7\pi}{2}; \frac{11\pi}{2}; ...\right\}$

$\Rightarrow x = \left\{(4k+3)\frac{\pi}{2} / k \in \mathbb{Z}\right\}$

Evaluando:

Para: $k = -1 \Rightarrow x = -\frac{\pi}{2} = -90°$

Para: $k = 0 \Rightarrow x = \frac{3\pi}{2} = 270°$

Para: $k = 1 \Rightarrow x = \frac{7\pi}{2} = 630°$

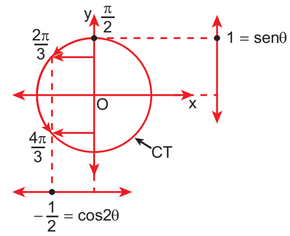
Por lo tanto, la menor solución positiva es 270°.

Clave D

6. $(\cos2\theta + \frac{1}{2})(\text{sen}\theta - 1) = 0$

$\Rightarrow \cos2\theta = -\frac{1}{2} \vee \text{sen}\theta = 1$

Analizando en el intervalo positivo $\langle 0; 2\pi\rangle$ en la CT:



Entonces: $\theta = \frac{\pi}{2} = 90°$

Además: $2\theta = \frac{2\pi}{3} \vee 2\theta = \frac{4\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{3} = 60° \vee \theta = \frac{2\pi}{3} = 120°$

Ordenando tenemos: $x = \{60°; 90°; 120°\}$

Por lo tanto, la segunda menor solución positiva es 90°.

Clave E

7. $\tan\left(5x - \frac{\pi}{12}\right) = 2 + \frac{3}{\sqrt{3}}$

$\tan\left(5x - \frac{\pi}{12}\right) = 2 + \sqrt{3}$

Sabemos: $\tan75° = 2 + \sqrt{3}$

$\Rightarrow \tan\left(\frac{5\pi}{12}\right) = 2 + \sqrt{3}$

$\Rightarrow \arctan(2 + \sqrt{3}) = \frac{5\pi}{12}$

Entonces: $VP = \arctan(2 + \sqrt{3}) = \frac{5\pi}{12}$

Usando la expresión general para la tangente:

$x_G = k\pi + VP$; $k \in \mathbb{Z}$

$\Rightarrow x_G = k\pi + \frac{5\pi}{12}$

$\Rightarrow \left(5x - \frac{\pi}{12}\right) = k\pi + \frac{5\pi}{12}$

$\Rightarrow x = \left\{\frac{k\pi}{5} + \frac{\pi}{10} / k \in \mathbb{Z}\right\}$

Luego, para obtener las soluciones positivas evaluamos:

Para: $k = 0 \Rightarrow x_1 = \frac{\pi}{10}$

Para: $k = 1 \Rightarrow x_2 = \frac{3\pi}{10}$

Piden:

$x_1 + x_2 = \frac{\pi}{10} + \frac{3\pi}{10} = \frac{4\pi}{10}$

$\therefore x_1 + x_2 = \frac{2\pi}{5}$

Clave B

8. $2\tan2x\,\text{sen}x = 0$

$\Rightarrow \tan2x = 0 \vee \text{sen}x = 0$

Entonces:

$\tan2x = 0 \Rightarrow VP = 0$

Usando la expresión general para la tangente:

$x_G = k\pi + VP$; $k \in \mathbb{Z}$

$\Rightarrow \underbrace{x_G}_{} = k\pi + (0)$

$2x = k\pi \Rightarrow x = \frac{k\pi}{2}$

$\Rightarrow x = \left\{\frac{k\pi}{2} / k \in \mathbb{Z}\right\}$ ...(I)

Luego:

$\text{sen}x = 0 \Rightarrow VP = 0$

Usando la expresión general para el seno:

$x_G = k\pi + (-1)^k VP;\ k \in \mathbb{Z}$

$\Rightarrow x_G = k\pi + (-1)^k(0)$

$x = k\pi$

$\Rightarrow\ x = \{k\pi\ /\ k \in \mathbb{Z}\}$ ...(II)

La solución de la ecuación será: (I) ∩ (II)

$\therefore x = \left\{\dfrac{k\pi}{2} / k \in \mathbb{Z}\right\}$

Clave B

9. $2\cos^2x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\Rightarrow \cos x = -\dfrac{1}{2}\ \vee\ \cos x = 1$

Entonces:

$\cos x = -\dfrac{1}{2} \Rightarrow VP = \arccos\left(-\dfrac{1}{2}\right) = \dfrac{2\pi}{3}$

Usando la expresión general para el coseno:

$x_G = 2k\pi \pm VP;\ k \in \mathbb{Z}$

$x_G = 2k\pi \pm \dfrac{2\pi}{3}$

$\Rightarrow\ x = \{2k\pi \pm \dfrac{2\pi}{3}\ /\ k \in \mathbb{Z}\}$ ...(I)

Luego:

$\cos x = 1 \Rightarrow VP = \arccos(1) = 0$

Usando la expresión general para el coseno:

$x_G = 2k\pi \pm VP;\ k \in \mathbb{Z}$

$x_G = 2k\pi \pm (0)$

$\Rightarrow\ x = \{2k\pi\ /\ k \in \mathbb{Z}\}$ ...(II)

La solución de la ecuación será: (I) ∪ (II)

$\therefore x = \{2k\pi \pm \dfrac{2\pi}{3}\} \cup \{2k\pi\};\ k \in \mathbb{Z}$

Clave A

10. $\dfrac{\cos x}{1+\cos 2x} - \dfrac{\text{sen}x}{1-\cos 2x} = 0$

Se debe tener en cuenta:

$1 + \cos 2x \neq 0 \quad \wedge \quad 1 - \cos 2x \neq 0$

$\cos 2x \neq -1 \quad \wedge \quad \cos 2x \neq 1$

Por dato: $0 < x < 2\pi$

$\Rightarrow 2x \neq \pi \quad \wedge \quad 2x \neq 0$

$x \neq \dfrac{\pi}{2} \quad \wedge \quad x \neq 0$

Luego, empleando las identidades del ángulo doble:

$\dfrac{\cos x}{2\cos^2x} - \dfrac{\text{sen}x}{2\text{sen}^2x} = 0$

$\dfrac{1}{2\cos x} = \dfrac{1}{2\text{sen}x}$

$\dfrac{\text{sen}x}{\cos x} = 1$

$\Rightarrow \tan x = 1$

Entonces: $VP = \arctan(1) = \dfrac{\pi}{4}$

Usando la expresión general para la tangente:

$x_G = k\pi + VP;\ k \in \mathbb{Z}$

$x_G = k\pi + \dfrac{\pi}{4}$

$\Rightarrow x = \{k\pi + \dfrac{\pi}{4}\ /\ k \in \mathbb{Z}\}$

Evaluando:

Para: $k = -1 \Rightarrow x = -\dfrac{3\pi}{4} \notin \langle 0;\ 2\pi\rangle$

Para: $k = 0 \Rightarrow x = \dfrac{\pi}{4} \in \langle 0;\ 2\pi\rangle$

Para: $k = 1 \Rightarrow x = \dfrac{5\pi}{4} \in \langle 0;\ 2\pi\rangle$

Para: $k = 2 \Rightarrow x = \dfrac{9\pi}{4} \notin \langle 0;\ 2\pi\rangle$

Además $\dfrac{\pi}{4}$ y $\dfrac{5\pi}{4}$ son diferentes de $\dfrac{\pi}{2}$.

$\therefore x = \dfrac{\pi}{4}\ \vee\ x = \dfrac{5\pi}{4}$

Clave A

11. $\text{sen}x - \csc x = \cos x - \sec x$

$\text{sen}x - \dfrac{1}{\text{sen}x} = \cos x - \dfrac{1}{\cos x};\ x \neq \dfrac{k\pi}{2};\ k \in \mathbb{Z}$

$\dfrac{\text{sen}^2x - 1}{\text{sen}x} = \dfrac{\cos^2x - 1}{\cos x}$

$\dfrac{\cos^2x}{\text{sen}x} = \dfrac{\text{sen}^2x}{\cos x}$

$\cos^3x = \text{sen}^3x$

$\cos x = \text{sen}x$

$\tan x = 1$

$\therefore\ x = \left\{\dfrac{\pi}{4};\ \dfrac{5\pi}{4}\right\}$

Clave E

12. Si: $x + y + z = \pi$

$\Rightarrow \tan x + \tan y + \tan z = \tan x\tan y\tan z$

Multiplicando por tanz

$\underbrace{\tan x\tan z}_{3} + \underbrace{\tan y\tan z}_{6} + \tan^2z = \underbrace{\tan x\tan z}_{(3)}\times\underbrace{\tan y\tan z}_{(6)}$

$9 + \tan^2z = 18$

$\tan^2z = 9 \Rightarrow \tan z = 3$

Luego en (I): $3\tan x = 3$

$\tan x = 1 \Rightarrow x = 45°$

$\therefore\ \tan\dfrac{x}{3} = \tan 15° = 2 - \sqrt{3}$

Clave C

13. $\tan\dfrac{\beta}{2} = \csc\beta - \text{sen}\beta$

$\csc\beta - \cot\beta = \csc\beta - \text{sen}\beta$

$\text{sen}\beta = \cot\beta$

$\text{sen}\beta = \dfrac{\cos\beta}{\text{sen}\beta}$

$\text{sen}^2\beta = \cos\beta$

$1 - \cos^2\beta = \cos\beta$

$\cos^2\beta + \cos\beta - 1 = 0$

$\cos\beta = \pm\left(\dfrac{\sqrt{5}-1}{2}\right)$

$VP = \arccos\left(\pm\left(\dfrac{\sqrt{5}-1}{2}\right)\right)$

$VP = \pm\arccos\left(\dfrac{\sqrt{5}-1}{2}\right)$

$x_G = 2k\pi \pm \arccos\left(\dfrac{\sqrt{5}-1}{2}\right)$

$x = 2k\pi \pm \arccos\left(\dfrac{\sqrt{5}-1}{2}\right)$

Clave B

14. $\cos 6x = \dfrac{1+\text{sen}6x}{\frac{1}{\cos 2x}}\left(\dfrac{1}{\text{sen}2x}\right);\ \text{sen}2x \neq 0\ \wedge\ \cos 2x \neq 0$

$\cos 6x = \dfrac{\cos 2x}{\text{sen}2x}(1 + \text{sen}6x)$

$\text{sen}2x\cos 6x = \cos 2x + \text{sen}6x\cos 2x$

$\text{sen}6x\cos 2x - \text{sen}2x\cos 6x + \cos 2x = 0$

$\text{sen}(6x - 2x) + \cos 2x = 0$

$\text{sen}4x + \cos 2x = 0$

$2\text{sen}2x\cos 2x + \cos 2x = 0$

$\cos 2x(2\text{sen}2x + 1) = 0$

$\cos 2x = 0 \vee \text{sen}2x = -\dfrac{1}{2};\ \text{sen}2x \neq 0 \wedge \cos 2x \neq 0$

$\Rightarrow\ \text{sen}2x = -\dfrac{1}{2}$

Luego. $0 \leq x \leq \pi\ \Rightarrow\ 0 \leq 2x \leq 2\pi$

Por lo tanto $2x = \dfrac{7\pi}{6}\ \vee\ 2x = \dfrac{11\pi}{6}$

$x = \dfrac{7\pi}{12}\ \vee\ x = \dfrac{11\pi}{12}$

Piden: $\dfrac{7\pi}{12} + \dfrac{11\pi}{12} = \dfrac{3\pi}{2}$

Clave A

PRACTIQUEMOS

Nivel 1 (página 78) Unidad 4

Comunicación matemática

1.

2.

Razonamiento y demostración

3. Piden, la suma de las dos primeras soluciones positivas de la ecuación:

$\text{sen}x = \dfrac{\sqrt{3}}{2}$

Empleando la expresión general para el seno:

$x_G = k\pi + (-1)^k VP;\ k \in \mathbb{Z}$

$x_G = k\pi + (-1)^k \arcsen\dfrac{\sqrt{3}}{2};\ k \in \mathbb{Z}$

$\Rightarrow x_G = k\pi + (-1)^k\ \dfrac{\pi}{3};\ k \in \mathbb{Z}$

Evaluando:

Para: $k = -1 \Rightarrow x = -\dfrac{4\pi}{3} = -240°$

Para: $k = 0 \Rightarrow x = \dfrac{\pi}{3} = 60°$

Para: $k = 1 \Rightarrow x = \dfrac{2\pi}{3} = 120°$

Luego, las dos primeras soluciones positivas son: 60° y 120°.

$\Rightarrow 60° + 120° = 180°$

Clave B

4. Piden, la suma de las dos primeras soluciones positivas de la ecuación:

$\text{sen}x = -\dfrac{\sqrt{2}}{2}$

Empleando la expresión general para el seno:

$x_G = k\pi + (-1)^k VP;\ k \in \mathbb{Z}$

$x_G = k\pi + (-1)^k \text{arcsen}\left(-\dfrac{\sqrt{2}}{2}\right);\ k \in \mathbb{Z}$

$x_G = k\pi + (-1)^k\left(-\frac{\pi}{4}\right); k \in \mathbb{Z}$

$\Rightarrow x_G = k\pi - (-1)^k\frac{\pi}{4}; k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow x = -\frac{\pi}{4} = -45°$

Para: $k = 1 \Rightarrow x = \frac{5\pi}{4} = 225°$

Para: $k = 2 \Rightarrow x = \frac{7\pi}{4} = 315°$

Luego, las dos primeras soluciones positivas son: 225° y 315°.

$\Rightarrow 225° + 315° = 540°$

Clave E

5. Piden, la suma de las dos primeras soluciones positivas de la ecuación:

$\cos x = \frac{1}{5}$

Empleando la expresión general para el coseno:

$x_G = 2k\pi \pm VP; k \in \mathbb{Z}$

$x_G = 2k\pi \pm \arccos\frac{1}{5}; k \in \mathbb{Z}$

$\Rightarrow x = 2k\pi \pm \arccos\frac{1}{5}; k \in \mathbb{Z}$

Evaluando:

Para: $k = 0$

$x = -\arccos\frac{1}{5} \lor x = \arccos\frac{1}{5}$

Para: $k = 1$

$x = 2\pi - \arccos\frac{1}{5} \lor x = 2\pi + \arccos\frac{1}{5}$

Luego, las dos primeras soluciones positivas son: $\arccos\frac{1}{5}$ y $2\pi - \arccos\frac{1}{5}$

$\Rightarrow \left(\arccos\frac{1}{5}\right) + \left(2\pi - \arccos\frac{1}{5}\right) = 2\pi = 360°$

Clave E

6. Piden, la suma de las dos primeras soluciones positivas de la ecuación:

$\cos x = -\frac{\sqrt{2}}{2}$

Usando la expresión general para el coseno:

$x_G = 2k\pi \pm VP; k \in \mathbb{Z}$

$x_G = 2k\pi \pm \arccos\left(-\frac{\sqrt{2}}{2}\right); k \in \mathbb{Z}$

$x_G = 2k\pi \pm \left(\frac{3\pi}{4}\right); k \in \mathbb{Z}$

$\Rightarrow x = 2k\pi \pm \frac{3\pi}{4}; k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow x = -\frac{3\pi}{4} \lor x = \frac{3\pi}{4}$

Para: $k = 1 \Rightarrow x = \frac{5\pi}{4} \lor x = \frac{11\pi}{4}$

Luego, las dos primeras soluciones positivas son: $\frac{3\pi}{4}$ y $\frac{5\pi}{4}$

$\Rightarrow \frac{3\pi}{4} + \frac{5\pi}{4} = 2\pi = 360°$

Clave D

7. Por dato: $\tan x = 1$

$\Rightarrow VP = \arctan 1 = \frac{\pi}{4}$

Empleando la expresión general para la tangente:

$x_G = k\pi + VP; k \in \mathbb{Z}$

$\Rightarrow x_G = k\pi + \frac{\pi}{4}; k \in \mathbb{Z}$

Evaluando:

Para: $k = -1 \Rightarrow x = -\frac{3\pi}{4} = -135°$

Para: $k = 0 \Rightarrow x = \frac{\pi}{4} = 45°$

Para: $k = 1 \Rightarrow x = \frac{5\pi}{4} = 225°$

Luego, las dos primeras soluciones positivas son: 45° y 225°.

Piden: la suma de las dos primeras soluciones positivas.

$\Rightarrow 45° + 225° = 270°$

Clave C

8. Por dato: $\tan x = -\sqrt{3}$

$\Rightarrow VP = \arctan(-\sqrt{3}) = -\frac{\pi}{3}$

Empleando la expresión general para la tangente:

$x_G = k\pi + VP; k \in \mathbb{Z}$

$\Rightarrow x = k\pi - \frac{\pi}{3}; k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow x = -\frac{\pi}{3} = -60°$

Para: $k = 1 \Rightarrow x = \frac{2\pi}{3} = 120°$

Para: $k = 2 \Rightarrow x = \frac{5\pi}{3} = 300°$

Luego, las dos primeras soluciones positivas son: 120° y 300°.

Piden: la suma de las dos primeras soluciones positivas.

$\Rightarrow 120° + 300° = 420°$

Clave E

9. Piden, la solución principal de la ecuación:

$\frac{\operatorname{sen}3x}{\operatorname{sen}x} = 1 \Rightarrow \frac{\operatorname{sen}x(2\cos 2x + 1)}{\operatorname{sen}x} = 1$

$2\cos 2x + 1 = 1; \operatorname{sen}x \neq 0$

$2\cos 2x = 0$

$\cos 2x = 0$

Empleando la expresión general para el coseno:

$x_G = 2k\pi \pm VP; k \in \mathbb{Z}$

$\Rightarrow 2x_G = 2k\pi \pm \arccos 0; k \in \mathbb{Z}$

$2x_G = 2k\pi \pm \frac{\pi}{2}; k \in \mathbb{Z}$

$\Rightarrow x_G = k\pi \pm \frac{\pi}{4}; k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow x = -\frac{\pi}{4} \lor x = \frac{\pi}{4}$

Para: $k = 1 \Rightarrow x = \frac{3\pi}{4} \lor x = \frac{5\pi}{4}$

Observamos que $\frac{\pi}{4}$ es el menor valor no negativo que satisface la igualdad original.

Por lo tanto, la solución principal de la ecuación es: $\frac{\pi}{4}$

Clave A

10. Piden, la suma de soluciones de la ecuación:

$(\tan 2x - 1)(\operatorname{sen}x - 1) = 0; \langle 0; \pi\rangle$

Igualando cada factor a cero:

- $\tan 2x - 1 = 0 \Rightarrow \tan 2x = 1$

  $x_G = k\pi + VP; k \in \mathbb{Z}$

  $\Rightarrow 2x_G = k\pi + \arctan 1 = k\pi + \frac{\pi}{4}$

  $\Rightarrow x_G = \frac{k\pi}{2} + \frac{\pi}{8}; k \in \mathbb{Z}$

  Evaluando:

  Para: $k = -1 \Rightarrow x = -\frac{3\pi}{8}$

  Para: $k = 0 \Rightarrow x = \frac{\pi}{8}$

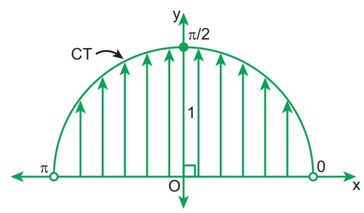
  Para: $k = 1 \Rightarrow x = \frac{5\pi}{8}$

  Para: $k = 2 \Rightarrow x = \frac{9\pi}{8}$

  En $\langle 0; \pi\rangle$ las soluciones son: $\frac{\pi}{8}$ y $\frac{5\pi}{8}$.

- $\operatorname{sen}x - 1 = 0 \Rightarrow \operatorname{sen}x = 1$

  Analizando en la CT, en $\langle 0; \pi\rangle$:



Observamos que $x = \frac{\pi}{2}$ es la única solución.

Entonces, para la ecuación original las soluciones en $\langle 0; \pi\rangle$, son: $\left\{\frac{\pi}{8}; \frac{\pi}{2}; \frac{5\pi}{8}\right\}$

$\Rightarrow \frac{\pi}{8} + \frac{\pi}{2} + \frac{5\pi}{8} = \frac{5\pi}{4}$

Clave E

Nivel 2 (página 78) Unidad 4

Comunicación matemática

11.

12.

Razonamiento y demostración

13. Piden, la suma de las dos primeras soluciones positivas de la ecuación:

$\operatorname{sen}2x\cos 2x = \frac{\sqrt{3}}{4}$

$2\operatorname{sen}2x\cos 2x = \frac{\sqrt{3}}{4}(2)$

$\operatorname{sen}4x = \frac{\sqrt{3}}{2}$

Usando la expresión general para el seno:

$x_G = k\pi + (-1)^k VP;\ k \in \mathbb{Z}$

$\Rightarrow 4x_G = k\pi + (-1)^k \text{arcsen}\frac{\sqrt{3}}{2};\ k \in \mathbb{Z}$

$4x_G = k\pi + (-1)^k \frac{\pi}{3};\ k \in \mathbb{Z}$

$\Rightarrow x_G = \frac{k\pi}{4} + (-1)^k \frac{\pi}{12};\ k \in \mathbb{Z}$

Evaluando:

Para: $k = -1 \Rightarrow x = -\frac{\pi}{3} = -60°$

Para: $k = 0 \quad \Rightarrow x = \frac{\pi}{12} = 15°$

Para: $k = 1 \quad \Rightarrow x = \frac{\pi}{6} = 30°$

Luego, las dos primeras soluciones positivas son: 15° y 30°.

$\Rightarrow 15° + 30° = 45°$

Clave B

14. Por dato:

$$\frac{\text{sen}7x - \text{sen}x}{\cos 4x} = \frac{1}{2}$$

$$\frac{2\cos\left(\frac{7x+x}{2}\right)\text{sen}\left(\frac{7x-x}{2}\right)}{\cos 4x} = \frac{1}{2}$$

$$\frac{2\cos 4x\text{sen}3x}{\cos 4x} = \frac{1}{2}$$

Luego:

$2\text{sen}3x = \frac{1}{2};\ \cos4x \neq 0$

$\text{sen}3x = \frac{1}{4}$

$3x = \text{arcsen}\frac{1}{4} \Rightarrow x = \frac{1}{3}\text{arcsen}\frac{1}{4}$

Cuando $x = \frac{1}{3}\text{arcsen}\frac{1}{4}$, el cos4x es diferente de cero.

$\therefore x = \frac{1}{3}\text{arcsen}\frac{1}{4}$

Clave B

15. Se tiene el sistema:

$2\text{sen}x + \cos^2 y = a \qquad ...(1)$

$\sqrt{3}\cos x + \sqrt{2}\cos y = b \qquad ...(2)$

Por dato: 30° y 45° son valores que toman x e y en el sistema.

Evaluando en (1):

$2\text{sen}30° + \cos^2 45° = a$

$2\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)^2 = a$

$1 + \frac{1}{2} = a$

$\Rightarrow a = \frac{3}{2}$

Evaluando en (2):

$\sqrt{3}\cos30° + \sqrt{2}\cos45° = b$

$\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) + \sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = b$

$\frac{3}{2} + 1 = b$

$\Rightarrow b = \frac{5}{2}$

Piden:

$a + b = \left(\frac{3}{2}\right) + \left(\frac{5}{2}\right) = 4$

$\therefore a + b = 4$

Clave C

16. Por dato: $x \in \left\langle \frac{\pi}{6}; \frac{\pi}{3} \right\rangle$

Además:

$\text{sen}x = \sqrt{2} - \cos x$

$\text{sen}x + \cos x = \sqrt{2}$

$\sqrt{2}\,\text{sen}\left(x + \frac{\pi}{4}\right) = \sqrt{2}$

$\Rightarrow \text{sen}\left(x + \frac{\pi}{4}\right) = 1$

Empleando la expresión general para el seno:

$x_G = k\pi + (-1)^k VP;\ k \in \mathbb{Z}$

$x_G = k\pi + (-1)^k \text{arcsen}1;\ k \in \mathbb{Z}$

$\left(x + \frac{\pi}{4}\right) = k\pi + (-1)^k \frac{\pi}{2};\ k \in \mathbb{Z}$

$\Rightarrow x = k\pi + (-1)^k \frac{\pi}{2} - \frac{\pi}{4};\ k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow x = \frac{\pi}{4} \in \left\langle \frac{\pi}{6}; \frac{\pi}{3} \right\rangle$

$\therefore x = \frac{\pi}{4}$

Clave A

17. Por dato: $x \in \langle 0°; 180° \rangle$

Además:

$\text{sen}(x + 40°) + \text{sen}(50° - x) = 0$

Por transformaciones trigonométricas se tiene:

$2\text{sen}45°\cos(x - 5°) = 0$

$\Rightarrow \cos\left(x - \frac{\pi}{36}\right) = 0$

Usando la expresión general para el coseno:

$x_G = 2k\pi \pm VP;\ k \in \mathbb{Z}$

$x_G = 2k\pi \pm \arccos 0;\ k \in \mathbb{Z}$

$\left(x - \frac{\pi}{36}\right) = 2k\pi \pm \frac{\pi}{2};\ k \in \mathbb{Z}$

$\Rightarrow x = 2k\pi \pm \frac{\pi}{2} + \frac{\pi}{36};\ k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow x = -\frac{17\pi}{36} \quad \vee \quad x = \frac{19\pi}{36}$

Para: $k = 1 \Rightarrow x = \frac{55\pi}{36} \quad \vee \quad x = \frac{91\pi}{36}$

El único valor que se encuentra en $\langle 0°; 180° \rangle$ es $\frac{19\pi}{36} = 95°$

$\therefore x = 95°$

Clave E

18. Piden, el número de soluciones de la ecuación.

$\text{sen}^4 x + \cos^4 x = \frac{3}{4};\ x \in \langle 0; 2\pi \rangle$

$1 - 2\text{sen}^2 x\cos^2 x = \frac{3}{4}$

$1 - \frac{3}{4} = 2\text{sen}^2 x\cos^2 x$

$\frac{1}{4} = 2\text{sen}^2 x\cos^2 x$

$\frac{1}{2} = 4\text{sen}^2 x\cos^2 x$

$\frac{1}{2} = (2\text{sen}x\cos x)^2$

$\frac{1}{2} = \text{sen}^2 2x$

$1 = 2\text{sen}^2 2x$

$1 = 1 - \cos 4x$

$\Rightarrow \cos 4x = 0$

Usando la expresión general para el coseno:

$x_G = 2k\pi \pm VP;\ k \in \mathbb{Z}$

$\Rightarrow 4x_G = 2k\pi \pm \arccos 0;\ k \in \mathbb{Z}$

$4x_G = 2k\pi \pm \frac{\pi}{2};\ k \in \mathbb{Z}$

$\Rightarrow x_G = \frac{k\pi}{2} \pm \frac{\pi}{8};\ k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow x = -\frac{\pi}{8} \quad \vee \quad x = \frac{\pi}{8}$

Para: $k = 1 \Rightarrow x = \frac{3\pi}{8} \quad \vee \quad x = \frac{5\pi}{8}$

Para: $k = 2 \Rightarrow x = \frac{7\pi}{8} \quad \vee \quad x = \frac{9\pi}{8}$

Para: $k = 3 \Rightarrow x = \frac{11\pi}{8} \quad \vee \quad x = \frac{13\pi}{8}$

Para: $k = 4 \Rightarrow x = \frac{15\pi}{8} \quad \vee \quad x = \frac{17\pi}{8}$

Las soluciones en $\langle 0; 2\pi \rangle$ son:

$x = \left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$

Por lo tanto, son 8 soluciones.

Clave B

19. Piden, la suma de las dos primeras soluciones positivas de la ecuación:

$\cos5x\cos x - \text{sen}5x\text{sen}x = \frac{1}{2}$

$\cos(5x + x) = \frac{1}{2}$

$\Rightarrow \cos 6x = \frac{1}{2}$

Usando la expresión general para el coseno:

$x_G = 2k\pi \pm VP;\ k \in \mathbb{Z}$

$\Rightarrow 6x_G = 2k\pi \pm \arccos\frac{1}{2};\ k \in \mathbb{Z}$

$6x_G = 2k\pi \pm \left(\frac{\pi}{3}\right);\ k \in \mathbb{Z}$

$\Rightarrow x_G = \frac{k\pi}{3} \pm \frac{\pi}{18};\ k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow x = -\frac{\pi}{18} \quad \vee \quad x = \frac{\pi}{18}$

Para: $k = 1 \Rightarrow x = \frac{5\pi}{18} \quad \vee \quad x = \frac{7\pi}{18}$

Luego, las dos primeras soluciones positivas son: $\frac{\pi}{18}$ y $\frac{5\pi}{18}$

$\Rightarrow \frac{\pi}{18} + \frac{5\pi}{18} = \frac{\pi}{3} = 60°$

Clave E

20. Por dato:

cosxtanx + 2senx = 1,5

$\Rightarrow \cos x\left(\frac{\text{sen}x}{\cos x}\right) + 2\text{sen}x = \frac{3}{2}$

$\text{sen}x + 2\text{sen}x = \frac{3}{2}; \cos x \neq 0$

$3\text{sen}x = \frac{3}{2}$

$\text{sen}x = \frac{1}{2}$

Empleando la expresión general para el seno:

$x_G = k\pi + (-1)^k VP; k \in \mathbb{Z}$

Donde: $VP = \text{arcsen}\frac{1}{2} = \frac{\pi}{6}$

$\Rightarrow x = k\pi + (-1)^k \frac{\pi}{6};\ k \in \mathbb{Z}$

Evaluando:

Para: $k = -1 \Rightarrow x = -\frac{7\pi}{6} = -210°$

Para: $k = 0 \Rightarrow x = \frac{\pi}{6} = 30°$

Para: $k = 1 \Rightarrow x = \frac{5\pi}{6} = 150°$

Luego, las dos primeras soluciones positivas son 30° y 150°, además en estos valores el coseno es diferente de cero.
Piden la suma de las dos primeras soluciones positivas.

$\Rightarrow 30° + 150° = 180°$

Clave C

Nivel 3 (página 79) Unidad 4

Comunicación matemática

21.

22.

Razonamiento y demostración

23. Por dato:

sen7x − senx = cos4x

Empleando transformaciones trigonométricas:

2cos4x . sen3x = cos4x

2cos4xsen3x − cos4x = 0

cos4x(2sen3x − 1) = 0

$\Rightarrow \cos 4x = 0 \ \vee \ \text{sen}3x = \frac{1}{2}$

Usando la expresión general para el coseno:

$4x = 2k\pi \pm \arccos 0; k \in \mathbb{Z}$

$4x = 2k\pi \pm \frac{\pi}{2}$

$\Rightarrow x = \frac{k\pi}{2} \pm \frac{\pi}{8}; k \in \mathbb{Z}$ ...(I)

Usando la expresión general para el seno:

$3x = k\pi + (-1)^k \text{arcsen}\frac{1}{2}; k \in \mathbb{Z}$

$3x = k\pi + (-1)^k \frac{\pi}{6}$

$\Rightarrow x = \frac{k\pi}{3} + (-1)^k \frac{\pi}{18}; k \in \mathbb{Z}$ ...(II)

Evaluando las expresiones (I) y (II):

Para: $k = 0 \Rightarrow x = \left\{-\frac{\pi}{8}; \frac{\pi}{8}; \frac{\pi}{18}\right\}$

Para. $k = 1 \Rightarrow x = \left\{\frac{5\pi}{18}; \frac{3\pi}{8}; \frac{5\pi}{8}\right\}$

Luego, las dos primeras soluciones positivas son: $\frac{\pi}{8}$ y $\frac{\pi}{18}$.

Piden la suma de las dos primeras soluciones positivas.

$\Rightarrow \frac{\pi}{8} + \frac{\pi}{18} = \frac{13\pi}{72}$

Clave C

24. Piden la suma de las dos primeras soluciones positivas de la ecuación:

cos5x + cosx = cos2x

$2\cos\left(\frac{5x + x}{2}\right)\cos\left(\frac{5x - x}{2}\right) = \cos 2x$

2cos3xcos2x = cos2x

2cos3xcos2x − cos2x = 0

cos2x(2cos3x − 1) = 0

$\Rightarrow \cos 2x = 0 \ \vee \ \cos 3x = \frac{1}{2}$

Usando la expresión general para el coseno en ambos casos se tiene:

$2x = 2k\pi \pm \arccos 0 \ \vee \ 3x = 2k\pi \pm \arccos\frac{1}{2}$

$2x = 2k\pi \pm \frac{\pi}{2} \ \vee \ 3x = 2k\pi \pm \frac{\pi}{3}$

$\Rightarrow x = k\pi \pm \frac{\pi}{4}; k \in \mathbb{Z} \ \vee \ x = \frac{2k\pi}{3} \pm \frac{\pi}{9}; k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow x = \left\{-\frac{\pi}{4}; -\frac{\pi}{9}; \frac{\pi}{9}; \frac{\pi}{4}\right\}$

Para: $k = 1 \Rightarrow x = \left\{\frac{5\pi}{9}; \frac{7\pi}{9}; \frac{3\pi}{4}; \frac{5\pi}{4}\right\}$

Luego, las dos primeras soluciones positivas son: $\frac{\pi}{9}$ y $\frac{\pi}{4}$.

$\Rightarrow \frac{\pi}{9} + \frac{\pi}{4} = \frac{13\pi}{36} = 65°$

Clave D

25. Piden, la suma de las dos primeras soluciones positivas de la ecuación:

$\text{sen}^2(x - 45°) - \text{sen}^2(x - 15°) = \frac{\sqrt{3}}{4}$

Sabemos por ángulo compuesto:

$\text{sen}(\alpha + \beta)\text{sen}(\alpha - \beta) = \text{sen}^2\alpha - \text{sen}^2\beta$

Entonces:

$\text{sen}(x - 45° + x - 15°)\text{sen}(x - 45° - x + 15°) = \frac{\sqrt{3}}{4}$

$\text{sen}(2x - 60°)\text{sen}(-30°) = \frac{\sqrt{3}}{4}$

$\text{sen}\left(2x - \frac{\pi}{3}\right)\left(-\frac{1}{2}\right) = \frac{\sqrt{3}}{4}$

$\Rightarrow \text{sen}\left(2x - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

Empleando la expresión general para el seno:

$x_G = k\pi + (-1)^k VP; k \in \mathbb{Z}$

$x_G = k\pi + (-1)^k \text{arcsen}\left(-\frac{\sqrt{3}}{2}\right)$

$x_G = k\pi + (-1)^k\left(-\frac{\pi}{3}\right)$

$\left(2x - \frac{\pi}{3}\right) = k\pi - (-1)^k \frac{\pi}{3}$

$\Rightarrow x = \frac{k\pi}{2} - (-1)^k \frac{\pi}{6} + \frac{\pi}{6}; k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow x = 0$

Para: $k = 1 \Rightarrow x = \frac{5\pi}{6} = 150°$

Para: $k = 2 \Rightarrow x = \pi = 180°$

Luego, las dos primeras soluciones positivas son: 150° y 180°.

$\Rightarrow 150° + 180° = 330°$

Clave E

26. Se tiene:

$16(1 - \text{sen}^2\theta)(1 - \cos^2\theta) - 1 = 0$

$16(\cos^2\theta)(\text{sen}^2\theta) - 1 = 0$

$4(2\text{sen}\theta\cos\theta)^2 - 1 = 0$

$4(\text{sen}2\theta)^2 - 1 = 0$

$2(2\text{sen}^2 2\theta) = 1$

$2(1 - \cos 4\theta) = 1$

$\Rightarrow \cos 4\theta = \frac{1}{2}$

Usando la expresión general para el coseno:

$\theta_G = 2k\pi \pm VP; k \in \mathbb{Z}$

$\theta_G = 2k\pi \pm \arccos\frac{1}{2}$

$4\theta = 2k\pi \pm \frac{\pi}{3}$

$\Rightarrow \theta = \frac{k\pi}{2} \pm \frac{\pi}{12}; k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow \theta = -\frac{\pi}{2} \ \vee \ \theta = \frac{\pi}{12}$

Para: $k = 1 \Rightarrow \theta = \frac{5\pi}{12} \ \vee \ \theta = \frac{7\pi}{12}$

Luego, las soluciones que pertenecen al intervalo de $\left\langle 0; \frac{\pi}{2}\right\rangle$ son: $\frac{\pi}{12}$ y $\frac{5\pi}{12}$.

Piden la suma de soluciones en $\left\langle 0; \frac{\pi}{2}\right\rangle$.

$\therefore \frac{\pi}{12} + \frac{5\pi}{12} = \frac{\pi}{2}$

Clave A

27. Piden, la suma de las soluciones de la ecuación:

$\tan 4x - \tan 2x = 0; x \in \langle 0; \pi\rangle$

Empleando las identidades del ángulo doble:

$\frac{2\tan 2x}{1 - \tan^2 2x} - \tan 2x = 0$

$\tan 2x\left[\frac{2}{1 - \tan^2 2x} - 1\right] = 0$

$\tan 2x\left[\frac{1 + \tan^2 2x}{1 - \tan^2 2x}\right] = 0$

$\tan 2x(\sec 4x) = 0$

$\frac{\tan 2x}{\cos 4x} = 0$

$\Rightarrow \tan 2x = 0; \cos 4x \neq 0 \Rightarrow x \neq (2k + 1)\frac{\pi}{8}; k \in \mathbb{Z}$

Usando la expresión general para la tangente:

$x_G = k\pi + VP; k \in \mathbb{Z}$

$2x_G = k\pi + \arctan 0$

$2x = k\pi + 0$

$\Rightarrow x = \frac{k\pi}{2}; k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow x = 0$

Para: $k = 1 \Rightarrow x = \frac{\pi}{2}$

Para: $k = 2 \Rightarrow x = \pi$

Observamos que solo existe una solución en el intervalo de $\langle 0; \pi \rangle$ y que satisface la igualdad original que es $\frac{\pi}{2}$.

Clave A

28. Piden, la suma de las dos primeras soluciones positivas de la ecuación:

$$\frac{\cos 2x + \mathrm{sen}^2 x}{\cos 2x - \cos^2 x} = -3$$

$$\frac{(1 - 2\mathrm{sen}^2 x) + \mathrm{sen}^2 x}{(2\cos^2 x - 1) - \cos^2 x} = -3$$

$$\frac{1 - \mathrm{sen}^2 x}{\cos^2 x - 1} = -3$$

$$1 - \mathrm{sen}^2 x = 3 - 3\cos^2 x$$

$$\cos^2 x = 3 - 3\cos^2 x$$

$$4\cos^2 x = 3$$

$$4\cos^2 x - 2 = 1$$

$$2(2\cos^2 x - 1) = 1$$

$$2(\cos 2x) = 1$$

$$\Rightarrow \cos 2x = \frac{1}{2}$$

Usando la expresión general para el coseno:

$x_G = 2k\pi \pm VP;\ k \in \mathbb{Z}$

$2x_G = 2k\pi \pm \arccos\frac{1}{2}$

$2x = 2k\pi \pm \frac{\pi}{3}$

$\Rightarrow x = k\pi \pm \frac{\pi}{6};\ k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow x = -\frac{\pi}{6} \quad \vee \quad x = \frac{\pi}{6}$

Para: $k = 1 \Rightarrow x = \frac{5\pi}{6} \quad \vee \quad x = \frac{7\pi}{6}$

Luego, las dos primeras soluciones positivas son $\frac{\pi}{6}$ y $\frac{5\pi}{6}$ y ambas son admisibles para la ecuación original.

$\Rightarrow \frac{\pi}{6} + \frac{5\pi}{6} = \pi = 180°$

Clave C

29. Piden, la suma de las dos primeras soluciones positivas del sistema:

$\mathrm{sen}x + \cos y = \frac{1}{2}$ ...(1)

$\mathrm{sen}x - \cos y = -\frac{1}{2}$ ...(2)

Sumando (1) y (2) se tiene:

$2\mathrm{sen}x = 0 \Rightarrow \mathrm{sen}x = 0$

Empleando la expresión general para el seno:

$x_G = k\pi + (-1)^k \mathrm{arcsen}0;\ k \in \mathbb{Z}$

$x = k\pi + (-1)^k(0)$

$\Rightarrow x = k\pi;\ k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow x = 0$

Para: $k = 1 \Rightarrow x = \pi$

Restando (1) y (2) se tiene:

$2\cos y = 1 \Rightarrow \cos y = \frac{1}{2}$

Empleando la expresión general para el coseno:

$y_G = 2k\pi \pm \arccos\frac{1}{2};\ k \in \mathbb{Z}$

$\Rightarrow y = 2k\pi \pm \frac{\pi}{3};\ k \in \mathbb{Z}$

Evaluando:

Para: $k = 0 \Rightarrow y = -\frac{\pi}{3} \quad \vee \quad y = \frac{\pi}{3}$

Para: $k = 1 \Rightarrow y = \frac{5\pi}{3} \quad \vee \quad y = \frac{7\pi}{3}$

Luego, las dos primeras soluciones positivas son: $x = \pi \ \wedge \ y = \frac{\pi}{3}$ respectivamente.

$\therefore x + y = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

Clave C

30. Piden, la suma de valores de y en $\langle 0; 2\pi \rangle$ del sistema:

$x + 2y = \frac{\pi}{2}$ ...(1)

$\mathrm{sen}(x + y) + \cos y = \frac{1}{3}$ ...(2)

Luego, de (1): $x = \frac{\pi}{2} - 2y$

Reemplazando en (2):

$$\mathrm{sen}\left(\frac{\pi}{2} - 2y + y\right) + \cos y = \frac{1}{3}$$

$$\mathrm{sen}\left(\frac{\pi}{2} - y\right) + \cos y = \frac{1}{3}$$

$$\cos y + \cos y = \frac{1}{3}$$

$$2\cos y = \frac{1}{3}$$

$$\Rightarrow \cos y = \frac{1}{6}$$

Empleando la expresión general para el coseno:

$y_G = 2k\pi \pm VP;\ k \in \mathbb{Z}$

$y_G = 2k\pi \pm \arccos\frac{1}{6}$

$\Rightarrow y = 2k\pi \pm \arccos\frac{1}{6};\ k \in \mathbb{Z}$

Evaluando:

Para: $k = 0$

$\Rightarrow y = -\arccos\frac{1}{6} \ \vee \ y = \arccos\frac{1}{6}$

Para: $k = 1$

$\Rightarrow y = 2\pi - \arccos\frac{1}{6} \vee y = 2\pi + \arccos\frac{1}{6}$

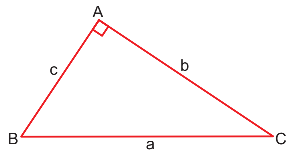
Luego, las soluciones para y en $\langle 0; 2\pi \rangle$ son: $\arccos\frac{1}{6}$ y $2\pi - \arccos\frac{1}{6}$.

$\therefore (\arccos\frac{1}{6}) + (2\pi - \arccos\frac{1}{6}) = 2\pi$

Clave B

APLICAMOS LO APRENDIDO (página 80) Unidad 4

1.



Por dato: $\frac{1}{b^2}+\frac{1}{c^2}=\frac{10}{a^2}$

$\Rightarrow \frac{c^2+b^2}{b^2c^2}=\frac{10}{a^2}$

Por el teorema de Pitágoras: $c^2+b^2=a^2$

$\Rightarrow \frac{(a^2)}{b^2c^2}=\frac{10}{a^2}\Rightarrow a^4=10b^2c^2$

$\Rightarrow a^2=\sqrt{10}\,bc$

Empleando ley de senos:

$(2R\text{senA})^2=\sqrt{10}\,(2R\text{senB})(2R\text{senC})$

$4R^2\text{sen}^2A=\sqrt{10}\,.\,4R^2\text{senBsenC}$

$\text{sen}^2A=\sqrt{10}\,\text{senBsenC}$

Pero: senA = sen90° = 1

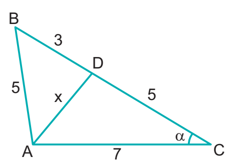
$\Rightarrow (1)^2=\sqrt{10}\,\text{senBsenC}$

$\frac{1}{\sqrt{10}}=\text{senBsenC}$

$\therefore \text{senBsenC}=\frac{\sqrt{10}}{10}$

Clave A

2.



En el ΔABC por ley de cosenos:

$5^2=8^2+7^2-2(8)(7)\cos\alpha$

$\Rightarrow 112\cos\alpha=88$

$\Rightarrow \cos\alpha=\frac{11}{14}$

En el ΔADC por ley de cosenos:

$x^2=5^2+7^2-2(5)(7)\cos\alpha$

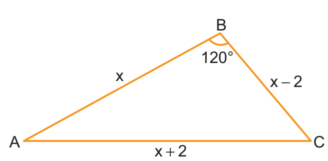
$\Rightarrow x^2=74-70\left(\frac{11}{14}\right)$

$x^2=19$

$\therefore x=\sqrt{19}$

Clave C

3.



Sea x: un número impar.

En el ΔABC por ley de cosenos:

$(x+2)^2=(x-2)^2+x^2-2(x-2)(x)\cos120°$

Luego:

$(x+2)^2-(x-2)^2=x^2-2(x-2)(x)\left(-\frac{1}{2}\right)$

$8x=x^2+(x-2)x$

$8=x+x-2$

$10=2x$

$\Rightarrow x=5$

La medida de los lados será: 3; 5 y 7.

Por lo tanto, el lado mayor mide 7.

Clave B

4. Por dato: abc = 32 cm^3

Además: $(\text{senA})(\text{senB})(\text{senC})=\frac{1}{2}$

De la ley de senos se tiene:

a = 2RsenA; b = 2RsenB; c = 2RsenC

Donde R es el circunradio del ΔABC.

Entonces:

(2RsenA)(2RsenB)(2RsenC) = 32

$8R^3\underbrace{(\text{senA})(\text{senB})(\text{senC})}_{\left(\frac{1}{2}\right)}=32$

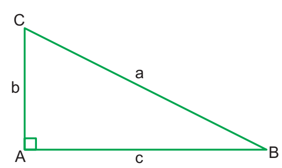
$\Rightarrow 8R^3\left(\frac{1}{2}\right)=32$

$4R^3=32\Rightarrow R^3=8$

$\therefore R=2$ cm

Clave A

5.



Del gráfico: A = 90° $\wedge$ B + C = 90°

Piden, expresar M en términos de los lados:

$M=\frac{\tan2B}{\cos(B-C)}=\frac{\tan2B}{\cos(B-(90°-B))}$

$M=\frac{\tan2B}{\cos(2B-90°)}=\frac{\tan2B}{\cos(90°-2B)}=\frac{\tan2B}{\text{sen}2B}$

$M=\left(\frac{\text{sen}2B}{\cos2B}\right)\left(\frac{1}{\text{sen}2B}\right)=\frac{1}{\cos2B}$

$M=\frac{1}{\cos^2B-\text{sen}^2B}=\frac{1}{\left(\frac{c}{a}\right)^2-\left(\frac{b}{a}\right)^2}$

$M=\frac{1}{\frac{c^2-b^2}{a^2}}$

$\therefore M=\frac{a^2}{c^2-b^2}$

Clave C

6. Por dato:

$a^2=b^2+c^2-\frac{2}{3}bc$ ...(1)

Por ley de cosenos:

$a^2=b^2+c^2-2bc\cos A$ ...(2)

Comparando (1) y (2):

$\Rightarrow -\frac{2}{3}bc=-2bc\cos A$

$\frac{2}{3}=2\cos A$

$\Rightarrow \cos A=\frac{1}{3}\Rightarrow \sec A=3$

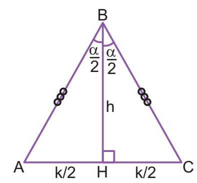
$\Rightarrow \sec^2A=9\Rightarrow 1+\tan^2A=9$

$\Rightarrow \tan^2A=8$

$\therefore \tan A=2\sqrt{2}$

Clave C

7.



En el ⊿AHB: $h=\frac{k}{2}\cot\left(\frac{\alpha}{2}\right)$

Piden:

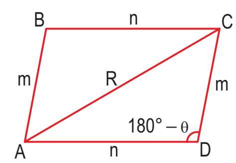
$A_{\Delta ABC}=\frac{(AC)(BH)}{2}=\frac{(k)(h)}{2}$

$\Rightarrow A_{\Delta ABC}=\frac{k\,.\,\frac{k}{2}\cot\left(\frac{\alpha}{2}\right)}{2}$

$\therefore A_{\Delta ABC}=\frac{k^2}{4}\cot\left(\frac{\alpha}{2}\right)$

Clave A

8.



Por dato: ABCD es un paralelogramo.

En el ΔADC por ley de cosenos:

$R^2=m^2+n^2-2mn\cos(180°-\theta)$

$R^2=m^2+n^2-2mn(-\cos\theta)$

$R^2=m^2+n^2+2mn\cos\theta$

$\therefore AC=\sqrt{m^2+n^2+2mn\cos\theta}$

Clave B

9. Piden:

$N=\frac{a\cos B+b\cos A}{b\cos C+c\cos B}$

Por la ley de proyecciones:

c = acosB + bcosA

a = bcosC + ccosB

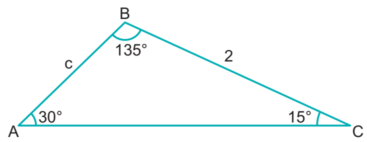
Reemplazando en la expresión N:

$N=\frac{a\cos B+b\cos A}{b\cos C+c\cos B}=\frac{c}{a}$

$\therefore N=\frac{c}{a}$

Clave C

10. Por dato:



Se cumple:

$m\angle A + m\angle B + m\angle C = 180°$

$30° + 135° + m\angle C = 180°$

$m\angle C = 15°$

Luego, por ley de senos:

$$\frac{c}{\text{sen}15°} = \frac{2}{\text{sen}30°} \Rightarrow \frac{c}{\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)} = \frac{2}{\left(\frac{1}{2}\right)}$$

$$\Rightarrow c = 4\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$$

$\therefore c = \sqrt{6}-\sqrt{2}$

Clave A

11. En un triángulo ABC, sus lados son:

$a = 33;\ b = 37;\ c = 40$

Piden: $m\angle B$

Entonces: $p = \dfrac{33+37+40}{2} = 55$

Luego:

$$\tan\frac{B}{2} = \sqrt{\frac{(p-a)(p-c)}{p(p-b)}}$$

$$\tan\frac{B}{2} = \sqrt{\frac{(55-33)(55-40)}{55(55-37)}}$$

$$\tan\frac{B}{2} = \sqrt{\frac{(22)(15)}{55(18)}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan\frac{B}{2} = \frac{\sqrt{3}}{3}$$

Sabemos: $\tan 30° = \dfrac{\sqrt{3}}{3}$

$$\Rightarrow \frac{B}{2} = 30° \Rightarrow B = 60°$$

$\therefore m\angle B = 60°$

Clave E

12. En un triángulo ABC, por dato:

$a = 3b \ \wedge \cot\left(\dfrac{A-B}{2}\right) = 2$

$$\Rightarrow \tan\left(\frac{A-B}{2}\right) = \frac{1}{2}$$

Por ley de tangentes:

$$\frac{a+b}{a-b} = \frac{\tan\left(\frac{A+B}{2}\right)}{\tan\left(\frac{A-B}{2}\right)}$$

$$\frac{(3b)+b}{(3b)-b} = \frac{\tan\left(\frac{A+B}{2}\right)}{\left(\frac{1}{2}\right)}$$

$$\left(\frac{4b}{2b}\right)\frac{1}{2} = \tan\left(\frac{A+B}{2}\right)$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = 1$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \tan 45°$$

$$\Rightarrow \left(\frac{A+B}{2}\right) = 45° \Rightarrow A+B = 90°$$

Además se cumple: $A + B + C = 180°$

$\Rightarrow (90°) + C = 180° \Rightarrow C = 90°$

$\therefore m\angle C = 90°$

Clave B

13. Por dato:

$m\angle A + m\angle B = 74° \wedge m\angle A - m\angle B = 53°$

Piden: $\dfrac{a+b}{a-b}$

Por ley de tangentes:

$$\frac{a+b}{a-b} = \frac{\tan\left(\frac{A+B}{2}\right)}{\tan\left(\frac{A-B}{2}\right)}$$

$$\Rightarrow \frac{a+b}{a-b} = \frac{\tan\left(\frac{74°}{2}\right)}{\tan\left(\frac{53°}{2}\right)} = \frac{\tan 37°}{\tan\frac{53°}{2}}$$

$$\Rightarrow \frac{a+b}{a-b} = \frac{\left(\frac{3}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{6}{4}$$

$$\therefore \frac{a+b}{a-b} = \frac{3}{2}$$

Clave B

14. En un triángulo ABC se cumple:

$$(a+b+c)(b+c-a) = \frac{bc}{4}$$

$$(b+c+a)(b+c-a) = \frac{bc}{4}$$

$$(b+c)^2 - a^2 = \frac{bc}{4}$$

$$b^2 + c^2 + 2bc - a^2 = \frac{bc}{4}$$

$$\Rightarrow b^2 + c^2 + \frac{7}{4}bc = a^2 \qquad ...(1)$$

Por ley de cosenos, se cumple:

$a^2 = b^2 + c^2 - 2bc\cos A \qquad ...(2)$

Comparando (1) y (2) se tiene:

$$\frac{7}{4}bc = -2bc\cos A$$

$$\frac{7}{4} = -2\cos A$$

$$\therefore \cos A = -\frac{7}{8}$$

Clave B

PRACTIQUEMOS

Nivel 1 (página 82) Unidad 4

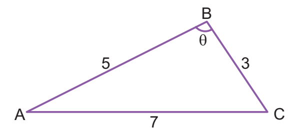
Comunicación matemática

1.

2.

Razonamiento y demostración

3.



Por ley de cosenos:

$7^2 = 3^2 + 5^2 - 2(3)(5)\cos\theta$

$\Rightarrow 30\cos\theta = -15$

$\therefore \cos\theta = -\dfrac{1}{2}$

Clave B

4. En un triángulo ABC:

$$N = \frac{ab\cos C + ac\cos B}{R\,\text{sen}A}$$

$$N = \frac{a(b\cos C + c\cos B)}{R\,\text{sen}A}$$

Por ley de proyecciones:

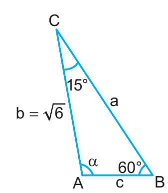
$a = b\cos C + c\cos B$

$$\Rightarrow N = \frac{a(a)}{R\,\text{sen}A} = \frac{a(2R\,\text{sen}A)}{R\,\text{sen}A}$$

$\therefore N = 2a$

Clave D

5.



En $\triangle ABC$ se cumple:

$\alpha + 15° + 60° = 180° \Rightarrow \alpha = 105°$

Por ley de senos:

$$\frac{a}{\text{sen}A} = \frac{b}{\text{sen}B} \Rightarrow \frac{a}{\text{sen}\alpha} = \frac{\sqrt{6}}{\text{sen}60°}$$

$$\Rightarrow a = \frac{\sqrt{6}\,\text{sen}105°}{\text{sen}60°} = \frac{\sqrt{6}\,\text{sen}(180° - 75°)}{\left(\frac{\sqrt{3}}{2}\right)}$$

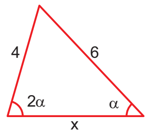
$$\Rightarrow a = 2\sqrt{2}\,\text{sen}75° = 2\sqrt{2}\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)$$

$$\Rightarrow a = \frac{4\sqrt{3}+4}{4}$$

$\therefore a = \sqrt{3} + 1$

Clave D

6.



Por ley de senos:

$$\frac{6}{\text{sen}2\alpha} = \frac{4}{\text{sen}\alpha} \Rightarrow \frac{6}{4} = \frac{\text{sen}2\alpha}{\text{sen}\alpha}$$

$$\Rightarrow \frac{2\text{sen}\alpha\,\cos\alpha}{\text{sen}\alpha} = \frac{3}{2} \Rightarrow \cos\alpha = \frac{3}{4}$$

Por ley de cosenos:

$$4^2 = 6^2 + x^2 - 2(6)(x)\cos\alpha$$

$$16 = 36 + x^2 - 12x\left(\frac{3}{4}\right)$$

Luego:

$$x^2 - 9x + 20 = 0$$

$$(x-5)(x-4) = 0$$

$$\Rightarrow x = 5 \ \vee \ x = 4$$

Si: $x = 4 \Rightarrow \alpha = 45° \Rightarrow \cos\alpha = \frac{\sqrt{2}}{2} \neq \frac{3}{4}$

$\therefore x = 5$

Clave E

7. En un triángulo ABC, simplificar:

$$N = \frac{b\cos B + c\cos C}{\cos(B-C)}$$

$$N = \frac{(2R\,\text{sen}B)\cos B + (2R\,\text{sen}C)\cos C}{\cos(B-C)}$$

$$N = \frac{R(2\text{sen}B\cos B + 2\text{sen}C\cos C)}{\cos(B-C)}$$

$$N = \frac{R(\text{sen}2B + \text{sen}2C)}{\cos(B-C)}$$

Empleando transformaciones trigonométricas:

$$N = \frac{R(2\text{sen}(B+C)\cos(B-C))}{\cos(B-C)}$$

$$\Rightarrow N = 2R\text{sen}(B+C) = 2R\text{sen}(180° - A)$$

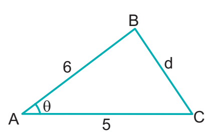
$$\Rightarrow N = 2R\text{sen}A$$

Por ley de senos: $a = 2R\text{sen}A$

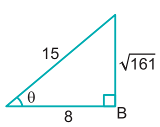
$\therefore N = a$

Clave A

8.



Por dato: $\tan\theta = \frac{\sqrt{161}}{8}$



$$\Rightarrow \cos\theta = \frac{8}{15}$$

En el $\triangle$ABC por ley de cosenos:

$$d^2 = 6^2 + 5^2 - 2(6)(5)\cos\theta$$

$$d^2 = 61 - 60\left(\frac{8}{15}\right) = 61 - 32$$

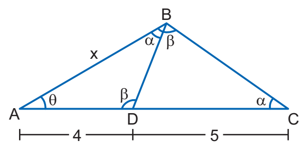
$$\Rightarrow d^2 = 29$$

$$\therefore d = \sqrt{29}$$

Clave C

Resolución de problemas

9. Del triángulo tenemos:



En el $\triangle$ABD (ley de senos)

$$\frac{x}{\text{sen}\beta} = \frac{4}{\text{sen}\alpha} \Rightarrow \frac{x}{4} = \frac{\text{sen}\beta}{\text{sen}\alpha}$$

En el $\triangle$ABC (ley de senos)

$$\frac{9}{\text{sen}\beta} = \frac{x}{\text{sen}\alpha} \Rightarrow \frac{9}{x} = \frac{\text{sen}\beta}{\text{sen}\alpha}$$

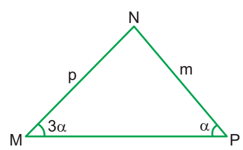
Luego tenemos:

$$\frac{x}{4} = \frac{9}{x} \Rightarrow x^2 = 36$$

$\therefore$ $x = 6$

Clave B

10. En el triángulo MNP, tenemos:



Por teorema de senos:

$$\frac{m}{\text{sen}3\alpha} = \frac{p}{\text{sen}\alpha} \Rightarrow \frac{m}{p} = \frac{\text{sen}3\alpha}{\text{sen}\alpha}$$

$$\frac{m}{p} = \frac{\text{sen}\alpha\,(2\cos2\alpha + 1)}{\text{sen}\alpha}$$

$$\frac{m}{p} = 2\cos2\alpha + 1$$

$$\frac{m-p}{p} = 2\cos2\alpha$$

$$\therefore\ \cos2\alpha = \frac{m-p}{2p}$$

Clave D

Nivel 2 (página 83) Unidad 4

Comunicación matemática

11. Tenemos:

$$\cos N = \frac{p^2 + m^2 - n^2}{2mp}$$

Si: $p^2 + m^2 - n^2 < 0 \ \Rightarrow\ \cos N < 0$

$$p^2 + m^2 < n^2$$

$\therefore$ N ángulo obtuso

Si: $p^2 + m^2 - n^2 = 0 \ \Rightarrow\ \cos N = 0$

$$p^2 + m^2 = n^2$$

$\therefore$ N ángulo recto

Si: $p^2 + m^2 - n^2 > 0 \ \Rightarrow\ \cos N > 0$

$$p^2 + m^2 > n^2$$

$\therefore$ N ángulo agudo

Entonces:

A) Ángulo agudo

B) Ángulo recto

C) Ángulo obtuso

12. Por teorema de proyecciones:

$a = b\cos B + c\cos C$ (F)

$b = a\cos C + c\cos A$ (V)

$c = a\cos B + b\cos A$ (V)

Por teorema de tangentes:

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{B-A}{2}\right)}{\tan\left(\frac{B+A}{2}\right)} \quad \text{(F)}$$

$$S = \left(\frac{ac}{2}\right)(\text{sen}B) \quad \text{(V)}$$

$\therefore$ Dos son falsas.

Clave E

Razonamiento y demostración

13. Por dato: $a^2 + b^2 + c^2 = 10$

Piden:

$E = ab\cos C + ac\cos B + bc\cos A$

De la ley de cosenos:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\Rightarrow ab\cos C = \frac{a^2 + b^2 - c^2}{2}$$

Análogamente:

$$ac\cos B = \frac{a^2 + c^2 - b^2}{2} \ \wedge\ bc\cos A = \frac{b^2 + c^2 - a^2}{2}$$

Reemplazando en E:

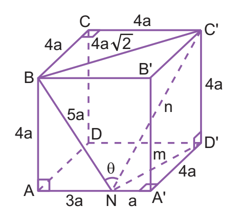
$$E = \frac{a^2 + b^2 - c^2}{2} + \frac{a^2 + c^2 - b^2}{2} + \frac{b^2 + c^2 - a^2}{2}$$

$$E = \frac{a^2 + b^2 + c^2}{2} = \frac{(10)}{2}$$

$\therefore E = 5$

Clave B

14.



En el ⊿NA'D' por el teorema de Pitágoras:

$$m^2 = a^2 + (4a)^2 \Rightarrow m^2 = 17a^2$$

En el ⊿ND'C' por el teorema de Pitágoras:

$$n^2 = m^2 + (4a)^2 = 17a^2 + 16a^2$$

$$n^2 = 33a^2 \Rightarrow n = \sqrt{33}\,a$$

En el $\triangle BNC'$ por ley de cosenos:

$$(4a\sqrt{2})^2 = (5a)^2 + (n)^2 - 2(5a)(n)\cos\theta$$

$$32a^2 = 25a^2 + n^2 - 10a(n)\cos\theta$$

$$7a^2 = (33a^2) - 10a(\sqrt{33}\,a)\cos\theta$$

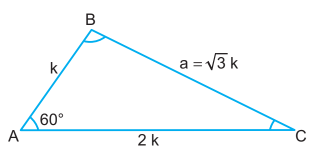
$$\Rightarrow 10\sqrt{33}\,a^2\cos\theta = 26a^2$$

$$\sqrt{33}\cos\theta = \frac{26}{10}$$

$$\therefore \sqrt{33}\cos\theta = \frac{13}{5}$$

Clave C

15.



Por ley de cosenos:

$$a^2 = k^2 + (2k)^2 - 2(k)(2k)\cos 60°$$

$$a^2 = k^2 + 4k^2 - 4k^2\left(\frac{1}{2}\right)$$

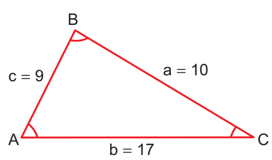
$$a^2 = 3k^2 \Rightarrow a = \sqrt{3}\,k$$

Se observa que el $\triangle ABC$ cumple con el teorema de Pitágoras, entonces:

$$m\angle C = 30° \wedge m\angle B = 90°$$

Clave C

16.



Por correspondencia triangular:

$$m\angle B > m\angle A > m\angle C$$

Piden: la tangente de la mitad del mayor ángulo.

$$\tan\frac{B}{2} = \sqrt{\frac{(p-a)(p-c)}{p(p-b)}}$$

$$p = \frac{a+b+c}{2} = \frac{10+17+9}{2} = 18$$

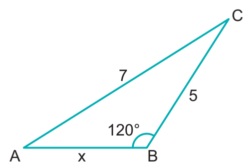
$$\Rightarrow \tan\frac{B}{2} = \sqrt{\frac{(18-10)(18-9)}{18(18-17)}}$$

$$\Rightarrow \tan\frac{B}{2} = \sqrt{\frac{(8)(9)}{18(1)}} = \sqrt{4}$$

$$\therefore \tan\frac{B}{2} = 2$$

Clave D

17.



En el $\triangle ABC$ por ley de cosenos:

$$7^2 = 5^2 + x^2 - 2(x)(5)\cos 120°$$

$$49 = 25 + x^2 - 10x\left(-\frac{1}{2}\right)$$

$$\Rightarrow x^2 + 5x - 24 = 0$$

$$(x+8)(x-3) = 0$$

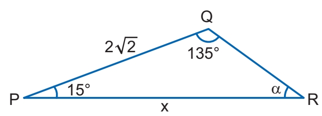
$$\Rightarrow x = -8 \;\; 0 \;\; x = 3$$

Como: $x > 0$

$\therefore x = 3$

Clave C

18.



Por suma de ángulos internos:

$$15° + 135° + \alpha = 180° \Rightarrow \alpha = 30°$$

Por ley de senos:

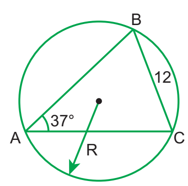
$$\frac{x}{\text{sen}135°} = \frac{2\sqrt{2}}{\text{sen}\alpha} \Rightarrow x = \frac{2\sqrt{2}\,\text{sen}(180° - 45°)}{\text{sen}30°}$$

$$\Rightarrow x = \frac{2\sqrt{2}\,\text{sen}45°}{\text{sen}30°} = \frac{2\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{1}{2}\right)} = 4$$

$\therefore x = 4$

Clave B

19.



En el $\triangle ABC$, por ley de senos se cumple:

$$BC = 2R\,\text{sen}A$$

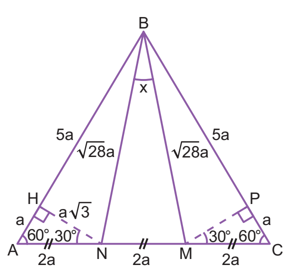
$$12 = 2R\,\text{sen}37°$$

$$12 = 2R\left(\frac{3}{5}\right) \Rightarrow 60 = 6R$$

$\therefore R = 10$

Clave D

20.



Del gráfico: el $\triangle ABC$ resulta ser equilátero.

Sea: $AC = 6a$

Luego en los triángulos rectángulos BHN y BPM por el teorema de Pitágoras, se obtiene:

$$BN = BM = \sqrt{28}\,a$$

En el $\triangle NBM$ por ley de cosenos:

$$(2a)^2 = (\sqrt{28}\,a)^2 + (\sqrt{28}\,a)^2 - 2(\sqrt{28}\,a)(\sqrt{28}\,a)\cos x$$

$$4a^2 = 28a^2 + 28a^2 - 2(28)a^2\cos x$$

$$\Rightarrow 56a^2\cos x = 52a^2$$

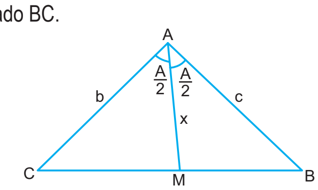
$$\cos x = \frac{52}{56}$$

$$\therefore \cos x = \frac{13}{14}$$

Clave E

Resolución de problemas

21. Sea x la longitud de la bisectriz interior relativa al lado BC.



Del gráfico tenemos:

$$S_{\triangle ABC} = S_{\triangle AMC} + S_{\triangle AMB}$$

$$\frac{bc}{2}\text{sen}A = \frac{bx}{2}\text{sen}\frac{A}{2} + \frac{cx}{2}\text{sen}\frac{A}{2}$$

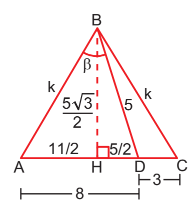
$$\frac{bc}{2}\left(2\text{sen}\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)\right) = \frac{bx}{2}\text{sen}\frac{A}{2} + \frac{cx}{2}\text{sen}\frac{A}{2}$$

$$2bc\cos\left(\frac{A}{2}\right) = x\,(b+c)$$

$$\therefore x = \frac{2bc}{b+c}\cos\left(\frac{A}{2}\right)$$

Clave C

22. De los datos, tenemos:



En el $\triangle AHD$ (teorema de Pitágoras):

$$k^2 = \left(\frac{11}{2}\right)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2 \Rightarrow k = 7$$

En el $\triangle ABD$ (ley de cosenos):

$$8^2 = k^2 + 5^2 - 2(k)(5)\cos\beta$$

$$8^2 = 7^2 + 5^2 - 2(7)(5)\cos\beta$$

$$\cos\beta = 1/7$$

$$\therefore k\cos^2\beta = 1/7$$

Clave A

Nivel 3 (página 83) Unidad 4

Comunicación matemática

23.

I. Sabemos:

$$\cos A = \frac{b^2+c^2-a^2}{bc} \quad ...(1)$$

$$\operatorname{sen}\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} \quad ...(2)$$

(1) en (2):

$$\operatorname{sen}\frac{A}{2} = \sqrt{\frac{1-\dfrac{b^2+c^2-a^2}{bc}}{2}}$$

$$\operatorname{sen}\frac{A}{2} = \sqrt{\frac{(a+c-b)(a+b-c)}{4bc}}$$

$$\operatorname{sen}\left(\frac{A}{2}\right) = \sqrt{\frac{(p-b)(p-c)}{bc}} \quad (V)$$

II. Sabemos:

$$\cos A = \frac{b^2+c^2-a^2}{bc} \quad ...(1)$$

$$\cos\left(\frac{A}{2}\right) = \sqrt{\frac{1+\cos A}{2}} \quad ...(2)$$

(1) en (2):

$$\cos\left(\frac{A}{2}\right) = \sqrt{\frac{1+\dfrac{b^2+c^2-a^2}{bc}}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \sqrt{\frac{(b+c+a)(b+c-a)}{4bc}} = \sqrt{\frac{p(p-a)}{bc}} \quad (F)$$

III. $S = \dfrac{bc}{2}\operatorname{sen}A = \dfrac{bc}{2} \times 2\operatorname{sen}\dfrac{A}{2}\cos\dfrac{A}{2}$

$$S = bc \times \frac{\sqrt{(p-b)(p-c)}}{\sqrt{bc}} \times \frac{\sqrt{p(p-a)}}{\sqrt{bc}}$$

$$S = \sqrt{p(p-a)(p-b)(p-c)} \quad (V)$$

24.

En I tenemos:

$a = 13 \wedge b + c = 15$, $b - c = 1$ $\Rightarrow$ $2b = 16$, $b = 8 \wedge c = 7$

En II tenemos:

$a + b + c = 28$, $c = 7 \Rightarrow a + b = 21$

Sabemos:

$a = b\cos C + c\cos B$

$b = a\cos C + c\cos A$

$c = b\cos A + a\cos B$

$a + b + c = (a + b)\cos C + (a + c)\cos B + (b + c)\cos A \quad ...(III)$

(I) en (III):

$13 + 8 + 7 = (13 + 8)\cos C + (13 + 7)\cos B + (8 + 7)\cos A$

$28 = 15\cos A + 20\cos B + 21\cos C$

(II) en (III):

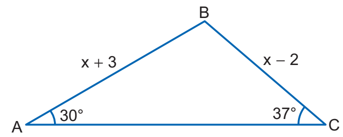
$28 = 21\cos C + (a + 7)\cos B + (b + 7)\cos A$

$\therefore$ Es necesario I, pero no II.

Clave B

Razonamiento y demostración

25.



Por ley de senos:

$$\frac{x+3}{\operatorname{sen}37°} = \frac{x-2}{\operatorname{sen}30°} \Rightarrow \frac{x+3}{\left(\frac{3}{5}\right)} = \frac{x-2}{\left(\frac{1}{2}\right)}$$

$\Rightarrow 5(x + 3) = 6(x - 2)$

$5x + 15 = 6x - 12$

$\Rightarrow x = 27$

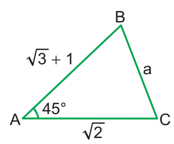
Piden:

$BC = x - 2 = 27 - 2 = 25$

$\therefore BC = 25$

Clave C

26.



En el ΔABC por ley de cosenos:

$$a^2 = (\sqrt{2})^2 + (\sqrt{3}+1)^2 - 2(\sqrt{2})(\sqrt{3}+1)\cos 45°$$

$$a^2 = 2 + 4 + 2\sqrt{3} - 2\sqrt{2}\,(\sqrt{3}+1)\left(\frac{\sqrt{2}}{2}\right)$$

Luego:

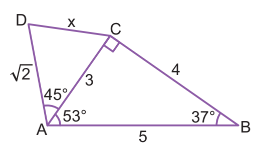
$a^2 = 6 + 2\sqrt{3} - 2\sqrt{3} - 2$

$\Rightarrow a^2 = 4$

$\therefore\ a = 2$

Clave C

27.



El $\triangle ACB$ es notable de 37° y 53°.

$\Rightarrow AC = 3 \wedge BC = 4$

En el ΔDAC por ley de cosenos:

$$x^2 = 3^2 + (\sqrt{2})^2 - 2(3)(\sqrt{2})\cos 45°$$

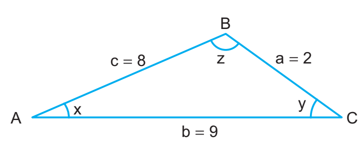
$$x^2 = 9 + 2 - 6\sqrt{2}\left(\frac{\sqrt{2}}{2}\right)$$

$\Rightarrow x^2 = 5$

$\therefore x = \sqrt{5}$

Clave C

28.



Por ley de senos:

$a = 2R\operatorname{sen}x$; $b = 2R\operatorname{sen}z$; $c = 2R\operatorname{sen}y$

Piden:

$$\frac{\operatorname{sen}^2 z}{\operatorname{sen}x\operatorname{sen}y} = \frac{\left(\dfrac{b}{2R}\right)^2}{\left(\dfrac{a}{2R}\right)\left(\dfrac{c}{2R}\right)} = \frac{\dfrac{b^2}{4R^2}}{\dfrac{ac}{4R^2}}$$

$$\Rightarrow \frac{\operatorname{sen}^2 z}{\operatorname{sen}x\operatorname{sen}y} = \frac{b^2}{ac} = \frac{(9)^2}{(2)(8)}$$

$$\therefore \frac{\operatorname{sen}^2 z}{\operatorname{sen}x\operatorname{sen}y} = \frac{81}{16}$$

Clave E

29. Por dato:

$$a^2 = b^2 + c^2 - \frac{3}{2}bc \quad ...(1)$$

Por ley de cosenos:

$$a^2 = b^2 + c^2 - 2bc\cos A \quad ...(2)$$

Comparando (1) y (2):

$-\dfrac{3}{2}bc = -2bc\cos A$

$\dfrac{3}{2} = 2\cos A$

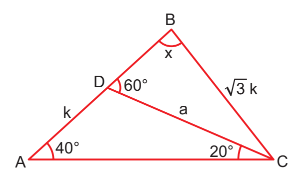
$\Rightarrow \cos A = \dfrac{3}{4} \Rightarrow \cos^2 A = \dfrac{9}{16}$

$\Rightarrow 1 - \operatorname{sen}^2 A = \dfrac{9}{16} \Rightarrow \operatorname{sen}^2 A = \dfrac{7}{16}$

$\therefore \operatorname{sen}A = \dfrac{\sqrt{7}}{4}$

Clave A

30.



En el ΔADC por ley de senos:

$$\frac{a}{\operatorname{sen}40°} = \frac{k}{\operatorname{sen}20°} \Rightarrow \frac{a}{k} = \frac{\operatorname{sen}40°}{\operatorname{sen}20°} \quad ...(1)$$

En el ΔDBC por ley de senos:

$$\frac{a}{\operatorname{sen}x} = \frac{\sqrt{3}\,k}{\operatorname{sen}60°} \Rightarrow \operatorname{sen}x = \frac{a\operatorname{sen}60°}{k\sqrt{3}}$$

$$\Rightarrow \operatorname{sen}x = \frac{a\left(\dfrac{\sqrt{3}}{2}\right)}{k\sqrt{3}} \Rightarrow \operatorname{sen}x = \frac{a}{2k} \quad ...(2)$$

Reemplazando (1) en (2):

$$\Rightarrow \operatorname{sen}x = \frac{1}{2}\left(\frac{\operatorname{sen}40°}{\operatorname{sen}20°}\right) = \frac{2\operatorname{sen}20°\cos 20°}{2\operatorname{sen}20°}$$

$\operatorname{sen}x = \cos 20° = \cos(90° - 70°)$

$\operatorname{sen}x = \operatorname{sen}70°$

$\Rightarrow x = 70° \ \vee\ x + 70° = 180°$

$\Rightarrow x = 110°$

$\therefore x = 70° \ \vee\ x = 110°$

Clave C

31. En un triángulo ABC, se cumple:
$A + B + C = \pi$ rad
Sea:
$H = bcsen(B + C)(cotB + cotC)$
$H = bcsen(\pi - A)\left(\frac{cosB}{senB} + \frac{cosC}{senC}\right)$
$H = bc(senA)\left(\frac{senC\,cosB + cosC\,senB}{senBsenC}\right)$
$H = bcsenA\left(\frac{sen(C + B)}{senBsenC}\right)$
$H = bcsenA\frac{sen(\pi - A)}{senBsenC}$
$\Rightarrow H = \frac{bcsenA(senA)}{senBsenC}$
Empleando ley de senos:
$\Rightarrow H = \frac{(2RsenB)(2RsenC)sen^2A}{senBsenC}$
$\Rightarrow H = 4R^2sen^2A = (2RsenA)^2$

Pero: $a = 2RsenA$
$\Rightarrow H = a^2$
$\therefore bcsen(B + C)(cotB + cotC) = a^2$

Clave E

32. En un triángulo ABC, se cumple:
$A + B + C = \pi$ rad
Piden:
$M = \left(\frac{a\cos A + b\cos B}{RsenC}\right)sec(B - A)$
$M = \left(\frac{(2RsenA)\cos A + (2RsenB)\cos B}{RsenC}\right)sec(B - A)$
$M = \left(\frac{2senA\cos A + 2senB\cos B}{senC}\right)sec(B - A)$
$M = \left(\frac{sen2A + sen2B}{senC}\right)sec(B - A)$
$M = \left(\frac{2sen(B + A)\cos(B - A)}{senC}\right)sec(B - A)$
$M = \frac{2sen(\pi - C)}{senC}\overbrace{\cos(B - A)\sec(B - A)}^{(1)}$
$M = \frac{2(senC)}{sen\,C}$
$\therefore M = 2$

Clave A

33. En un triángulo ABC, por dato:
$a = 5b$ y $m\angle C = 120°$

Como: $A + B + C = 180°$
$\Rightarrow A + B + 120° = 180° \Rightarrow A + B = 60°$

Por ley de tangentes:

$$\frac{a - b}{a + b} = \frac{\tan\left(\frac{A - B}{2}\right)}{\tan\left(\frac{A + B}{2}\right)}$$

Entonces:

$$\frac{(5b) - b}{(5b) + b} = \frac{\tan\left(\frac{A - B}{2}\right)}{\tan\left(\frac{60°}{2}\right)}$$

$$\tan\left(\frac{A - B}{2}\right) = \left(\frac{4b}{6b}\right)\tan 30°$$

$$\tan\left(\frac{A - B}{2}\right) = \frac{2}{3}\left(\frac{\sqrt{3}}{3}\right) = \frac{2\sqrt{3}}{9}$$

$$\Rightarrow \tan\left(\frac{A - B}{2}\right) = \frac{2\sqrt{3}}{9}$$

$$\Rightarrow \cot\left(\frac{A - B}{2}\right) = \frac{9}{2\sqrt{3}} = \frac{3\sqrt{3}}{2}$$

Por identidad del ángulo doble:

$2\csc 2\theta = \cot\theta + \tan\theta$

$$\Rightarrow 2\csc(A - B) = \cot\left(\frac{A - B}{2}\right) + \tan\left(\frac{A - B}{2}\right)$$

$$2\csc(A - B) = \frac{3\sqrt{3}}{2} + \frac{2\sqrt{3}}{9}$$

$$2\csc(A - B) = \frac{31\sqrt{3}}{18}$$

$$\csc(A - B) = \frac{31\sqrt{3}}{36}$$

$$\therefore \csc^2(A - B) = \frac{961}{432}$$

Clave A

Resolución de problemas

34. Por teorema de senos tenemos:
$\frac{m}{senM} = \frac{n}{senN} = \frac{o}{senO}$

Pero:
$\frac{senM}{5} = \frac{senN}{6} = \frac{senO}{7}$

Entonces:
$\frac{m}{5} = \frac{n}{6} = \frac{o}{7} = k$
$\Rightarrow m = 5k$
$n = 6k$
$o = 7k$

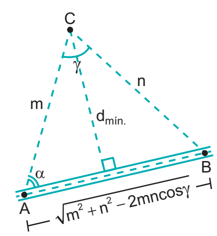
Recordemos:
$S_{\Delta MNO} = \sqrt{p(p - m)(p - n)(p - o)}$
Donde: $2p = m + n + o$

Entonces:
$90\sqrt{3}\ cm^2 = \sqrt{(9k)(4k)(3k)(2k)}$
$90^2 \times 3 = 9 \times 4 \times 3 \times 2 \times k^4$
$k^4 = \frac{90^2}{72} \Rightarrow k = \sqrt[4]{\frac{225}{2}}$

El lado opuesto a N es:
$n = 6k$
$n = 6\sqrt[4]{\frac{225}{2}}$
$n = 3\sqrt[4]{1800}$
$n = 3\sqrt{30\sqrt{2}}$

Clave E

35. De los datos tenemos:



$d_{mín.} = msen\alpha$ ... (1)

Por ley de senos:

$$\frac{n}{sen\alpha} = \frac{\sqrt{m^2 + n^2 - 2mn\cos\gamma}}{sen\gamma}$$

$$sen\alpha = \frac{n\,sen\gamma}{\sqrt{m^2 + n^2 - 2mn\cos\gamma}} \quad ... (2)$$

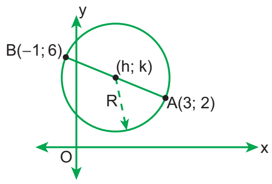
Reemplazamos (2) en (1):

$$d_{mín.} = \frac{m \times n \times sen\gamma}{\sqrt{m^2 + n^2 - 2mn\cos\gamma}}$$

Clave A

APLICAMOS LO APRENDIDO (página 85) Unidad 4

1.



Por punto medio de $\overline{AB}$, se tiene:

$h = \dfrac{-1+3}{2} \Rightarrow h = 1$

$k = \dfrac{6+2}{2} \Rightarrow k = 4$

Por distancia entre dos puntos:

$(2r)^2 = (-1-3)^2 + (6-2)^2 \Rightarrow r^2 = 8$

Nos piden: $(x-1)^2 + (y-4)^2 = 8$

Clave B

2. Para hallar la ecuación de la circunferencia necesitamos el centro y la medida del radio. En este caso solo calcularemos el valor del radio.

$r = \sqrt{(6-3)^2 + (4-4)^2}$

$r = \sqrt{3^2} = 3$

Sabemos que la ecuación de la circunferencia es:

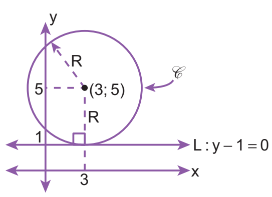
$(x-h)^2 + (y-k)^2 = r^2$

Reemplazamos los valores:

$(x-6)^2 + (y-4)^2 = 9$

Clave C

3.



Del gráfico: $R = 5 - 1 \Rightarrow R = 4$

El centro de la circunferencia es: $(h; k) = (3; 5)$

Piden la ecuación de la circunferencia $\mathscr{C}$.

$\mathscr{C} : (x-h)^2 + (y-k)^2 = R^2$

$\mathscr{C} : (x-3)^2 + (y-5)^2 = (4)^2$

$\therefore \mathscr{C} : (x-3)^2 + (y-5)^2 = 16$

Clave A

4. Por dato:

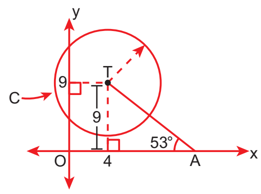
$\mathscr{C} : x^2 + y^2 - 8x - 18y - 24 = 0$

Completando términos:

$x^2 - 2x(4) + 4^2 + y^2 - 2y(9) + 9^2 = 24 + 4^2 + 9^2$

$\Rightarrow (x-4)^2 + (y-9)^2 = 121$

Luego, las coordenadas del centro serán: (4; 9)



Del gráfico: TA = 9csc53°

$\Rightarrow TA = 9\left(\dfrac{5}{4}\right) = \dfrac{45}{4} = 11{,}25$

$\therefore TA = 11{,}25$

Clave B

5. Según los vértices notamos que la elipse tiene su centro en el origen de coordenadas.

Luego: $a = 5 \Rightarrow a^2 = 25$

$c = 4 \Rightarrow c^2 = 16$

Además se cumple:

$b^2 = a^2 - c^2$

$b^2 = 25 - 16$

$b^2 = 9$

Por lo tanto, la ecuación de la elipse es:

$\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

Clave E

6. Por dato: C(−4; −3)

Además:

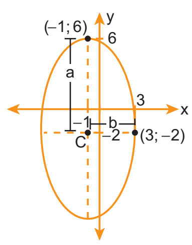
$V_1V_2 = 2a = 34 \Rightarrow a = 17$

$V_1 = (-4; 17-3) = (-4, 14)$

$V_2 = (-4; -17-3) = (-4; -20)$

Clave A

7. Graficamos la elipse:



Del gráfico: C = (−1; −2)

Observando la gráfica:

$a = 6 + 2 = 8 \Rightarrow a^2 = 64$

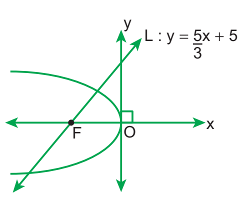
$b = 1 + 3 = 4 \Rightarrow b^2 = 16$

Luego, la ecuación de la elipse es:

$\dfrac{(x+1)^2}{16} + \dfrac{(y+2)^2}{64} = 1$

Clave C

8.



Por dato: F es el foco de la parábola.

Además: F(x; y) = F(p; 0); p < 0

Como la recta L pasa por el punto F, entonces:

$0 = \dfrac{5}{3}(p) + 5 \Rightarrow -15 = 5p$

$p = -3$

Piden la ecuación de la parábola:

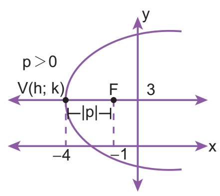
$y^2 = 4px$

$\Rightarrow y^2 = 4(-3)x$

$\therefore y^2 = -12x$

Clave C

9.



Por dato: F es el foco de la parábola.

Además: V(h; k) = V(−4; 3)

Del gráfico, el eje focal es paralelo al eje x.

Luego: $|p| = |-4-(-1)| = |-4+1|$

$\Rightarrow |p| = |-3|$; como $p > 0 \Rightarrow p = 3$

Piden la ecuación de la parábola:

$(y-k)^2 = 4p(x-h)$

$\Rightarrow (y-3)^2 = 4(3)(x-(-4))$

$\therefore (y-3)^2 = 12(x+4)$

Clave A

10. Por dato:

$\mathscr{C} : x^2 + y^2 - 8x - 6y = 0$

Completando términos:

$x^2 - 2x(4) + 4^2 + y^2 - 2y(3) + 3^2 = 4^2 + 3^2$

$\Rightarrow (x-4)^2 + (y-3)^2 = 25$

Entonces, las coordenadas del centro de la circunferencia serán: (h; k) = (4; 3)

Piden la distancia (d) del centro de la circunferencia al origen de coordenadas (0; 0).

Empleando la fórmula de distancia entre dos puntos:

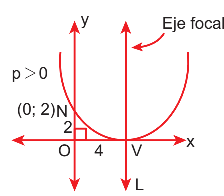
$d = \sqrt{(h-0)^2 + (k-0)^2}$

$\Rightarrow d = \sqrt{(4)^2 + (3)^2} = \sqrt{25}$

$\therefore d = 5$

Clave C

11.



Por dato: V es el vértice de la parábola.

Del gráfico:
$V(h; k) = V(4; 0) \ \wedge \ N(x; y) = N(0; 2)$

Piden la ecuación de la parábola:

$(x - h)^2 = 4p(y - k)$

$(x - 4)^2 = 4p(y - 0)$

$\Rightarrow (x - 4)^2 = 4py$

Como la parábola pasa por el punto N, entonces:
$(0 - 4)^2 = 4p(2)$
$16 = 8p \Rightarrow p = 2$

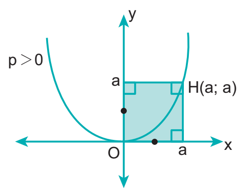
Finalmente, la ecuación de la parábola será:

$(x - 4)^2 = 4(2)y$

$\therefore (x - 4)^2 = 8y$

Clave A

12.



Por dato: $A_{\square} = 16\ m^2$

$a^2 = 16 \Rightarrow a = 4$

El vértice de la parábola se encuentra en el origen de coordenadas (0; 0), luego su ecuación será:

$x^2 = 4py$ ...(I)

La parábola pasa por el punto H, entonces:

$(a)^2 = 4p(a)$

$(4)^2 = 4p(4) \Rightarrow p = 1$

Reemplazando en (I):

$x^2 = 4(1)y$

$\therefore x^2 = 4y$

Clave B

13. Por dato:

L_1: $3x - 2y - 24 = 0$

L_2: $2x + 7y + 9 = 0$

Además: $\overleftrightarrow{L_1} \cap \overleftrightarrow{L_2} = (h; k)$

Luego:

$3h - 2k - 24 = 0 \Rightarrow 3h - 2k = 24$ ...(I)

$2h + 7k + 9 = 0 \Rightarrow 2h + 7k = -9$ ...(II)

Resolviendo el sistema formado por (I) y (II), se obtiene: $h = 6 \wedge k = -3$

Piden la ecuación de la circunferencia $\mathscr{C}$ que pasa por el origen de coordenadas y de centro (h; k):

$\mathscr{C}$: $(x - h)^2 + (y - k)^2 = R^2$

$\mathscr{C}$: $(x - 6)^2 + (y - (-3))^2 = R^2$

$\Rightarrow \mathscr{C}$: $(x - 6)^2 + (y + 3)^2 = R^2$

Evaluando el punto O(0; 0)

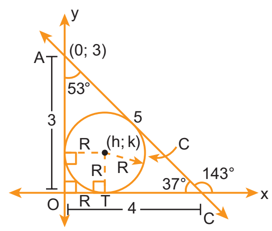
$(0 - 6)^2 + (0 + 3)^2 = R^2$

$\Rightarrow R^2 = 45$

$\therefore \mathscr{C}$: $(x - 6)^2 + (y + 3)^2 = 45$

Clave A

14.



Del $\triangle$AOC (notable de 37° y 53°):
$OC = 4 \ \wedge \ AC = 5$

Por el teorema de Poncelet:
$OA + OC = AC + 2R$
$3 + 4 = 5 + 2R$
$2 = 2R \Rightarrow R = 1$

Luego, las coordenadas del centro de la circunferencia serán: $(h; k) = (R; R) = (1; 1)$.
Piden la ecuación de la circunferencia $\mathscr{C}$.
Empleando la ecuación ordinaria:

$\mathscr{C}$: $(x - h)^2 + (y - k)^2 = R^2$

$\therefore \mathscr{C}$: $(x - 1)^2 + (y - 1)^2 = 1$

Clave C

PRACTIQUEMOS

Nivel 1 (página 87) Unidad 4

Comunicación matemática

1.

A) $x^2 + y^2 = 6^2$
$x^2 + y^2 = 36$

B) $(x - 1)^2 + (y - 0)^2 = 3^2$
$(x - 1)^2 + y^2 = 9$

C) $(x - 0)^2 + (y - 3)^2 = 2^2$
$x^2 + (y - 3)^2 = 4$

D) $(x - (-1))^2 + (y - 2)^2 = 2^2$
$(x + 1)^2 + (y - 2)^2 = 4$

2.

A) $\dfrac{(x - 0)^2}{4^2} + \dfrac{(y - 0)^2}{3^2} = 1$

$\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

B) $\dfrac{(x - 0)^2}{2^2} + \dfrac{(y - 0)^2}{3^2} = 1$

$\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

C) $\dfrac{(x - 0)^2}{4^2} + \dfrac{\left(y - (-1)\right)^2}{3^2} = 1$

$\dfrac{x^2}{16} + \dfrac{(y + 1)^2}{9} = 1$

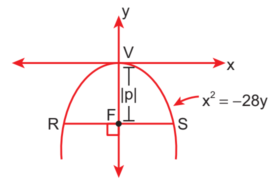
Razonamiento y demostración

3. Se tiene la parábola: $x^2 + 28y = 0$

$\Rightarrow x^2 = -28y$

Es de la forma: $x^2 = 4py$

$\Rightarrow -28 = 4p \Rightarrow p = -7 \ (p < 0)$



Las coordenadas del foco serán:

$F(x; y) = F(0; p)$

$\Rightarrow F(x; y) = F(0; -7)$

El lado recto es: $\overline{RS}$

Por propiedad: $RS = 4VF$

$\Rightarrow RS = 4|p| = 4|-7|$

$\therefore \ RS = 4 \times 7 = 28$ u

Clave C

4. Se tiene la circunferencia:

$\mathscr{C}: x^2 + y^2 - 8x - 6y - 11 = 0$

Completando términos:

$x^2 - 2x(4) + 4^2 + y^2 - 2y(3) + 3^2 = 11 + 4^2 + 3^2$

$\Rightarrow (x - 4)^2 + (y - 3)^2 = 36$

Entonces las coordenadas de su centro serán: (4; 3)

Además: $R^2 = 36$ (donde R es el radio)

$\Rightarrow \quad R = \sqrt{36} \quad \therefore R = 6$

Clave C

5. Se tiene la parábola:

$(x + 6)^2 = -7(y + 1)$ ...(I)

Es de la forma: $(x - h)^2 = 4p(y - k)$ ...(II)

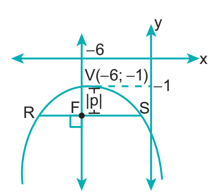
Donde su vértice es: (h; k)

Comparando (I) y (II): $h = -6 \wedge k = -1$

$\Rightarrow V(h; k) = V(-6; -1)$

Además: $4p = -7$

$p = -\dfrac{7}{4} \ (p < 0)$



Luego, las coordenadas del foco serán:

$F(x; y) = F(-6; -1 + p)$

$F(x; y) = F\left(-6; -1 + \left(-\frac{7}{4}\right)\right)$

$\Rightarrow F(x; y) = F\left(-6; -\frac{11}{4}\right)$

El lado recto es: $\overline{RS}$

Por propiedad: $RS = 4VF$

$\Rightarrow RS = 4|p| = 4\left|-\frac{7}{4}\right|$

$RS = 4 \times \frac{7}{4}$

$\therefore RS = 7$ u

Clave C

6. Se tiene la parábola:

$(x - 1)^2 = 2(y + 2)$

Pide: los puntos de intersección de la parábola con el eje de abscisas.

Luego, los puntos de intersección con el eje de abscisas tienen la forma:

$(x; y) = (x; 0)$

Entonces, haciendo $y = 0$ tenemos:

$(x - 1)^2 = 2(0 + 2) = 4$

$|x - 1| = 2$

$\Rightarrow x - 1 = 2 \lor x - 1 = -2$

$x = 3 \qquad x = -1$

Por lo tanto, los puntos de intersección serán: $(3; 0)$ y $(-1; 0)$

Clave E

7. Se tiene la circunferencia:

$\mathscr{C}: x^2 + y^2 - 4x + 12y - 20 = 0$

Llevamos la ecuación de la circunferencia a su forma ordinaria:

$(x - h)^2 + (y - k)^2 = R^2$

Donde su centro es $(h; k)$ y su radio es R.

Luego, completando términos:

$x^2 - 2x(2) + 2^2 + y^2 + 2y(6) + 6^2 = 20 + 2^2 + 6^2$

$\Rightarrow (x - 2)^2 + (y + 6)^2 = 60$

Comparando se obtiene que las coordenadas de su centro serán: $(h; k) = (2; -6)$

Clave E

8. Sabemos que el centro es el punto medio del segmento que une los vértices:

$C = (h; k) = \left(\frac{7 + 7}{2}; \frac{-3 + 9}{2}\right) = (7; 3)$

Además:

$V_1V_2 = 2a = 9 - (-3) = 12$

$\Rightarrow a = 6 \Rightarrow a^2 = 36$

Por dato sabemos: $LR = \frac{2b^2}{a} = 10 \Rightarrow b^2 = 30$

La ecuación de la elipse es: $\frac{(x - 7)^2}{30} + \frac{(y - 3)^2}{36} = 1$

Clave A

9. Por el enunciado, tenemos:

$F'(0; -3); F(0; 3) \Rightarrow F'F = 2c = 3 - (-3) = 6$

$c = 3 \Rightarrow c^2 = 9$

$V_1(0, -8)$ y $V_2(0; 8) \Rightarrow V_1V_2 = 2a = 8 - (-8) = 16$

$a = 8 \Rightarrow a^2 = 64$

Luego: $b^2 = a^2 - c^2 \Rightarrow b^2 = 8^2 - 3^2 = 55$

La ecuación de la elipse es: $\frac{x^2}{55} + \frac{y^2}{64} = 1$

Clave B

Resolución de problemas

10. Por dato:

Eje mayor $\rightarrow 2a$

Distancia entre focos $\rightarrow 2c$

$\Rightarrow 2a = 2(2c)$

$a = 2c$

Sabemos:

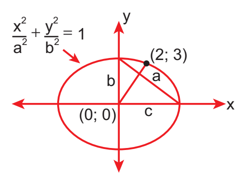
$a^2 = b^2 + c^2$

$(2c)^2 = b^2 + c^2$

$4c^2 = b^2 + c^2$

$3c^2 = b^2$

Luego, tenemos:



$\frac{2^2}{a^2} + \frac{3^2}{b^2} = 1$

$\frac{4}{(2c)^2} + \frac{9}{3c^2} = 1$

$\frac{1}{c^2} + \frac{3}{c^2} = 1$

$\frac{4}{c^2} = 1 \Rightarrow c^2 = 4 \Rightarrow c = 2$

$\therefore a = 4 \land b = 2\sqrt{3}$

De la ecuación:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{4^2} + \frac{y^2}{(2\sqrt{3})^2} = 1$

$\frac{x^2}{16} + \frac{y^2}{12} = 1$

$\therefore 3x^2 + 4y^2 = 48$

Clave B

11. Tenemos el centro de la circunferencia: $(h; k)$

Entonces: $\sqrt{h^2 + k^2} = 5$

$h^2 + k^2 = 25 \qquad \ldots (I)$

$(x - h)^2 + (y - k)^2 = 2^2$

$(-5 - h)^2 + (4 - k)^2 = 4$

$25 + 10h + h^2 + 16 - 8k + k^2 = 4$

$37 + \underbrace{h^2 + k^2}_{(I)} + 10h = 8k$

$37 + 25 + 10h = 8k$

$\frac{31 + 5h}{4} = k \qquad \ldots (II)$

Reemplazamos (II) en (I):

$h^2 + \left(\frac{31 + 5h}{4}\right)^2 = 25$

$16h^2 + 961 + 310h + 25h^2 = 400$

$41h^2 + 310h + 561 = 0$

41h   187
h   3

$(41h + 187)(h + 3) = 0$

$\Rightarrow h = -3$

Reemplazamos en (II):

$\frac{31 + 5(-3)}{4} = k$

$\Rightarrow k = 4$

Luego, la ecuación es:

$(x - h)^2 + (y - k)^2 = r^2$

$(x - (-3))^2 + (y - 4)^2 = (2)^2$

$(x + 3)^2 + (y - 4)^2 = 4$

Clave B

Nivel 2 (página 88) Unidad 4

Comunicación matemática

12. A) $x^2 = 4py$

B) $y^2 = 4px$

C) $x^2 = 4py$

D) $(x - h)^2 = 4p(y - k)$

13. A) $(x - 1)^2 + (y - 2)^2 = 3^2$

$x^2 - 2x + 1 + y^2 - 4y + 4 = 9$

$\therefore x^2 - 2x + y^2 - 4y - 4 = 0$

B) Del gráfico:

$a = 5 - 2 \land b = 2$

$a = 3$

$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

$\frac{(x - 2)^2}{9} + \frac{(y - 0)^2}{4} = 1$

$4(x - 2)^2 + 9y^2 = 36$

$4x^2 - 8x + 16 + 9y^2 = 36$

$4x^2 - 8x + 9y^2 - 20 = 0$

C) $p > 0$

$\Rightarrow (x - h)^2 = 4p(y - k)$

$(x - 2)^2 = 4(3)(y - (-2))$

$x^2 - 4x + 4 = 12y + 24$

$\therefore x^2 - 4x - 12y - 20 = 0$

Razonamiento y demostración

14. Se tiene la parábola: $3x^2 - 16y = 0$

$\Rightarrow x^2 = \frac{16}{3}y$

Es de la forma: $x^2 = 4py$

$\Rightarrow \frac{16}{3} = 4p \Rightarrow p = \frac{4}{3}\ (p > 0)$

Se trata de una parábola con eje focal vertical que se abre hacia arriba, con vértice en el origen (0; 0).

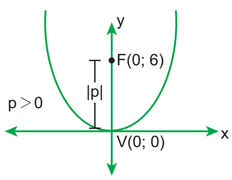
Luego, su directriz ($\overleftrightarrow{L_D}$) es de la forma:

$y = -p \Rightarrow y = -\frac{4}{3} \Rightarrow 3y + 4 = 0$

$\therefore L_D$: $3y + 4 = 0$

Clave A

15. Por dato: el vértice de la parábola es (0; 0) y su foco (0; 6).



Del gráfico: $|p| = 6$

Como: $p > 0 \Rightarrow p = 6$

Piden la ecuación de la parábola.

Empleando la ecuación canónica: $x^2 = 4py$

$\Rightarrow x^2 = 4(6)y$

$\therefore x^2 = 24y$

Clave A

16. Se tiene la circunferencia:

$\mathscr{C}: x^2 + y^2 - 8x + 8y - 9 = 0$

Piden un punto de intersección con el eje coordenado y.

Luego, los puntos de intersección con el eje **y** tienen la forma: (x; y) = (0; y)

Entonces, haciendo x = 0, tenemos:

$(0)^2 + y^2 - 8(0) + 8y - 9 = 0$

$y^2 + 8y - 9 = 0$

$(y + 9)(y - 1) = 0$

$\Rightarrow y = -9 \lor y = 1$

Por lo tanto, los puntos de intersección serán: (0; –9) y (0; 1)

Clave A

17. Piden la ecuación general de la circunferencia que pasa por los puntos:

(2; –2), (–1; 4) y (4; 6).

Empleamos la ecuación general:

$\mathscr{C}: x^2 + y^2 + Dx + Ey + F = 0$

Evaluando en los puntos dados:

$(2)^2 + (-2)^2 + D(2) + E(-2) + F = 0$

$\Rightarrow 2D - 2E + F = -8$ ...(I)

$(-1)^2 + (4)^2 + D(-1) + E(4) + F = 0$

$\Rightarrow -D + 4E + F = -17$ ...(II)

$(4)^2 + (6)^2 + D(4) + E(6) + F = 0$

$\Rightarrow 4D + 6E + F = -52$ ...(III)

Resolviendo el sistema formado por (I), (II) y (III) se obtiene:

$D = -\frac{16}{3}$; $E = -\frac{25}{6}$; $F = -\frac{17}{3}$

Reemplazando en la ecuación general:

$x^2 + y^2 - \frac{16}{3}x - \frac{25}{6}y - \frac{17}{3} = 0$

$\therefore \mathscr{C}: 6x^2 + 6y^2 - 32x - 25y - 34 = 0$

Clave A

18. Por dato: el centro de la circunferencia es (5; –1) y el radio mide 7 u.

$\Rightarrow (h; k) = (5; -1) \land R = 7$

Empleando la ecuación ordinaria de la circunferencia:

$(x - h)^2 + (y - k)^2 = R^2$

$\Rightarrow (x - 5)^2 + (y - (-1))^2 = (7)^2$

$\Rightarrow (x - 5)^2 + (y + 1)^2 = 49$

Desarrollando la ecuación ordinaria, obtenemos:

$x^2 - 10x + 25 + y^2 + 2y + 1 = 49$

$\therefore \mathscr{C}: x^2 + y^2 - 10x + 2y - 23 = 0$

Clave A

19. Por dato: el centro de la circunferencia es (–2; 3) y además esta pasa por el punto (4; 5).

$\Rightarrow (h; k) = (-2; 3)$

Empleando la ecuación ordinaria, de la circunferencia:

$(x - h)^2 + (y - k)^2 = R^2$

$(x - (-2))^2 + (y - 3)^2 = R^2$

$\Rightarrow (x + 2)^2 + (y - 3)^2 = R^2$ ...(I)

Evaluando en el punto (4; 5):

$(4 + 2)^2 + (5 - 3)^2 = R^2$

$36 + 4 = R^2$

$\Rightarrow R^2 = 40$

Reemplazando en (I):

$\Rightarrow (x + 2)^2 + (y - 3)^2 = 40$

Desarrollando la ecuación ordinaria, obtenemos:

$x^2 + 4x + 4 + y^2 - 6y + 9 = 40$

$\therefore \mathscr{C}: x^2 + y^2 + 4x - 6y - 27 = 0$

Clave B

20. Dada la ecuación:

$5x^2 + 2y^2 - 10x - 12y + 13 = 0$

Completamos cuadrados y obtenemos:

$5x^2 - 10x + 5 + 2y^2 - 12y + 18 = -13 + 5 + 18$

$5(x^2 - 2x + 1) + 2(y^2 - 6y + 9) = 10$

$5(x - 1)^2 + 2(y - 3)^2 = 10$

$$\frac{(x-1)^2}{2} + \frac{(y-3)^2}{5} = 1$$

De la ecuación ordinaria de la elipse notamos que el eje focal es paralelo al eje y.

$\left.\begin{array}{l} a^2 = 5 \\ b^2 = 2 \end{array}\right\} \begin{array}{l} c^2 = a^2 - b^2 \\ c^2 = 5 - 2 \Rightarrow c = \sqrt{3} \end{array}$

Las coordenadas de los focos son:

$F'(1; 3 - \sqrt{3})$; $F = (1; 3 + \sqrt{3})$

Clave A

21. Sabemos que el centro de la elipse es (–2; –5) y al eje focal es paralelo al eje y, luego:

$$\frac{(x+2)^2}{b^2} + \frac{(y+5)^2}{a^2} = 1$$

Longitud del eje mayor:

$2a = 24 \Rightarrow a = 12$

Excentricidad:

$e = \frac{c}{a} = \frac{\sqrt{5}}{3} \Rightarrow \frac{c}{12} = \frac{\sqrt{5}}{3} \Rightarrow c = 4\sqrt{5}$

Además:

$a^2 = b^2 + c^2 \Rightarrow b^2 = (12)^2 - (4\sqrt{5})^2 = 64$

$b = 8$

Entonces la ecuación de la elipse es:

$$\frac{(x+2)^2}{64} + \frac{(y+5)^2}{144} = 1$$

Clave B

22. La ecuación de la elipse es de la forma:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Ya que los puntos ($\sqrt{6}$; –1) y (2; $\sqrt{2}$) pertenecen a la elipse, tenemos:

$$\left.\begin{array}{l} \frac{(\sqrt{6})^2}{a^2} + \frac{(-1)^2}{b^2} = 1 \\ \Rightarrow \frac{6}{a^2} + \frac{1}{b^2} = 1 \\ \frac{(2)^2}{a^2} + \frac{(\sqrt{2})^2}{b^2} = 1 \\ \Rightarrow \frac{4}{a^2} + \frac{2}{b^2} = 1 \end{array}\right\} a^2 = 8 \land b^2 = 4$$

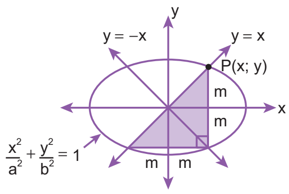
Por lo tanto, la ecuación de la elipse es:

$\frac{x^2}{8} + \frac{y^2}{4} = 1$

Clave A

Resolución de problemas

23. Del gráfico tenemos:



$$A_{\blacktriangle}=\frac{2m(2m)}{2}=2m^2 \qquad \text{... (I)}$$

En el punto P: x = y = m

$$\frac{m^2}{a^2}+\frac{m^2}{b^2}=1$$

$$b^2m^2+a^2m^2=a^2b^2$$

$$m^2(a^2+b^2)=a^2b^2$$

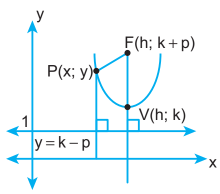
$$m^2=\frac{a^2b^2}{a^2+b^2} \qquad \text{... (II)}$$

Reemplazamos (II) en (I):

$$A_{\blacktriangle}=2m^2=\frac{2a^2b^2}{a^2+b^2}$$

Clave D

24. Graficamos en base al vértice y la directriz:



De los datos tenemos:

V(h; k) = (3; 2)

$\Rightarrow$ h = 3 $\wedge$ k = 2

y = k − p = 1

2 − p = 1 $\Rightarrow$ p = 1

Luego, la ecuación será:

$(x-h)^2=4p(y-k)$

$(x-3)^2=4(1)(y-2)$

$(x-3)^2=4(y-2)$

Clave A

Nivel 3 (página 90) Unidad 4

Comunicación matemática

25.

I. V

II. F
El plano no debe ser paralelo a ninguna de sus generatrices.

III. F
Ec.: $x^2+y^2+Cx+Dy+E=0$

IV. V

$\therefore$ VFFV

Clave C

26.

M: $y^2+2x-10y+27=0$

$$y^2-10y+25=-2(x+1)$$

$$(y-5)^2=-2(x+1)$$

$$(y-5)^2=4\left(-\frac{1}{2}\right)(x+1)$$

$\Rightarrow$ p = −1/2

$$LR=4|p|=4\left|-\frac{1}{2}\right|=4\left(\frac{1}{2}\right)$$

$\therefore$ LR = 2 u

M = 2 u

N: $y^2-2y+x^2+4x-11=0$

$y^2-2y+1+x^2+4x+4-16=0$

$(y-1)^2+(x+2)^2=4^2$

$\therefore$ r = 4 u $\Rightarrow$ N = 4 u

$\Rightarrow$ 2M = N

Clave D

Razonamiento y demostración

27. Se tiene la parábola: $x^2+9y=0$

Los puntos A(3; a) y B(b; −4) pertenecen a la parábola.

Evaluando en dichos puntos, tenemos:

$(3)^2+9(a)=0 \Rightarrow a=-1$

$(b)^2+9(-4)=0 \Rightarrow b^2=36$

$\Rightarrow b=6 \ \vee\ b=-6$

Por dato: B $\in$ IIIC $\Rightarrow$ b < 0

$\therefore$ b = −6

Entonces: A(3; −1) y B(−6; −4)

Piden la longitud del segmento AB.

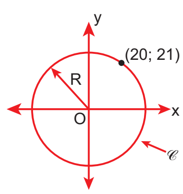
$$AB=\sqrt{[3-(-6)]^2+[-1-(-4)]^2}$$

$$AB=\sqrt{(9)^2+(3)^2}=\sqrt{90}$$

$\therefore AB=3\sqrt{10}$ u

Clave E

28.



Del gráfico, el centro de la circunferencia es (0; 0).
Empleando la ecuación canónica:

$\mathscr{C}: x^2+y^2=R^2$ (donde R es el radio)

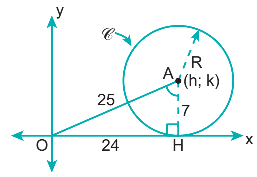
Como la circunferencia pasa por el punto (20; 21), evaluamos en dicho punto:

$20^2+21^2=R^2 \quad \Rightarrow R^2=841$

$\therefore$ R = 29 u

Clave C

29.



Por dato: R = 7 $\wedge$ OA = 25

En el $\triangle$OHA por el teorema de Pitágoras:

OH = 24

Del gráfico: (h; k) = (24; 7)

Piden la ecuación de la circunferencia $\mathscr{C}$.

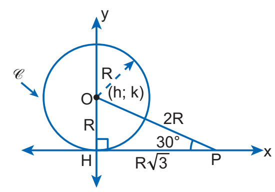
Empleando la ecuación ordinaria:

$\mathscr{C}: (x-h)^2+(y-k)^2=R^2$

$\therefore \mathscr{C}: (x-24)^2+(y-7)^2=7^2$

Clave D

30.



Por dato: OP = 12

$\Rightarrow$ 2R = 12 $\Rightarrow$ R = 6

Del gráfico: (h; k) = (0; R)

$\Rightarrow$ (h; k) = (0; 6)

Piden la ecuación de la circunferencia $\mathscr{C}$.

Empleando la ecuación ordinaria:

$\mathscr{C}: (x-h)^2+(y-k)^2=R^2$

$\Rightarrow (x-0)^2+(y-6)^2=6^2$

Desarrollando la ecuación ordinaria, obtenemos:

$x^2+y^2-12y+36=36$

$\therefore \mathscr{C}: x^2+y^2-12y=0$

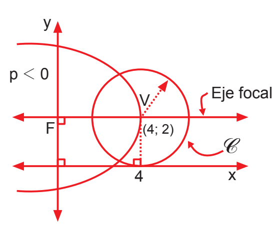
Clave C

31. Por dato:

$\mathscr{C}: (x-4)^2+(y-2)^2=4$

Luego, las coordenadas del centro de la circunferencia serán: (h; k) = (4; 2)

Además, F es el foco de la parábola.



Del gráfico, el centro de la circunferencia coincide con el vértice de la parábola:

$\Rightarrow$ V(h; k) = V(4; 2)

También: |p| = FV = 4

Como: p < 0 $\Rightarrow$ p = −4

Piden la ecuación de la parábola:

$(y-k)^2 = 4p(x-h)$

$\Rightarrow (y-2)^2 = 4(-4)(x-4)$

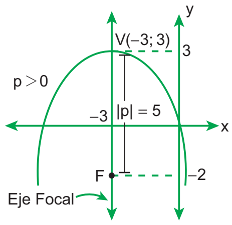
$\therefore (y-2)^2 = -16(x-4)$

Clave B

32. Por dato: el lado recto de la parábola mide 20.

$\Rightarrow 4|p| = 20 \Rightarrow |p| = 5$

Además: las coordenadas de su foco son (–3; –2) y su vértice está arriba del foco.



Del gráfico, se deduce que las coordenadas del vértice de la parábola son (–3; 3) y p = –5.

Piden la ecuación de la parábola.

Empleando la ecuación ordinaria:

$(x-h)^2 = 4p(y-k)$;

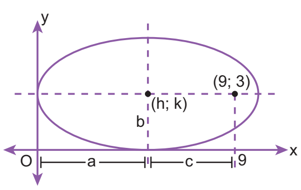
donde (h; k) = (–3; 3)

$\Rightarrow (x-(-3))^2 = 4(-5)(y-3)$

$\therefore (x+3)^2 = -20(y-3)$

Clave B

33. Del gráfico, tenemos:



$b = 3 \Rightarrow a + c = 9$

Luego: $b^2 = a^2 - c^2$

$9 = (a-c)(a+c) = (a-c)\,9$

$a - c = 1$

$\Rightarrow a = 5 \text{ y } c = 4$

Además: C = (h, k) = (5; 3)

La ecuación de la elipse es:

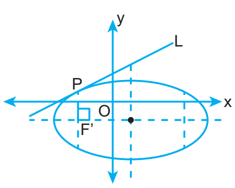
$$\frac{(x-5)^2}{25} + \frac{(y-3)^2}{9} = 1$$

Desarrollando la ecuación tenemos la ecuación general:

$9x^2 + 25y^2 - 90x - 150y + 225 = 0$

Clave C

34.



De la ecuación dada, tenemos:

$$\frac{(x-1)^2}{16} + \frac{(y+1)^2}{7} = 1$$

$$\left.\begin{aligned} &\Rightarrow\ C(h;k) = (+1; -1) \\ &a^2 = 16 \Rightarrow a = 4 \\ &b^2 = 7 \Rightarrow b = \sqrt{7} \end{aligned}\right\} c^2 = a^2 - b^2 \Rightarrow c = 3$$

Lado recto:

$$\frac{2b^2}{a} = \frac{2(7)}{4} = \frac{7}{2} \Rightarrow PF' = \frac{7}{4}$$

Además: F' = (–2; –1)

Calculamos la ordenada de P:

$$\frac{7}{4} - 1 = \frac{3}{4}$$

$\Rightarrow P = (-2; 3/4)$

Por último, la ecuación de la recta tangente es:

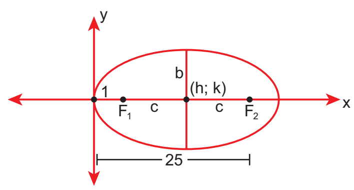
$$7(-2-1)(x-1) + 16\left(\frac{3}{4} + 1\right)(y+1) = 112$$

$$3x - 4y + 15 = 0$$

Clave E

Resolución de problemas

35.



$1 + 2c = 25$

$2c = 24 \Rightarrow c = 12$

$1 + c = a$

$1 + 12 = a \Rightarrow a = 13$

$a^2 = b^2 + c^2$

$13^2 = b^2 + 12^2 \Rightarrow b^2 = 25$

$\therefore b = 5$

$h = 1 + c$

$h = 1 + 12 = 13 \wedge k = 0$

Luego, la ecuación es:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-13)^2}{169} + \frac{(y-0)^2}{25} = 1$$

$$\frac{(x-13)^2}{169} + \frac{y^2}{25} = 1$$

Clave A

36. $y^2 - 4x - 10y + 17 = 0$

$y^2 - 10y + 25 = 4x + 8$

$(y-5)^2 = 4(1)(x-(-2))$

$(y-k)^2 = 4(p)(x-h)$

Luego tenemos que:

$h = -2;\ p = 1$

L_D: $x = h - p$

L_D: $x = -2 - 1$

$\therefore x = -3$

Clave B

APLICAMOS LO APRENDIDO (página 92) Unidad 4

1. $A = \lim\limits_{x \to 0} \text{sen}\left(\frac{\pi}{2} + x\right)$

Por reducción al primer cuadrante:

$\text{sen}\left(\frac{\pi}{2} + x\right) = \cos x$

$\Rightarrow A = \lim\limits_{x \to 0} \cos x = \cos 0 = 1$

$\therefore A = 1$

Clave C

2. $\lim\limits_{x \to 0} \frac{\text{sen}\,9x}{x} = \lim\limits_{x \to 0} \frac{9\text{sen}\,9x}{9x}$

$\lim\limits_{x \to 0} \frac{\text{sen}\,9x}{x} = 9.\underbrace{\lim\limits_{x \to 0} \frac{\text{sen}\,9x}{9x}}_{(1)}$

$\Rightarrow \lim\limits_{x \to 0} \frac{\text{sen}\,9x}{x} = 9(1) = 9$

$\therefore \lim\limits_{x \to 0} \frac{\text{sen}\,9x}{x} = 9$

Clave B

3. $B = \lim\limits_{x \to 0} \frac{\text{sen}^2 x}{x^2 \cos x}$

$B = \lim\limits_{x \to 0} \frac{\text{sen}x}{x} \cdot \frac{\text{sen}x}{x \cos x}$

$B = \lim\limits_{x \to 0} \frac{\text{sen}x}{x} \cdot \frac{\tan x}{x}$

$\Rightarrow B = \underbrace{\lim\limits_{x \to 0} \frac{\text{sen}x}{x}}_{(1)} \cdot \underbrace{\lim\limits_{x \to 0} \frac{\tan x}{x}}_{(1)}$

$\therefore B = (1)\,(1) = 1$

Clave C

4. $C = \lim\limits_{x \to 0} \frac{p\tan px}{q\tan qx}$

$C = \lim\limits_{x \to 0} \frac{\frac{p\tan px}{(pqx)}}{\frac{q\tan qx}{(pqx)}} = \lim\limits_{x \to 0} \frac{\frac{p}{q} \cdot \frac{\tan px}{px}}{\frac{q}{p} \cdot \frac{\tan qx}{qx}}$

$C = \frac{\lim\limits_{x \to 0} \frac{p}{q}\left(\frac{\tan px}{px}\right)}{\lim\limits_{x \to 0} \frac{q}{p}\left(\frac{\tan qx}{qx}\right)} = \frac{\frac{p}{q} \cdot \lim\limits_{x \to 0} \frac{\tan px}{px}}{\frac{q}{p} \cdot \lim\limits_{x \to 0} \frac{\tan qx}{qx}}$

$\Rightarrow C = \frac{\frac{p}{q}(1)}{\frac{q}{p}(1)} = \frac{\frac{p}{q}}{\frac{q}{p}} = \frac{p^2}{q^2}$

$\therefore C = \frac{p^2}{q^2}$

Clave C

5. Piden: $\lim\limits_{x \to 1} \frac{\text{sen}(x-1)}{x^3 - 1}$

Sea:

$H = \lim\limits_{x \to 1} \frac{\text{sen}(x-1)}{x^3 - 1}$

Entonces:

$H = \lim\limits_{x \to 1} \frac{\text{sen}(x-1)}{(x-1)(x^2+x+1)}$

$H = \underbrace{\lim\limits_{x \to 1} \frac{\text{sen}(x-1)}{(x-1)}}_{A} \cdot \underbrace{\lim\limits_{x \to 1} \frac{1}{(x^2+x+1)}}_{B}$

$\Rightarrow H = A \,.\, B$ ...(I)

La expresión A se puede escribir como:

$A = \lim\limits_{x - 1 \to 0} \frac{\text{sen}(x-1)}{(x-1)} = 1$

$\Rightarrow A = 1$

Además:

$B = \lim\limits_{x \to 1} \frac{1}{x^2 + x + 1} = \frac{1}{(1)^2 + (1) + 1} = \frac{1}{3}$

$\Rightarrow B = \frac{1}{3}$

Reemplazando en (I):

$H = (1)\left(\frac{1}{3}\right)$

$\Rightarrow H = \frac{1}{3}$

$\therefore \lim\limits_{x \to 1} \frac{\text{sen}(x-1)}{x^3 - 1} = \frac{1}{3}$

Clave D

6. $f(x) = \frac{67x}{\text{sen}\,2010x}$

Piden: $\lim\limits_{x \to 0} f(x)$

Entonces:

$\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \frac{67x}{\text{sen}\,2010x}$

$\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \frac{67}{\left(\frac{\text{sen}\,2010x}{x}\right)}$

$\Rightarrow \lim\limits_{x \to 0} f(x) = \frac{\lim\limits_{x \to 0} 67}{\lim\limits_{x \to 0}\left(\frac{2010\text{sen}\,2010x}{2010x}\right)}$

$\Rightarrow \lim\limits_{x \to 0} f(x) = \frac{67}{2010 \,.\, \underbrace{\lim\limits_{x \to 0} \frac{\text{sen}\,2010x}{2010x}}_{(1)}}$

$\Rightarrow \lim\limits_{x \to 0} f(x) = \frac{67}{2010\,(1)} = \frac{1}{30}$

$\therefore \lim\limits_{x \to 0} f(x) = \frac{1}{30}$

Clave A

7. $M = \lim\limits_{x \to 0} \frac{1 - \cos(1 - \cos x)}{x}$

Evaluando $x = 0$ se obtiene: $\frac{0}{0}$ (indeterminado).

Aplicando la regla de L' Hospital:

$M = \lim\limits_{x \to 0} \frac{[1 - \cos(1 - \cos x)]'}{(x)'}$

$M = \lim\limits_{x \to 0} \frac{[0 - (-\text{sen}(1 - \cos x))(1 - \cos x)']}{(1)}$

$M = \lim\limits_{x \to 0} \text{sen}(1 - \cos x)(0 - (-\text{sen}x))$

$M = \lim\limits_{x \to 0} \text{sen}(1 - \cos x)\,\text{sen}x$

Evaluando el límite:

$M = \text{sen}(1 - \cos 0)\,\text{sen}0$

$M = \text{sen}(1 - 1)\,\text{sen}0$

$M = (\text{sen}0)^2 = (0)^2$

$\therefore M = 0$

Clave B

8. $f(x) = \text{sen}^2 x$

Piden: $f'\left(\frac{127°}{2}\right)$

Si u es una función diferenciable en x, y n es un entero positivo o negativo, entonces:

$\frac{d(u^n)}{dx} = nu^{n-1}\left(\frac{du}{dx}\right)$

Luego:

$\underbrace{\frac{d(\text{sen}^2 x)}{dx}}_{f'(x)} = 2(\text{sen}x)^{2-1}\left(\frac{d\text{sen}x}{dx}\right)$

$\Rightarrow f'(x) = 2\text{sen}x(\cos x) = \text{sen}2x$

$\Rightarrow f'(x) = \text{sen}2x$

Evaluando en $f'(x)$ para $x = \frac{127°}{2}$ tenemos:

$\Rightarrow f'\left(\frac{127°}{2}\right) = \text{sen}2\left(\frac{127°}{2}\right)$

$\Rightarrow f'\left(\frac{127°}{2}\right) = \text{sen}127° = \text{sen}53° = \frac{4}{5}$

$\therefore f'\left(\frac{127°}{2}\right) = \frac{4}{5}$

Clave C

9. $f(x) = \text{sen}2x\text{sen}3x$

Entonces:

$f'(x) = (\text{sen}2x)'\,\text{sen}3x + \text{sen}2x\,(\text{sen}3x)'$

$f'(x) = (\cos 2x \,.\, 2)\,\text{sen}3x + \text{sen}2x\,(\cos 3x \,.\, 3)$

$\Rightarrow f'(x) = 2\cos 2x\text{sen}3x + 3\text{sen}2x\cos 3x$

Piden: $f'\left(\frac{\pi}{2}\right)$

$\Rightarrow f'\left(\frac{\pi}{2}\right) = 2\cos\frac{2\pi}{2}\text{sen}\frac{3\pi}{2} + 3\text{sen}\frac{2\pi}{2}\cos\frac{3\pi}{2}$

$\Rightarrow f'\left(\frac{\pi}{2}\right) = 2\cos\pi\text{sen}\frac{3\pi}{2} + 3\text{sen}\pi\cos\frac{3\pi}{2}$

$\Rightarrow f'\left(\frac{\pi}{2}\right) = 2(-1)(-1) + 3(0)(0) = 2$

$\therefore f'\left(\frac{\pi}{2}\right) = 2$

Clave A

10. $f(x) = \text{sen}x + \text{sen}3x + \text{sen}5x + ... + \text{sen}21x$

Piden: $f'(0)$

Entonces:

$f'(x) = (\text{sen}x)' + (\text{sen}3x)' + (\text{sen}5x)' + ... + (\text{sen}21x)'$

$f'(x) = (\cos x) + (\cos 3x \cdot 3) + (\cos 5x \cdot 5) + ... + (\cos 21x \cdot 21)$

$f'(x) = \cos x + 3\cos 3x + 5\cos 5x + ... + ... + 21\cos 21x$

Evaluando $f'(x)$ para $x = 0$, tenemos:

$f'(0) = \cos 0 + 3\cos 0 + 5\cos 0 + ... + 21\cos 0$

$f'(0) = (1) + 3(1) + 5(1) + ... + 21(1)$

$f'(0) = 1 + 3 + 5 + ... + 21$

Se obtiene una suma de números impares.

$\Rightarrow f'(0) = n^2$; donde: $2n - 1 = 21 \Rightarrow n = 11$

$\therefore f'(0) = 11^2 = 121$

Clave E

11. Sea: $T = \lim\limits_{x \to \pi} \dfrac{\tan 4x - \tan 2x}{\tan 3x - \tan x}$

Por ángulos compuestos:

$\tan\alpha - \tan\theta = \dfrac{\text{sen}(\alpha - \theta)}{\cos\alpha \cos\theta}$

Entonces:

$$T = \lim_{x \to \pi} \frac{\dfrac{\text{sen}(4x - 2x)}{\cos 4x \cos 2x}}{\dfrac{\text{sen}(3x - x)}{\cos 3x \cos x}}$$

$$T = \lim_{x \to \pi} \frac{\text{sen}2x \cos 3x \cos x}{\text{sen}2x \cos 4x \cos 2x}$$

$$T = \lim_{x \to \pi} \frac{\cos 3x \cos x}{\cos 4x \cos 2x}$$

Evaluando el límite:

$$\Rightarrow T = \frac{\cos 3\pi \cos \pi}{\cos 4\pi \cos 2\pi} = \frac{(-1)(-1)}{(1)(1)} \Rightarrow T = 1$$

$$\therefore \lim_{x \to \pi} \frac{\tan 4x - \tan 2x}{\tan 3x - \tan x} = 1$$

Clave B

12. A) $y = \cot x - \tan x = 2\cot 2x$

$y' = 2(\cot 2x)' = 2(-\csc^2 2x \cdot 2)$

$y' = -4\csc^2 2x$

B) $y = 3\text{sen}x - 4\text{sen}^3 x = \text{sen}3x$

$y' = (\text{sen}3x)' = (\cos 3x \cdot 3)$

$y' = 3\cos 3x$

C) $y = \csc x - \cot x = \tan\dfrac{x}{2}$

$y' = \left(\tan\dfrac{x}{2}\right)' = \left(\sec^2\dfrac{x}{2} \cdot \dfrac{1}{2}\right)$

$y' = \dfrac{1}{2}\sec^2\dfrac{x}{2}$

D) $y = \cos x(2\cos 2x - 1) = \cos 3x$

$y' = (\cos 3x)' = (-\text{sen}3x \cdot 3)$

$y' = -3\text{sen}3x$

E) $y = \cos^4 x - \text{sen}^4 x = \cos 2x$

$y' = (\cos 2x)' = (-\text{sen}2x \cdot 2)$

$y' = -2\text{sen}2x \neq 2\text{sen}2x$

Clave E

13. Hacemos $x = y + 1$, si $x \to 1$, $y \to 0$

$$\lim_{y \to 0} (-y)\tan\frac{\pi}{2}(y + 1) = -\lim_{y \to 0} y \tan\frac{\pi}{2}(y + 1)$$

$$-\lim_{y \to 0} y \tan\left(\frac{\pi}{2}y + \frac{\pi}{2}\right)$$

$$-\lim_{y \to 0} y \frac{\text{sen}\left(\dfrac{\pi}{2}y + \dfrac{\pi}{2}\right)}{\cos\left(\dfrac{\pi y}{2} + \dfrac{\pi}{2}\right)}$$

$$-\lim_{y \to 0} y \frac{\text{sen}\dfrac{\pi y}{2}\cos\dfrac{\pi}{2} + \cos\dfrac{\pi y}{2}\text{sen}\dfrac{\pi}{2}}{\cos\dfrac{\pi y}{2}\cos\dfrac{\pi}{2} - \text{sen}\dfrac{\pi y}{2}\text{sen}\dfrac{\pi}{2}}$$

$$-\lim_{y \to 0} y \frac{\cos\dfrac{\pi y}{2}}{-\text{sen}\dfrac{\pi y}{2}} = \frac{2}{\pi}$$

Clave C

14. Haciendo $x = y + \dfrac{\pi}{4}$, si $x \to \dfrac{\pi}{4}$, $y \to 0$

$$\lim_{y \to 0} \frac{\tan\left(y + \dfrac{\pi}{4}\right) - 1}{y}$$

$$= \lim_{y \to 0} \frac{\dfrac{\tan y + \tan\dfrac{\pi}{4}}{1 - \tan y \tan\dfrac{\pi}{4}} - 1}{y}$$

$$= \lim_{y \to 0} \frac{2\tan y}{y(1 - \tan y)}$$

$$= 2\lim_{y \to 0} \frac{\tan y}{y} \cdot \lim_{y \to 0} \frac{1}{1 - \tan y}$$

$$= 2 \cdot 1 \cdot \frac{1}{1 - 0} = 2$$

Clave D

PRACTIQUEMOS

Nivel 1 (página 94) Unidad 4

Comunicación matemática

1.

2.

Razonamiento y demostración

3. Sea: $E = \lim\limits_{x \to a} \dfrac{\cos x - \cos a}{\text{sen}\dfrac{x}{2} - \text{sen}\dfrac{a}{2}}$

Evaluando:

$x = a$ se obtiene $\dfrac{0}{0}$ (indeterminado).

Aplicando la regla de L' Hospital:

$$E = \lim_{x \to a} \frac{(\cos x - \cos a)'}{\left(\text{sen}\dfrac{x}{2} - \text{sen}\dfrac{a}{2}\right)'}$$

$$E = \lim_{x \to a} \frac{(-\text{sen}x) - 0}{\cos\dfrac{x}{2} \cdot \dfrac{1}{2} - 0} = \lim_{x \to a} \frac{-2\text{sen}x}{\cos\dfrac{x}{2}}$$

Evaluando el límite:

$$\Rightarrow E = \frac{-2\text{sen}a}{\cos\dfrac{a}{2}} = \frac{-2\left(2\text{sen}\dfrac{a}{2}\cos\dfrac{a}{2}\right)}{\cos\dfrac{a}{2}}$$

$$\Rightarrow E = -4\text{sen}\frac{a}{2}$$

$$\therefore \lim_{x \to a} \frac{\cos x - \cos a}{\text{sen}\dfrac{x}{2} - \text{sen}\dfrac{a}{2}} = -4\text{sen}\frac{a}{2}$$

Clave A

4. Sea:

$$M = \lim_{x \to 0} \frac{\text{sen}4x}{x} + \lim_{x \to 0} \frac{4x}{\text{sen}x}$$

$$M = \lim_{x \to 0} \frac{4\text{sen}4x}{4x} + \lim_{x \to 0} \frac{4}{\dfrac{\text{sen}x}{x}}$$

$$M = 4\left(\lim_{x \to 0} \frac{\text{sen}4x}{4x}\right) + \frac{\lim\limits_{x \to 0} 4}{\left(\lim\limits_{x \to 0} \dfrac{\text{sen}x}{x}\right)}$$

$$\Rightarrow M = 4(1) + \frac{(4)}{(1)} = 4 + 4 = 8$$

$$\therefore \lim_{x \to 0} \frac{\text{sen}4x}{x} + \lim_{x \to 0} \frac{4x}{\text{sen}x} = 8$$

Clave A

5. Sea: $N = \lim\limits_{x \to \frac{\pi}{3}} \dfrac{\cos x - \dfrac{1}{2}}{x - \dfrac{\pi}{3}}$

Evaluando:

$x = \dfrac{\pi}{3}$ se obtiene $\dfrac{0}{0}$ (indeterminado).

Aplicando la regla de L' Hospital:

$$N = \lim_{x \to \frac{\pi}{3}} \frac{\left(\cos x - \dfrac{1}{2}\right)'}{\left(x - \dfrac{\pi}{3}\right)'}$$

$$N = \lim_{x \to \frac{\pi}{3}} \frac{(-\text{sen}x) - 0}{1 - 0} = \lim_{x \to \frac{\pi}{3}} (-\text{sen}x)$$

Evaluando el límite:

$$\Rightarrow N = -\text{sen}\frac{\pi}{3} = -\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow N = -\frac{\sqrt{3}}{2}$$

$$\therefore \lim_{x \to \frac{\pi}{3}} \frac{\cos x - \dfrac{1}{2}}{x - \dfrac{\pi}{3}} = -\frac{\sqrt{3}}{2}$$

Clave A

6. Sea: $P = \lim\limits_{x \to \frac{\pi}{2}} \dfrac{x - 0{,}5\pi}{\cos x}$

Evaluando:

$x = \dfrac{\pi}{2}$ se obtiene $\dfrac{0}{0}$ (indeterminado)

Aplicando la regla de L' Hospital:

$$P = \lim_{x \to \frac{\pi}{2}} \frac{(x - 0{,}5\pi)'}{(\cos x)'}$$

$$P = \lim_{x \to \frac{\pi}{2}} \frac{1 - 0}{(-\text{sen}x)} = \lim_{x \to \frac{\pi}{2}} (-\csc x)$$

Evaluando el límite: $P = -\csc\frac{\pi}{2} = -(1)$

$\Rightarrow P = -1$

$\therefore \lim\limits_{x \to \frac{\pi}{2}} \dfrac{x - 0,5\pi}{\cos x} = -1$

Clave B

7. Sea: $E = \lim\limits_{x \to 0} \dfrac{\operatorname{sen}x - \operatorname{sen}2x}{\operatorname{sen}x}$

Luego:

$E = \lim\limits_{x \to 0} \dfrac{\operatorname{sen}x - 2\operatorname{sen}x\cos x}{\operatorname{sen}x}$

$E = \lim\limits_{x \to 0} \dfrac{\operatorname{sen}x(1 - 2\cos x)}{\operatorname{sen}x}$

$E = \lim\limits_{x \to 0} (1 - 2\cos x)$

Evaluando el límite:
$E = 1 - 2\cos 0 = 1 - 2(1)$
$\Rightarrow E = -1$

$\therefore \lim\limits_{x \to 0} \dfrac{\operatorname{sen}x - \operatorname{sen}2x}{\operatorname{sen}x} = -1$

Clave A

8. Sea: $B = \lim\limits_{x \to a} \dfrac{\operatorname{sen}x - \operatorname{sen}a}{\tan x - \tan a}$

Evaluando: $x = a$ se obtiene $\frac{0}{0}$ (indeterminado).

Aplicando la regla de L' Hospital:
$B = \lim\limits_{x \to a} \dfrac{(\operatorname{sen}x - \operatorname{sen}a)'}{(\tan x - \tan a)'}$

$B = \lim\limits_{x \to a} \dfrac{\cos x - 0}{\sec^2 x - 0} = \lim\limits_{x \to a} \cos^3 x$

Evaluando el límite:
$\Rightarrow B = \cos^3 a$
$\therefore \lim\limits_{x \to a} \dfrac{\operatorname{sen}x - \operatorname{sen}a}{\tan x - \tan a} = \cos^3 a$

Clave A

9. $G(x) = 1 - 2\operatorname{sen}^2 x\cos^2 x$

$G(x) = 1 - \dfrac{4\operatorname{sen}^2 x\cos^2 x}{2}$

$G(x) = 1 - \dfrac{(2\operatorname{sen}x\cos x)^2}{2}$

$G(x) = 1 - \dfrac{(\operatorname{sen}2x)^2}{2}$

Piden:

$G'(x) = 0 - \dfrac{2(\operatorname{sen}2x)^{2-1}}{2}.(\cos 2x . 2)$

$G'(x) = -2\operatorname{sen}2x\cos 2x = -\operatorname{sen}4x$

$\therefore G'(x) = -\operatorname{sen}4x$

Clave B

10. $f(x) = 1 + \operatorname{sen}x + \cos x$

Luego:
$f'(x) = 0 + \cos x + (-\operatorname{sen}x)$
$\Rightarrow f'(x) = \cos x - \operatorname{sen}x$
$f''(x) = (-\operatorname{sen}x) - (\cos x)$
$\Rightarrow f''x) = -\operatorname{sen}x - \cos x$

$f'''(x) = -(\cos x) - (-\operatorname{sen}x)$
$\Rightarrow f'''(x) = -\cos x + \operatorname{sen}x$
$\therefore f(x) + f'(x) + f''x) + f'''(x) = 1$

Clave A

Nivel 2 (página 94) Unidad 4

Comunicación matemática

11.

12.

Razonamiento y demostración

13. Sea: $E = \lim\limits_{x \to 1} \dfrac{\cos\pi x + 1}{(x-1)^2}$

Evaluando: $x = 1$ se obtiene $\frac{0}{0}$ (indeterminado).

Aplicando la regla de L' Hospital:
$E = \lim\limits_{x \to 1} \dfrac{(\cos\pi x + 1)'}{(x^2 - 2x + 1)'}$

$E = \lim\limits_{x \to 1} \dfrac{(-\operatorname{sen}\pi x)\pi + 0}{2x - 2 + 0} = \lim\limits_{x \to 1} \dfrac{-\pi\operatorname{sen}\pi x}{2x - 2}$

Evaluando otra vez se obtiene $\frac{0}{0}$ (indeterminado).

Aplicando nuevamente L' Hospital:
$E = -\frac{\pi}{2} . \lim\limits_{x \to 1} \dfrac{\operatorname{sen}\pi x}{x - 1} = -\frac{\pi}{2} . \lim\limits_{x \to 1} \dfrac{(\operatorname{sen}\pi x)'}{(x-1)'}$

$E = -\frac{\pi}{2} . \lim\limits_{x \to 1} \dfrac{(\cos\pi x . \pi)}{1 - 0} = -\frac{\pi}{2} . \lim\limits_{x \to 1}(\pi\cos\pi x)$

Evaluando el límite:
$\Rightarrow E = -\frac{\pi}{2}(\pi\cos\pi) = -\frac{\pi^2}{2}(-1)$

$\Rightarrow E = \frac{\pi^2}{2}$

$\therefore \lim\limits_{x \to 1} \dfrac{\cos\pi x + 1}{(x-1)^2} = \frac{\pi^2}{2}$

Clave A

14. Sea: $E = \lim\limits_{x \to a} \dfrac{\operatorname{sen}^2 x - \operatorname{sen}^2 a}{\operatorname{sen}(2x - a) - \operatorname{sen}a}$

Por identidad del ángulo compuesto:
$\operatorname{sen}^2 x - \operatorname{sen}^2 a = \operatorname{sen}(x + a)\operatorname{sen}(x - a)$

Por transformaciones trigonométricas:
$\operatorname{sen}(2x - a) - \operatorname{sen}a = 2\operatorname{sen}(x - a)\cos x$

Entonces:
$E = \lim\limits_{x \to a} \dfrac{\operatorname{sen}(x + a)\operatorname{sen}(x - a)}{2\operatorname{sen}(x - a)\cos x}$

$E = \lim\limits_{x \to a} \dfrac{\operatorname{sen}(x + a)}{2\cos x}$

Evaluando el límite:
$\Rightarrow E = \dfrac{\operatorname{sen}(a + a)}{2\cos a} = \dfrac{\operatorname{sen}2a}{2\cos a}$

$\Rightarrow E = \dfrac{2\operatorname{sen}a\cos a}{2\cos a} = \operatorname{sen}a$

$\therefore \lim\limits_{x \to a} \dfrac{\operatorname{sen}^2 x - \operatorname{sen}^2 a}{\operatorname{sen}(2x - a) - \operatorname{sen}a} = \operatorname{sen}a$

Clave E

15. Sea: $R = \lim\limits_{x \to 0^+} \dfrac{\sqrt{1 - \cos x}}{\tan 5x}$

Por identidad del ángulo doble:
$1 - \cos x = 2\operatorname{sen}^2\frac{x}{2}$

Entonces:

$R = \lim\limits_{x \to 0^+} \dfrac{\sqrt{2\operatorname{sen}^2\frac{x}{2}}}{\tan 5x} = \lim\limits_{x \to 0^+} \dfrac{\sqrt{2}\left|\operatorname{sen}\frac{x}{2}\right|}{\tan 5x}$

Como: $x \to 0^+ \Rightarrow x > 0 \wedge x \in IC$

Luego:

$R = \lim\limits_{x \to 0^+} \dfrac{\sqrt{2}\left(\operatorname{sen}\frac{x}{2}\right)}{\tan 5x} = \lim\limits_{x \to 0^+} \dfrac{\dfrac{\frac{\sqrt{2}}{2}\operatorname{sen}\frac{x}{2}}{\frac{x}{2}}}{\dfrac{5\tan 5x}{5x}}$

$R = \dfrac{\frac{\sqrt{2}}{2}\left[\lim\limits_{x \to 0^+} \dfrac{\operatorname{sen}\frac{x}{2}}{\frac{x}{2}}\right]}{5\left[\lim\limits_{x \to 0^+} \dfrac{\tan 5x}{5x}\right]} = \dfrac{\frac{\sqrt{2}}{2}(1)}{5(1)}$

$\Rightarrow R = \frac{\sqrt{2}}{10}$

$\therefore \lim\limits_{x \to 0^+} \dfrac{\sqrt{1 - \cos x}}{\tan 5x} = \frac{\sqrt{2}}{10}$

Clave C

16. $F(x) = 4\operatorname{sen}^3\left(x - \frac{\pi}{4}\right)$

$F'(x) = 4\left[\operatorname{sen}^3\left(x - \frac{\pi}{4}\right)\right]'$

$F'(x) = 4\left[3\operatorname{sen}^2\left(x - \frac{\pi}{4}\right)\cos\left(x - \frac{\pi}{4}\right)1\right]$

$F'(x) = 6\left[2\operatorname{sen}\left(x - \frac{\pi}{4}\right)\cos\left(x - \frac{\pi}{4}\right)\right]\operatorname{sen}\left(x - \frac{\pi}{4}\right)$

$F'(x) = 6\left[\operatorname{sen}\left(2x - \frac{\pi}{2}\right)\right]\operatorname{sen}\left(x - \frac{\pi}{4}\right)$

$F'(x) = 6\left[-\operatorname{sen}\left(\frac{\pi}{2} - 2x\right)\right]\operatorname{sen}\left(x - \frac{\pi}{4}\right)$

$F'(x) = -6(\cos 2x)\operatorname{sen}\left(x - \frac{\pi}{4}\right)$

$\therefore F'(x) = -6\cos 2x\operatorname{sen}\left(x - \frac{\pi}{4}\right)$

Clave A

17. $H(x) = -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + 1$

Piden:
$H'(x) = -(-\operatorname{sen}x) + \frac{2}{3} . 3\cos^2 x . (-\operatorname{sen}x) - \frac{1}{5}$.
$5\cos^4 x(-\operatorname{sen}x) + 0$
$H'(x) = \operatorname{sen}x - 2\operatorname{sen}x\cos^2 x + \operatorname{sen}x\cos^4 x$
$H'(x) = \operatorname{sen}x(1 - 2\cos^2 x + \cos^4 x)$
$H'(x) = \operatorname{sen}x(1 - \cos^2 x)^2 = \operatorname{sen}x(\operatorname{sen}^2 x)^2$
$\Rightarrow H'(x) = \operatorname{sen}x(\operatorname{sen}^4 x) = \operatorname{sen}^5 x$
$\therefore H'(x) = \operatorname{sen}^5 x$

Clave E

18. $G(x) = x^3\text{senx} + 3x^2\cos x - 6\cos x - 6x\text{senx}$

$G'(x) = (x^3\text{senx})' + 3(x^2\cos x)' - 6(-\text{senx}) - 6(x\text{senx})'$

$G'(x) = (x^3\text{senx})' + 3(x^2\cos x)' - 6(x\text{senx})' + 6\text{senx}$

Luego:

$(x^3\text{senx})' = (x^3)'\text{senx} + x^3(\text{senx})'$

$\Rightarrow (x^3\text{senx})' = 3x^2\text{senx} + x^3\cos x$

$(x^2\cos x)' = (x^2)'\cos x + x^2(\cos x)'$

$\Rightarrow (x^2\cos x)' = 2x\cos x - x^2\text{senx}$

$(x\text{senx})' = (x)'\text{senx} + x(\text{senx})'$

$\Rightarrow (x\text{senx})' = \text{senx} + x\cos x$

Entonces:

$G'(x) = (3x^2\text{senx} + x^3\cos x) + 3(2x\cos x - x^2\text{senx}) - 6(\text{senx} + x\cos x) + 6\text{senx}$

Reduciendo se obtiene: $G'(x) = x^3\cos x$

Clave A

19. $J(x) = \text{sen}^6x + \cos^6x + 1$

$J'(x) = 6\text{sen}^5x(\cos x) + 6\cos^5x(-\text{senx}) + 0$

$J'(x) = 6\text{sen}^5x\cos x - 6\cos^5x\text{senx}$

$J'(x) = 6\text{senx}\cos x(\text{sen}^4x - \cos^4x)$

$J'(x) = 3\text{sen}2x\underbrace{(\text{sen}^2x + \cos^2x)}_{(1)}\underbrace{(\text{sen}^2x - \cos^2x)}_{(-\cos 2x)}$

$J'(x) = -3\text{sen}2x\cos 2x$

$J'(x) = \dfrac{-3(2\text{sen}2x\cos 2x)}{2} = -\dfrac{3}{2}(\text{sen}4x)$

$\Rightarrow J'(x) = -\dfrac{3}{2}\text{sen}4x$

Por dato: $J'(x) = A\text{sen}4x$

$\Rightarrow J'(x) = A\text{sen}4x = -\dfrac{3}{2}\text{sen}4x$

$\therefore A = -\dfrac{3}{2}$

Clave A

20. $G(x) = \dfrac{x^2}{2} + \dfrac{\cos 4x}{16}$

$G'(x) = \dfrac{1}{2}(x^2)' + \dfrac{1}{16}(\cos 4x)'$

$G'(x) = \dfrac{1}{2}(2x) + \dfrac{1}{16}(-\text{sen}4x \,.\, 4)$

$G'(x) = x - \dfrac{1}{4}\text{sen}4x$

Luego:

$G''(x) = (x)' - \dfrac{1}{4}(\text{sen}4x)'$

$G''(x) = 1 - \dfrac{1}{4}(\cos 4x \,.\, 4)$

$\Rightarrow G''(x) = 1 - \cos 4x = 2\text{sen}^2 2x$

$\therefore G''(x) = 2\text{sen}^2 2x$

Clave C

Nivel 3 (página 95) Unidad 4

Comunicación matemática

21.

22.

Razonamiento y demostración

23. Sea: $A = \lim\limits_{x \to 0} \dfrac{1}{x}\left(\dfrac{1}{\text{senx}} - \cot x\right)$

Luego: $A = \lim\limits_{x \to 0} \dfrac{1}{x}(\csc x - \cot x)$

Por identidad del ángulo mitad:

$\tan\dfrac{\theta}{2} = \csc\theta - \cot\theta$

Entonces:

$A = \lim\limits_{x \to 0} \dfrac{1}{x}\left(\tan\dfrac{x}{2}\right)$

$A = \lim\limits_{x \to 0} \dfrac{\tan\frac{x}{2}}{2 \,.\, \frac{x}{2}} = \dfrac{1}{2}\left|\lim\limits_{x \to 0} \dfrac{\tan\frac{x}{2}}{\frac{x}{2}}\right|$

$\Rightarrow A = \dfrac{1}{2}(1) = \dfrac{1}{2}$

$\therefore \lim\limits_{x \to 0} \dfrac{1}{x}\left(\dfrac{1}{\text{senx}} - \cot x\right) = \dfrac{1}{2}$

Clave C

24. Sea: $L = \lim\limits_{x \to 0} \dfrac{\text{sen}^2 4x}{x\text{sen}3x}$

Luego:

$L = \lim\limits_{x \to 0} \dfrac{\frac{\text{sen}^2 4x}{x}}{\text{sen}3x} = \lim\limits_{x \to 0} \dfrac{\frac{\text{sen}^2 4x}{x^2}}{\frac{\text{sen}3x}{x}}$

$L = \lim\limits_{x \to 0} \dfrac{\frac{16\text{sen}^2 4x}{16x^2}}{\frac{3\text{sen}3x}{3x}} = \lim\limits_{x \to 0} \dfrac{16\left(\frac{\text{sen}4x}{4x}\right)^2}{3\left(\frac{\text{sen}3x}{3x}\right)}$

$L = \dfrac{16\left(\lim\limits_{x \to 0} \frac{\text{sen}4x}{4x}\right)^2}{3\left(\lim\limits_{x \to 0} \frac{\text{sen}3x}{3x}\right)} = \dfrac{16(1)^2}{3(1)}$

$\Rightarrow L = \dfrac{16}{3}$

$\therefore \lim\limits_{x \to 0} \dfrac{\text{sen}^2 4x}{x\text{sen}3x} = \dfrac{16}{3}$

Clave C

25. Sea: $B = \lim\limits_{x \to \theta} \dfrac{\cos x - \cos\theta}{\text{senx} - \text{sen}\theta}$

Aplicando transformaciones trigonométricas:

$B = \lim\limits_{x \to \theta} \dfrac{-2\text{sen}\left(\frac{x+\theta}{2}\right)\text{sen}\left(\frac{x-\theta}{2}\right)}{2\text{sen}\left(\frac{x-\theta}{2}\right)\cos\left(\frac{x+\theta}{2}\right)}$

$B = \lim\limits_{x \to \theta} \dfrac{-\text{sen}\left(\frac{x+\theta}{2}\right)}{\cos\left(\frac{x+\theta}{2}\right)} = \lim\limits_{x \to \theta} -\tan\left(\dfrac{x+\theta}{2}\right)$

Evaluando el límite:

$\Rightarrow B = -\tan\left(\dfrac{\theta+\theta}{2}\right) = -\tan\theta$

$\therefore \lim\limits_{x \to \theta} \dfrac{\cos x - \cos\theta}{\text{senx} - \text{sen}\theta} = -\tan\theta$

Clave A

26. $F(x) = \text{senx}\,\text{sen}2x\,\text{sen}3x$

$F(x) = \dfrac{\text{sen}2x}{2}(2\text{sen}3x\text{senx})$

$F(x) = \dfrac{\text{sen}2x}{2}[\cos(3x - x) - \cos(3x + x)]$

$F(x) = \dfrac{\text{sen}2x\cos 2x}{2} - \dfrac{\text{sen}2x\cos 4x}{2}$

$F(x) = \dfrac{2\text{sen}2x\cos 2x}{4} - \dfrac{2\text{sen}2x\cos 4x}{4}$

$F(x) = \dfrac{\text{sen}4x}{4} - \dfrac{\text{sen}6x + \text{sen}(-2x)}{4}$

$F(x) = \dfrac{\text{sen}4x}{4} - \dfrac{\text{sen}6x}{4} + \dfrac{\text{sen}2x}{4}$

Luego:

$F'(x) = \dfrac{1}{4}(\text{sen}4x)' - \dfrac{1}{4}(\text{sen}6x)' + \dfrac{1}{4}(\text{sen}2x)'$

$F'(x) = \dfrac{\cos 4x \,.\, 4}{4} - \dfrac{\cos 6x \,.\, 6}{4} + \dfrac{\cos 2x \,.\, 2}{4}$

$F'(x) = \cos 4x - \dfrac{3\cos 6x}{2} + \dfrac{\cos 2x}{2}$

Piden: $F'\left(\dfrac{\pi}{2}\right)$

$F'\left(\dfrac{\pi}{2}\right) = \cos 2\pi - \dfrac{3\cos 3\pi}{2} + \dfrac{\cos\pi}{2}$

$F'\left(\dfrac{\pi}{2}\right) = (1) - \dfrac{3(-1)}{2} + \dfrac{(-1)}{2}$

$F'\left(\dfrac{\pi}{2}\right) = 1 + \dfrac{3}{2} - \dfrac{1}{2} = 2$

$\therefore F'\left(\dfrac{\pi}{2}\right) = 2$

Clave C

27. $F(x) = 16\text{sen}^5x - 20\text{sen}^3x$

Luego:

$F'(x) = 16(\text{sen}^5x)' - 20(\text{sen}^3x)'$

$F'(x) = 16(5\text{sen}^4x\cos x) - 20(3\text{sen}^2x\cos x)$

$F'(x) = 80\text{sen}^4x\cos x - 60\text{sen}^2x\cos x$

$F'(x) = 20\text{senx}\cos x(4\text{sen}^3x - 3\text{senx})$

$F'(x) = 10(2\text{senx}\cos x)(-\text{sen}3x)$

$F'(x) = -5(2\text{sen}2x\text{sen}3x)$

$F'(x) = -5[\cos(3x - 2x) - \cos(3x + 2x)]$

$F'(x) = -5\cos x + 5\cos 5x$ ...(I)

Por dato: $F'(x) = A\cos x + B\cos 5x$ ...(II)

Comparando (I) y (II): $A = -5 \wedge B = 5$

Piden:

$A + B = (-5) + (5) = 0$

$\therefore A + B = 0$

Clave A

28. $f(x) = \text{sen}x + 2\text{sen}2x + 3\text{sen}3x + ... + n\text{sen}nx$

$f'(x) = \cos x + 2(\cos 2x \,.\, 2) + 3(\cos 3x \,.\, 3) + ...$
$+ n(\cos nx \,.\, n)$

$f'(x) = \cos x + 2^2\cos 2x + 3^2\cos 3x + ... + n^2\cos nx$

Piden: $f'(0)$

$f'(0) = \cos 0 + 2^2\cos 0 + 3^2\cos 0 + ... + n^2\cos 0$

Pero: $\cos 0 = 1$

$\Rightarrow f'(0) = 1 + 2^2 + 3^2 + ... + n^2$

Se obtiene una suma de números cuadrados perfectos.

$\therefore f'(0) = \dfrac{n(n+1)(2n+1)}{6}$

Clave A

29. $$S = \lim_{n \to \infty}\left[\frac{\pi}{n}\left(\text{sen}\frac{\pi}{n} + \text{sen}\frac{2\pi}{n} + \text{sen}\frac{3\pi}{n} + ... + \text{sen}\frac{(n-1)\pi}{n}\right)\right]$$

Por series trigonométricas (para el seno):

$$S = \lim_{n \to \infty}\left[\frac{\pi}{n}\left(\frac{\text{sen}(n-1)\frac{\pi}{2n}}{\text{sen}\frac{\pi}{2n}} \,.\, \text{sen}\left(\frac{\frac{\pi}{n} + (n-1)\frac{\pi}{n}}{2}\right)\right)\right]$$

$$S = \lim_{n \to \infty}\left[\frac{\pi}{n}\left(\frac{\text{sen}\left(\frac{\pi}{2} - \frac{\pi}{2n}\right)}{\text{sen}\frac{\pi}{2n}} \,.\, \text{sen}\frac{\pi}{2}\right)\right]$$

$$S = \lim_{n \to \infty}\left[\frac{\pi}{n}\left(\frac{\cos\frac{\pi}{2n}}{\text{sen}\frac{\pi}{2n}} \,.\, 1\right)\right]$$

$$S = \lim_{n \to \infty}\left(\frac{\pi}{n}\cot\frac{\pi}{2n}\right) = \lim_{n \to \infty}\left(\frac{\frac{\pi}{n}}{\tan\frac{\pi}{2n}}\right)$$

Sea: $x = \dfrac{\pi}{2n} \Rightarrow 2x = \dfrac{\pi}{n}$

Si: $n \to \infty \Rightarrow x \to 0$

Entonces:

$$S = \lim_{x \to 0}\frac{2x}{\tan x} = \lim_{x \to 0}\frac{2}{\left(\frac{\tan x}{x}\right)}$$

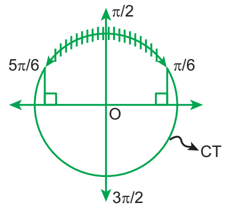
$$\Rightarrow S = \frac{\lim\limits_{x \to 0} 2}{\lim\limits_{x \to 0}\left(\frac{\tan x}{x}\right)} = \frac{2}{(1)} = 2$$

$\therefore S = 2$

Clave B

MARATÓN MATEMÁTICA (página 96)

1. Analizamos en la CT:



$\dfrac{\pi}{6} < x < \dfrac{5\pi}{6}$

$\dfrac{1}{2} < \text{sen}x \leq 1$

$\therefore\ \text{sen}x \in \left\langle \dfrac{1}{2}; 1\right]$

Clave B

2. Hallamos el dominio:

$-1 \leq 2x - 5 \leq 1$

$4 \leq 2x \leq 6$

$2 \leq x \leq 3$

Entonces: $\text{Dom}(f) = [2; 3]$

Clave E

3. $-\pi/4 < x < \pi/4$

$-1 < \tan x < 1$

$\arccos(-1) > \arccos(\tan x) > \arccos(1)$

$\pi > F(x) > 0$

$\text{Ran}(F) = \langle 0; \pi\rangle$

Clave C

4. $\dfrac{x}{3} = 2n\pi;\ n \in \mathbb{Z}$

$x = 6n\pi;\ n \in \mathbb{Z}$

Clave A

5. La ecuación general de la parábola es:

$(x-h)^2 = 4p(y-k)$

$x^2 - 2xh + h^2 = 4py - 4pk$

$x^2 - 4py - 2hx + h^2 + 4pk = 0$

Luego, tenemos:

$-4p = -3;\ -2h = -6$

$p = \dfrac{3}{4};\quad h = 3$

$3^2 + 4\left(\dfrac{3}{4}\right)k = -9$

$9 + 3k = -9$

$k = -6$

$V(h; k) = (3; -6)$

Clave E

6. $V = (3; 2) = (h; k)$

$y = -1 = k - p = 2 - p$

$h = 3;\ k = 2;\ p = 3$

$(x-h)^2 = 4p(y-k)$

$(x-3)^2 = 4(3)(y-2)$

$x^2 - 6x + 9 = 12y - 24$

$x^2 - 6x - 12y + 33 = 0$

Clave D

7. $d(V_1; V_2) = 9 - 1$

$2a = 8$

$a = 4$

Por otra parte, $C(h; k) \in L: y = x + 2$

$k = h + 2$

Luego, las coordenadas de los vértices y el centro son:

$V_1(1; h+2)$, $V_2(9; h+2)$ y $C(h; h+2)$.

Pero: $CV_2 = 9 - h$

$a = 9 - h$

$4 = 9 - h \Rightarrow h = 5$

Así, la ecuación de la elipse centrada en $C(5; 7)$ es de la forma:

$\dfrac{(x-5)^2}{16} + \dfrac{(y-7)^2}{b^2} = 1$

Como el punto $P(2; 6)$ está sobre la elipse, tenemos:

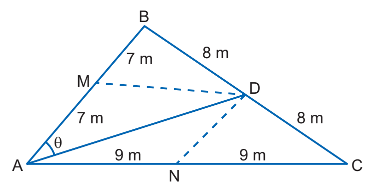
$\dfrac{(2-5)^2}{16} + \dfrac{(6-7)^2}{b^2} = 1$

$b^2 = \dfrac{16}{7}$

$\therefore\ E: \dfrac{(x-5)^2}{16} + \dfrac{(y-7)^2}{16/7} = 1$

Clave A

8. Piden $\cos\theta$:



Para $\triangle ABC$

$\cos A = \dfrac{14^2 + 18^2 - 16^2}{2(14)(18)}$

$\cos A = \dfrac{11}{21}$

Luego, tenemos:

$4(AD)^2 = 14^2 + 18^2 + 2(14)(18)\left(\dfrac{11}{21}\right)$

$AD = 14$

En $\triangle AMD$:

$\cos\theta = \dfrac{7^2 + (AD)^2 - 9^2}{2(7)(AD)}$

$\cos\theta = \dfrac{7^2 + (14)^2 - 9^2}{2(7)(14)}$

$\cos\theta = \dfrac{41}{49}$

Clave C